What should I look for with Mathematical Practices 7 & 8?

Agenda

Examine mathematical practices 7 and 8.
Examine our own thinking and name some of the ways we approach problems.
Examine student thinking.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
“Is the low number of observed instances for Mathematical Practice 7 because it isn’t being seen or because we aren’t sure what to look for?”
### Does this help elucidate the difference between the two?

<table>
<thead>
<tr>
<th>SMP7: Look for and make use of structure.</th>
<th>Look for patterns.</th>
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<tr>
<td>SMP 8: Look for and make use of repeated reasoning.</td>
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### Standards for Mathematical Practice

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### Is it good?
THINK

In what ways do we want students to think?

Develop patterns of thinking

“(Teachers) often find that when they begin to notice and name students' thinking and positive learning moves, their students begin to exhibit more of those behaviors.”

-Ron Ritchhart
Adding Multiples

Nina was finding multiples of 6. She said,

"18 and 42 are both multiples of 6, and when I add them, I also get a multiple of 6: 18 + 42 = 60."

Explain to Nina why adding two multiples of 6 will always result in another multiple of 6.

Open Array

6 x 3 + 6 x 7 = 6(3 + 7)
What is the smallest possible value?

What is the minimum value taken by the expression

\[(x - 4)^2 + 6\]

How does the structure of the expression help to see why?

Using structures in equations

\[\frac{9x - 5}{7} + 3 = 10\]

Using structures in equations

\[7 + 3 = 10\]
Using structures in equations

\[
\frac{9x - 5}{7} + 3 = 10
\]

\[
\frac{9x - 5}{7} = 7
\]

Using structures in equations

\[
\frac{9x - 5}{7} + 3 = 10
\]

\[
\frac{49}{7} = 7
\]

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Using structures in equations

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\]

\[
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\]

\[
54 - 5 = 49
\]

Using structures in equations

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\frac{9x - 5}{7} + 3 = 10
\]

\[
\frac{9x - 5}{7} = 7
\]

\[
9x - 5 = 49
\]

\[
9x = 54
\]

\[
54
\]
Using structures in equations

\[
\frac{9x - 5}{7} + 3 = 10
\]

\[
\frac{9x - 5}{7} = 7
\]

\[
9x - 5 = 49
\]

\[
9x = 54
\]

\[
x = 6
\]

NUMBER OF THE DAY

Build expressions

Write 12 as the sum of two terms.  
\[4 + 8\]

Take the second term and write it as the product of two factors.  
\[4 + 2 \times 4\]

Take the second factor in the second term and write it as the sum of two terms.

\[4 + 2(3 + 1)\]
Building expressions

Write 12 as the sum of two terms.

Take the second term and write it as the product of two factors.

Take the second factor in the second term and write it as the sum of two terms.

What if a student has arrived at $4 + 2 \times 3 + 1$?

How would you do…

![Image](310-299)

Recently…

![Image](310-299-310-299-310)

Procedure without understanding.
Use problems to elicit thinking and illuminate structure

99 + 97

Which has the greater area?

A: Side lengths of 13, 13, and 10.

B: Side lengths of 13, 13, and 24
Which triangle has the greater area?

A: Side lengths of 13, 13, and 10.
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A: Side lengths of 13, 13, and 10.
B: Side lengths of 13, 13, and 24

Using Heron's formula...

A: \( \sqrt{\frac{(18)(5)(5)(8)}{2}} \)
B: \( \sqrt{\frac{(25)(12)(12)(1)}{2}} \)

Which has the greater area?

A: Side lengths of 13, 13, and 10.
B: Side lengths of 13, 13, and 24

Using Heron's formula...

A: \( \sqrt{\frac{(18)(5)(5)(8)}{2}} \), \( \sqrt{\frac{(25)(12)(12)(1)}{2}} \)
B: \( \sqrt{\frac{(25)(12)(12)(1)}{2}} \)

SMP 8: Look for and make use of repeated reasoning.
Purposeful Number Strings

15 + 15
15 + 16
17 + 15
15 + 18

• What is the intent behind this number string?

Purposeful Number Strings

7 + 19
16 + 29
19 + 18
29 + 33

• What is the intent behind this number string?
Purposeful Strings

Solve each equation:

a) $5x = 2x + 21$

b) $5(x - 100) = 2(x - 100) + 21$

c) $5(x - 40) = 2(x - 40) + 21$

d) $5(x - 90) = 2(x - 90) + 21$

e) $5(x - 8) = 2(x - 8) + 21$

f) $5(x + 100) = 2(x + 100) + 21$

g) Find a pattern in describing the relationship between the calculations and the results.

What is the intent behind this problem string?

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Organization

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Write an expression...

Jose is ordering lunches for a party. The restaurant charges a $5 delivery fee. Each lunch costs $6.50. Write an expression that will determine the total amount Jose must pay for any number $n$ attending the party.
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How much will he pay if...

Jose is ordering lunches for a party. The restaurant charges a $5 delivery fee. Each lunch costs $6.50. If there will be 10 total people at the party, how much should Jose expect to pay?

What if 15 people were attending?

What if 20 people were attending?
How much will he pay if…

Jose is ordering lunches for a party. The restaurant charges a $5 delivery fee. Each lunch costs $6.50. If there will be 10 total people at the party, how much should Jose expect to pay?

What if 15 people were attending?

What if 20 people were attending?

How are you calculating the total cost?

Could you find total cost for any number of people? How would you do that?

Write an expression...

Jose is ordering lunches for a party. The restaurant charges a $5 delivery fee. Each lunch costs $6.50. If there will be 10 total people at the party, how much should Jose expect to pay?

What if 15 people were attending?

What if 20 people were attending?

How are you calculating the total cost?

Could you find total cost for any number of people? How would you do that? Can you write it as an algebraic expression?

Leverage numerical understanding of quantities to build algebraic structure.
Once there was a farmer who had an unfortunate accident. He was laid up in bed with a broken leg. To make matters worse, he had lost his memory.

As things settled down after the accident, he decided it was time to get back to taking care of his farm, even if it was from the confines of the master bedroom of his house. He called the feed store.

-How many bags of feed would you like, sir?
- I'm not too sure...
-Well, sir, what type of animals do you have?
In a panic, he asked his 6-year old daughter, who had been practicing her counting, what type of animals they raised.
- Chickens and Cows!!
- Well, how many of each?
- I counted 173 heads and 482 feet!!

He thought about it for a minute, then placed his order over the phone, feeling like he could still take care of his farm in spite of his injury.

How many cows and how many chickens did the farmer have on his farm?

Give me a wrong answer.
Check it.
Do you need more chickens or more cows?

Looking back on solved problem spaces.

Pile patterns and sequences...
Pile patterns and sequences...

4x + 2  6 + 4(x - 1)  4(x - 2) + 10

tables 1 2 3 4 5 ...

chairs 6 10 14 18 22 ...

Noticing Structure through Repeated Reasoning...

If we have 18 blocks and we need to put them in groups, how many go in each group?

[Diagram of 18 blocks]
If we have 18 blocks and we need to put them in groups, how many go in each group?
Noticing Structure through Repeated Reasoning...

That looks like what you’ve been doing in multiplication. Can you tell me how all of the numbers fit?
Gregory writes several problems involving the sums of odd numbers:

5 + 5 = 10
11 + 15 = 26
99 + 3 = 102
45 + 83 = 128
1 + 7 = 8
13 + 15 = 28
97 + 25 = 122
65 + 3 = 68

Gregory looks over his various examples and concludes that the sum of two odd numbers is always even.

Terrence considers what he knows about odd numbers and writes the following:

If I have two odd numbers, a and b, that means that each is always one more than an even number (since they are odd).

\[ a = 2n + 1 \]
\[ b = 2m + 1 \]

\[ a + b = 2n + 1 + 2m + 1 \]
\[ 2n + 2m + 1 + 1 = 2n + 2m + 2 = 2(n + m + 1) \]

Since \( a + b \) is equal to \( 2(n + m + 1) \), the sum of a and b also has a factor of two.

Terrence concludes that the sum of two odd numbers is always even.

Felipe was modeling with cubes and noticed that every time he added two odds, no matter what, the "extra" piece from one matched up with the "extra" piece from the other.

Felipe concludes that the sum of two odd numbers is always even.

How do you approach this problem?

Which has the greater shaded area?
Follow up question…

Is that true for any size array of circles?

What about…
Is this true for spheres in cubes?

Reframing Questions to Elicit Student Thinking

Reframing Questions

<table>
<thead>
<tr>
<th>Original</th>
<th>Is a square a trapezoid?</th>
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<tbody>
<tr>
<td>Reframed</td>
<td>Why is a square a trapezoid?</td>
</tr>
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</table>
**Reframing Questions**

**Original**
What is a prime number?

**Reframed**
Why is 17 a prime number and 15 not?

**Sources**

Other resources:
- Math Forum
- Illustrative Mathematics
- Brilliant.org
- Alcumus from Art of Problem Solving
- EDC

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Send your text message to this Phone Number: 37607

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Thank You

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