Outcomes

• Develop an array of well orchestrated instructional strategies to support all students as they increase their fluency with addition and subtraction facts through

• Gain an understanding of the phases of fluency and how to assess it.

• Learn how students develop understanding and fluency through activities and games to use in your classroom throughout the year.
What is fluency?

In what ways do you develop and assess student fluency?

What is fluency in mathematics?

“According to CCSSM, fluency is “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO 2010, p. 6). Thus, far from just being a measure of speed, fluency with multiplication facts involves flexibly and accurately using an appropriate strategy to find the answer efficiently.”

- Three Steps to Mastering Multiplication Facts by Gina Kling and Jennifer Bay-Williams
What is fluency in mathematics?

Procedural fluency is a critical component of mathematical proficiency. Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop procedural fluency, students need experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through distributed practice.

- NCTM Position, July 2014

Mastering Basic Facts

Phase 1: Counting
(Counts or counts on with objects or mentally)

Phase 2: Deriving
(Reasoning strategies based on known facts)

Phase 3: Mastery
(Efficient production of answers)

Adapted from Baroody, 2006
"Enriching Addition and Subtraction Fact Mastery Through Games" by Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics Volume 21, No. 4, November 2014

CCSSM Standards for Fluency

K
K.OA.5 Add/subtract within 5

1st
1.OA.6 Add/subtract within 10

2nd
2.OA.2 Single-digit sums and differences (sums by memory by end of grade)
2.NBT.5 Add/subtract within 100

3rd
3.NBT.2 Add/subtract within 1000

4th
4.NBT.4 Add/subtract within 1,000,000

3rd
3.OA.7 Single-digit products and quotients (products by memory by end of grade)

5th
5.NBT.5 Multi-digit multiplication

6th
6.NS.2, 3 Multi-digit division
Multi-digit decimal operations
Two Ways to Learn Basic Facts

- Strategies
- Memorization

Research

“...research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.”

— Boaler, Page 3

Research Findings

Study of students learning math facts in two ways – through strategies or memorization.

“Importantly the study...found that those who learned through strategies achieved ‘superior performance’ over those who memorized, they solved problems at the same speed, and showed better transfer to new problems.” (Delazer et al, 2005)
Research Findings

Data from 13 million 15-year olds on the International PISA mathematics test.

“...the lowest achieving students are those who focus on memorization and who believe that memorizing is important when studying mathematics. The highest achievers in the world are those who focus on big ideas in mathematics and connections between ideas.

From “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts” by Jo Boaler, 2015, page 5
Ways to Develop Fluency

<table>
<thead>
<tr>
<th>Number Talks</th>
<th>Games</th>
<th>Meaningful Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quick Images/Dot Cards</td>
<td>Targeted Practice (focus on a particular group of facts)</td>
<td>Story Problems</td>
</tr>
<tr>
<td>Rekenreks</td>
<td>General Practice (all facts for a particular operation)</td>
<td>Contextual problems</td>
</tr>
<tr>
<td>Computational problems</td>
<td>Computer games/Apps</td>
<td>Ten Frames/Arrays</td>
</tr>
<tr>
<td>Number strings (series of related computational problems)</td>
<td></td>
<td>Visual representations</td>
</tr>
</tbody>
</table>

“Enriching Addition and Subtraction Fact Mastery Through Games” by Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics Volume 21, No. 4, November 2014

Number Talks

- Intentional focus on specific strategies
- Develop connections between strategies
- Compare strategies to deepen understanding and increase efficiency
- Opportunities for students to hear about and utilize new strategies

How might you use this information to strategically plan your Number Talks?
Math Talks & Routines

Other Math Talks

Number Talk

Routines

Counting Routines

• **Choral Counting** – Students count a number sequence aloud together.
• **Count Around the Clock** – Choose a counting sequence. One at a time, students count around the circle.
• **Start and Stop Counting** – Similar to Choral Counting with a “Start” and “Stop” number. Students may be able to identify the difference between the numbers.

Adapted from *Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3*, Jessica F. Shumway, pages 24-25
Context

• Can you create a math story for this picture?
• How many apples?
• How can you see $1 + 3 = 4$ in this picture?
• How can you see $4 - 1 = 3$ in this picture?

Quick Images

Elham Kazemi and Allison Hintz, Intentional Talk
T - F Equations

Decide if the equations are true or false. Explain your answer:

a. $2 + 5 = 6$

b. $3 + 4 = 2 + 5$

c. $8 = 4 + 4$

d. $3 + 4 + 2 = 4 + 5$

e. $5 + 3 = 8 + 1$

f. $1 + 2 = 12$

g. $12 = 10 + 2$

h. $3 + 2 = 2 + 3$

i. $32 = 23$

Which one doesn’t belong?

4  8  9  12
Fluency Games

Why use games?
• Are engaging.
• Provide opportunities for strategy discussion and assessment.
• Should be sequenced developmentally.
• Can be targeted practice or general practice.
• Lend to differentiation.

Developing and Assessing Fact Fluency, Amanda Ruch and Gina Kling, NCTM 2015

Which card is greater?

1. Each player shuffles their cards.
2. Each player turns over the top card from their deck.
3. The player with the most dots on it wins the round and gets the cards. If there is a tie, players keep their own cards.
4. Play continues until there are no remaining cards in the stack.
5. Players count the total number of dots on the cards (or cards) they have at the end of the game, and the player with the largest number wins.

Variation: Players could compare and the player with the least amount of buttons wins the round.

Card options: 10 frames (without number), 10 frames (with numbers), Deck of cards (face cards removed), Digit Cards

Building Conceptual Understanding and Fluency Through Games, for the CCSSM, Grade K
North Carolina Department of Public Instruction, http://www.ncpublicschools.org/curriculum/mathematics/
Snap It

• All students start with a given number of linker cubes in a train.
• On the signal “Snap,” children break their trains into two parts and hold one hand behind their back.
• Children take turns going around the circle showing their remaining cubes. The other children work out the full number combination.

High Roller

• Students take turns rolling two dice.
• During each turn, a student rolls both dice.
• Then the students selects the die with the greater number rolled and puts it to the side.
• The student rerolls the second die.
• The student finds the sum of the two dice counting on from the first number to find the total.
• The student records the total and the other student begin his/her turn.

“Enriching Addition and Subtraction Fact Mastery Through Games” by Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics Volume 21, No. 4, November 2014
Race to The Top
(Roll and Total)

• A student rolls the number die and then rolls the dot die.
• The students adds the two by starting with the numeral die and counting on for the number rolled using the dot die.
• The student records the total (sum) in the column.
• Play continues until one sum column reaches the top.

Double It

• Students take turns selecting a number card or rolling a die 1 – 10.
• The student doubles the number selected/rolled.
• The student records the total (sum) of the double in the correct column.
• Play continues until one sum column reaches the top.
**Nutty Buddies**

1. Each player places all of their game markers on any number on their gameboard. There may be more than one marker on a number.
2. Each player takes a turn rolling the dice and finding the sum.
3. The player may remove one cube from the sum that was rolled.
4. If there is not a marker to take off the gameboard, the player loses the turn.
5. The player that clears their gameboard first is the winner.

Variation/Extension: Players can roll the dice and subtract the sum from 14.

---

**Salute**

- Digit Cards 0 – 9, with ten-frames
- 3 students: 1 leader, 2 players
- The leader hands each player a card.
- The leader says “Salute!”
- The players put their cards on their foreheads.
- The leader says the sum/total of the two cards.
- The players work to determine the number on their forehead. Once both players have done so, they look at their cards and then students rotate roles.

“Enriching Addition and Subtraction Fact Mastery Through Games” by Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics Volume 21, No. 4, November 2014
Greg Tang Math Games

Phase 1 Games
- Ten Frame Mania

Phase 2 Games
- Breakapart

Phase 3 Games
- Kakooma
- Missing

http://gregtangmath.com/games.html

How do we know what they’re thinking?

https://flic.kr/p/djaFxF
Mastering Basic Facts

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Adapted from Baroody, 2006
“Enriching Addition and Subtraction Fact Mastery Through Games” by Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics Volume 21, No. 4, November 2014

Discuss

• How will you support and develop fluency in the classroom?
• What ideas or activities are you most excited to try with your students? Why?
Assessing Fluency

What can we learn from this assessment in regards to student flexibility, accuracy, efficiency, and appropriate strategy use?

Too often we hear that the kids don’t know their basic facts . . . Instead ask: Which kids? And Which facts?

In order to know which kids and which facts, we need to assess and monitor.
Assessing All Aspects of Fluency

Procedural fluency is “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO 2010, p. 6).

What do timed tests assess?

The student can add and subtract:

- **Flexibly**
- **Accurately**
- **Efficiently**
- ** Appropriately**

“Enriching Addition and Subtraction Fact Mastery Through Games” by Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics Volume 21, No. 4, November 2014
Assessing Fluency

- Observation
- Interviews
- Writing prompts
- Strategy quizzes
- Self-assessment
Observation Logs

• Use your observation log as you monitor students during fluency games, practice, and math tasks.
• Mark the facts that students know indicating which level they are at.
• Make notes about what students say they are thinking/doing.
• Use your log to decide which games to play, facts to practice, and small groups to work with.

Interviews

Focus on fluency:
1. Write 4 + 3 on a card. (point at card) What does 4 + 3 mean?
2. What is your solution to 4 + 3?
3. How did you find your solution? Can you find it another way?
4. If your friend was having trouble solving this problem, what strategy might you suggest to him/her?
### Interviews

<table>
<thead>
<tr>
<th>Flexibility</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve 6 + 7 using one strategy. Now try solving it using a different strategy.</td>
<td>What is the answer to 7 + 8? How do you know it is correct? How might you check it?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>Appropriate Strategy Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which facts did you just know (know in your mind)? Which facts did you use a strategy? Which strategy did you use and why?</td>
<td>Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not?</td>
</tr>
</tbody>
</table>

---

**Strategy Quizzes**

- Students solve problems and indicate how they solved them.
- Add the information to your observation log.
- Connect to student self assessment.

---

(a) Quiz questions can be used to see if students “just know” foundational facts. Solve these problems and tell how you solved them.

<table>
<thead>
<tr>
<th>4 x 5 =</th>
<th>Check one:</th>
<th>I used this strategy:</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 x 6 =</td>
<td>Check one:</td>
<td>I just knew:</td>
</tr>
<tr>
<td>6 x 2 =</td>
<td>Check one:</td>
<td>I used this strategy:</td>
</tr>
<tr>
<td>5 x 3 =</td>
<td>Check one:</td>
<td>I just knew:</td>
</tr>
<tr>
<td>2 x 9 =</td>
<td>Check one:</td>
<td>I used this strategy:</td>
</tr>
<tr>
<td>3 x 10 =</td>
<td>Check one:</td>
<td>I just knew:</td>
</tr>
<tr>
<td>5 x 7 =</td>
<td>Check one:</td>
<td>I used this strategy:</td>
</tr>
<tr>
<td>8 x 10 =</td>
<td>Check one:</td>
<td>I just knew:</td>
</tr>
</tbody>
</table>

(b) A quiz assesses if students recognize the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies.

On completion, say to class, “Circle the row that was easiest for you to solve. If they were both the same, write ‘same’.”

Solve these addition problems.

<table>
<thead>
<tr>
<th>ROW A</th>
<th>9 + 1 =</th>
<th>5 + 1 =</th>
<th>3 + 1 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Rowe B</td>
<td>1 + 8 =</td>
<td>1 + 7 =</td>
<td>1 + 9 =</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

---

Fact Assessments

• Use Traditional Fact Assessments as strategy quizzes.
• For example,
  o Solve all the facts you know in your mind. Skip the others. 😊
  o Solve only the facts you need to work out.
  o Solve only the doubles facts/near doubles facts.
  o Solve facts where you can make a ten to solve.

Discuss

• How will you support and develop fluency in the classroom?
• What ideas or activities are you most excited to try with your students? Why?
Resources

• Developing and Assessing Fact Fluency, Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, NCTM 2015
• Kling, Gina and Jennifer M. Bay-Williams. 2014. Assessing Basic Fact Fluency. Teaching Children Mathematics, Volume 20, Number 8, 488-497.
• Building Conceptual Understanding and Fluency Through Games, for the CCSSM, North Carolina Department of Public Instruction, http://maccss.ncdpi.wikispaces.net/Elementary

Thank you!

Enjoy the rest of your conference.

Christine Roberts

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🐦 @tcoechristine
Your feedback is appreciated

Send your text message to: 37607

Type: 13211 (0-3)(0-3)(0-3)_other comments or feedback in words

Engaging and effective
Well prepared and knowledgeable
Session matched description

Strongly Disagree (0) Disagree (1) Agree (2) Strongly Agree (3)