<table>
<thead>
<tr>
<th>Mathematics Content</th>
<th>Mathematical Practices</th>
<th>ELD Standards</th>
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**Language & Learning Objective:**

Consider the opportunities and structures for students to read, write, listen, and speak about mathematics throughout your lesson. Indicate these (r, w, l, s) in your plan.

**Launch**

**Explore**

**Summarize**
Lesson Title: 4.3 Interpret the Remainder
Chapter/Unit: Chapter 4 Divide by 1 – Digit Numbers

<table>
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<tr>
<td>4.OA.3 Solve word problems and make sense of the remainders in context of the situation.</td>
<td>SMP 1 – Make Sense of Problems and Persevere in Solving Them – Solution pathways, reasonableness</td>
<td>ELD.PI.4.11a Support opinions (thoughts) by expressing reasons w/evidence. (Productive)</td>
</tr>
<tr>
<td></td>
<td>SMP 2 – Reason Abstractly and Quantitatively - Contextualize and decontextualize</td>
<td>ELD.PI.4.6 Combine clauses to make connections between and join ideas in sentences. Such as, but, so (for, and, nor, but, or, yet, so).</td>
</tr>
<tr>
<td></td>
<td>SMP 3 – Construct Viable Arguments and Critique the Reasoning of Others – Create arguments and support with reasons.</td>
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</tr>
</tbody>
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Language & Learning Objective:

Students will make sense of remainders in context, orally supporting their thinking with reasons and evidence from their work and from the text (word problems).

Launch

p. 152 Read & flip w/van problem
• 1st Read – Choral, Retell
• 2nd Read – T. fluency read, Ss mark the text, Create answer statements
• 3rd Read – Independently read, discuss 1st step & why
• Solve

Group Discussion Sentence Frames:
• The answer is __________, so they will need __________ because __________.
• I got________, so ______________.
• My solution is __________, but ______.

Explore

p. 153 Work with a partner to complete #’s 4-5. After each problem, explain your solution, what the remainder means in context, and your reasoning.

Summarize

p. 154 Complete #11 on your own T. circulates and selects student to share. 2-3 students share their thinking and use a sentence frame as needed to make sense of the remainder in the problem. *Continue to use coordinating conjunction sentence frames.
### Day 1 Build and Compare

**Math Standards:**
3.NF.3a-3d, SMP 3, 5

**ELD Standards:**
ELD.PI.3.11, ELD.PII.3.4

**Language Objective:** Students will orally compare two fractions and provide justification for their thinking.

**Launch** - Chorally Count by $\frac{1}{2}$ to $6/2$, list, place on # line

**Explore** - Build & Compare Partner Activity
- S. build each fraction using a fraction tool such as fraction tiles/strips, circles, etc (R).
- S. describe the fraction card they selected and how to create it with their tool (D).

**Summarize** - Have students orally describe the comparison of their last round with their partner (D).
- Partners will rehearse these oral comparisons with one another.
- S. write their fraction comparison in words and as an inequality in their math journal.
- T. will listen to the partners and select 2-3 pairs to share.

### Day 2 Sketch and Compare

**Math Standards:**
3.NF.3a-3d, SMP 3

**ELD Standards:**
ELD.PI.3.3, ELD.PII.3.4

**Language Objective:** Students will orally & in writing compare fractions to determine whether they agree or disagree with a statement, providing justification for their thinking.

**Launch** - Chorally Count by $\frac{1}{8}$ to $12/8$, list, place on # line

**Explore** - Sketch & Compare Partner Activity
- S. sketch each fraction using rectangles (R).
- Students describe the fraction card they drew and shaded their fraction (D).
- Students write each inequality statement and justify the comparison (W).

**Summarize** - Students will meet up with a new partner to share written descriptions.
- S. A will read their inequality and description.
- S. B will listen, and say "I agree w/what you said about ___ because ___" or "I disagree w/ your statement that ___ because ___.
- Switch roles and repeat.

### Day 3 Comparing Fractions

**Game** - Chorally Count by $\frac{1}{4}$ to $10/4$, list, place on # line

**Explore** - Comparing Fractions Game (6 rounds)
- S. turn over a fraction card (R).
- Each student places their inequality symbol (R).
- S. discuss whether they agree or disagree with the symbol w/justification (D).

**Summarize** - Have students complete the following question.
- S. will share their explanation with a partner.
- T. selects 3 groups to share (T. intentionally selects groups to share one of their comparisons so that all three strategies will be shared) (D).
- 3 Strategies: Using common numerators, Using common denominators, Comparing to a benchmark fraction
- T. creates a chart summarizing the three strategies after each pair shares.

### Day 4 Comparing Fractions

**Game** - Chorally Count by $\frac{1}{3}$ to $11/3$, list, place on # line

**Explore** - Comparing Fractions Game (4 rounds) (R, D)

**Summarize** - T. facilitates a conversation to help students formalize the strategies that they are using to compare fractions.
- S. will explain why they placed the fraction card in the strategy they chose (D).}

**Summarize** - Teacher facilitates a conversation to help students formalize the strategies that they are using to compare fractions.
- S. are asked to describe each of the three strategies (D).
- S. describe how using common numerators and common denominators to compare fractions are alike and different (D).
Background
Mrs. Verners’ 30 fourth graders have been learning about place value during the first few weeks since school began. They are currently toward the end of their place value unit. Students have been engaged in lessons and math routines focused on their grade level standards for Number and Operations in Base Ten that are focused on place value. This will be one of their first experiences with a larger task focused on the same concepts. Students will work independently and collaboratively with their table groups during the task.

The students at the school are predominately hispanic and over half of the students are English Learners. Almost 90% of the students receive free and reduced lunch. Mrs. Verners has 11 ELs with 4 at the Emerging level, 5 at the Expanding level, and 2 at the Bridging level. Students with disabilities are included in all mathematics instruction. The fourth grade team of teachers at this school meet weekly to discuss and plan their math lessons, discussing instructional strategies and resources that they are using.

Lesson Context
During the place value unit, students have explored place value through daily math lessons and routines. Students are able to identify the place value of given digits, and can write numbers in standard, word, and expanded form. Students compare numbers using their understanding of place value and inequality symbols. They have had some experiences describing these comparisons orally and through writing. Mrs. Verners is working to develop student understanding of how the places within the place value system are related through multiplying and dividing by ten. Students have analyzed the relationship between the value of a digit in two locations within a number. For instance, they understand that in the number 5,500, the 5 in the thousands place is ten times greater than the 5 in the hundreds place. In this task, they will explore the relationship between values of a common digit in as they compare several different numbers.

Lesson Excerpts
Mrs. Verner's lesson provides students the opportunity to apply what they have learned about the relationships within the base ten place value system and comparing numbers within the context of a real-world situation. Students will engage independently and collaboratively with their small group to deepen their understanding of the relationship between the value of a digit located in different places within numbers. Students will have developed a foundation for their work on this task through previous class lessons focused on place value concepts. Mrs. Verners and her grade level team had identified during their collaborative planning that students would need an opportunity to develop background knowledge regarding the places described within the task before beginning the math portion. They decided to add a map and introductory activity during Social Studies to discuss and identify the location within the task on the map. The learning target and clusters of CA CCLS for Mathematics and CS ELD Standards in focus for today’s lesson are the following:

**Learning Target:** The students will organize 4th grade population data for different locations across the United States in order to compare and describe the relationships between the values of digits within the number.

**CCSS for Mathematics:**
4.NBT.1 - Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700/70 = 10 by applying concepts of place value and division; 4.NBT.2 - Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers
based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons; SMP 1 - Make sense of problems and persevere in solving them; SMP 7 - Look for and make use of structure.

CA ELD Standards (Expanding):  ELD.PI.4.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics ELD.PI.4.10 - Writing literary and informational texts to present, describe, and explain ideas and information.

Task
There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States. We write 4 million as 4,000,000. There are about 4 thousand fourth graders in Washington, D.C.

Use the approximate populations given to solve.

a) How many times more fourth graders are there in Texas than in Mississippi?
b) How many times more fourth graders are there in the United States than in Texas?
c) How many times more fourth graders are there in the United States than in Washington, D.C.?

Source: Adapted from “Thousands and Millions of Fourth Graders,” Illustrative Mathematics, https://www.illustrativemathematics.org/content-standards/tasks/1808

Day 1
During social studies, Mrs. Verners introduces the math task to her students saying that tomorrow they will be exploring populations in different locations in the United State. She gives students the task handout with a map of the United States on the top. She begins the conversation with her class by asking students what state they live in. She refers to a copy of the map under the document camera to serve as a visual. Students discuss with their small groups and share their ideas with the whole class. She asks students to shade California yellow. Next, she asks them to discuss what city they live in and where they think it is located in California. Mrs. Verners models how to place a dot to represent their city in its approximate location in California. The teacher points to the section labeled “key” on their handout. Mrs. Verners states that key is a multiple meaning word and asks students if they know of another way this word is used. Students respond that keys are used to unlock things. Mrs. Verners makes a connection between a key, like a house key, and the key on their map, which is used to help you understand the symbols and colors used on the map. The conversation continues as she helps students to identify the United States, Texas, Mississippi, and Washington D.C. on the map and represent them on the key. Mrs. Verners tells her students that they will use this map tomorrow during math as they explore populations of 4th graders in the different locations they identified.

Day 2
The next day, Mrs. Verners launches the math lesson by revisiting the map and telling students that they will be talking about approximate populations of 4th graders in these different locations. She asks the students to write other words that mean the same thing as estimate or approximate on their whiteboards. After showing their whiteboards, Mrs. Verners asks students to share with their partners the words they wrote down. She lists several of the words that she saw students write on the whiteboard for the class to see. Mrs. Verners says that these words (pointing to her list on the whiteboard) are synonyms that mean about or close to. She explains that when we use numbers that are not exact, we sometimes use the words almost or about to say that these numbers are estimates or approximations. She says that the word approximate in English is approximado in Spanish and that these words mean the same thing.

Next, she asks the students to estimate the number of fourth grade students at their school. Students make individual estimates and records them on their whiteboards. Students share their
Mrs. Verners tells the students that they will be looking at the population of fourth grade students in the different locations, the places they identified on their maps. She tells the students that she is going to read the task aloud and wants the students to listen carefully and point to each location on the map when she reads it in the task. Students are asked to reread the task silently, underlining or circling important ideas in the task to help them make sense of what they are reading. Students take turns sharing something that they underlined or circled with their small group.

Next, students are asked to individually complete the data table by writing the 4th grade population of each location in standard form in order to organize the population data that they were given in the task. Mrs. Verners explains that table is a multiple meaning word. She explains that there are different types of tables. In math, tables are used to record information and organize data. She shows students the t-table on their task handout and says that this is an example of a table that we use in math. After asking her students to begin working independently, Mrs. Verners asks for several of her students to meet her at her small group table. Here, she works with her Emerging ELs to collaboratively complete the t-table. She facilitates the conversation using the following types of questions:

- Where can you find the population of each location in the text? How is the population written?
- How can we rewrite the populations from word form to standard form?
- What are the digits in this number? What digits do we use in our base ten number system?
- What do you notice about the location of the digit 4 in the numbers in your table?
- What does the location of the digit 4 tell you about its value?

After working together to discuss and create their data tables, the teacher excuses her small group to return to their groups. Mrs. Verners brings the class back together and describes how they will work with their small group during the next portion of the task to answer several questions.
comparing the population of fourth graders in the different locations and explaining these comparisons in writing. She shows the class two sentence frames that she has written on the board, reads them to the class, and tells them that they may use these frames as they are writing or they may create sentences on their own. Her sentence frames are:

- The number of 4th grades in ______ is _____ times as many as the 4th graders as in ______.
- There are ____ times as many 4th graders in ______ than ______.

Students are asked to complete a and b collaboratively with their group, saving c to complete on their own so that Mrs. Verners can use this information to check the level of student understanding.

The teacher circulates as students are working in small groups and ask questions to support and extend student thinking. She poses the following types of questions:

- What do you notice about the numbers/populations listed in your table?
- Do you see a relationship between these numbers?
- Do you notice a pattern in the place value of the digit 4?
- What tools might help you as you're trying to represent the place value of the 4 in each of these numbers? (base ten blocks, place value chart, etc.)
- How would you describe the relationship between the digit 4 in these numbers?
- You noticed that each place value is x 10 from the place before it. How might you group those to find the relationship between 4,000 and 4,000,000?

Mrs. Verners selects 2 - 3 groups that will share their explanation from question a. Within each group, she selects one student to represent the group and present to the whole class. She considers students that have recently presented and intentionally selects students who have not had an opportunity to present their thinking to the whole class recently. She also considers the ability levels of her students in her continued efforts to support their class norm that all students have good math ideas and selects students that represent a range of ability levels. Mrs. Verners asks the students who have been selected to practice what they will say to their table groups before presenting in front of the whole class. After the students share their group’s explanation, Mrs. Verners asks questions to deepen student understanding and make connections between the different explanations that were presented. Next, she asks all students to reread their explanations in part a and provides them time to add on to their explanation to make it stronger or to revise their thinking. Students are given time to add and/or revise.

Mrs. Verners asks the students to think about the explanations they have heard and practiced with their partner. She asks them to use what they have learned from their work on parts a and b of the task to complete part c independently. She tells the students that she is interested in looking at their work and reading their writing in part c so that she can learn about what students understand about comparing numbers. Students write their explanations independently.

**Teacher Reflection and Next Steps**

Mrs. Verners collects the student work and reviews their independent work and explanation from part c. As she reads, she analyzes whether or not students were able to generalize their place value understanding to describe the relationship between the digit 4 in the population of fourth graders in Washington D.C. and the United States. Students have had experience describing the relationship between a digit in a given place value and the place to its right or left; however, this question asks them to describe the relationship of a digit three places to the left. As Mrs. Verners analyzes the student work, she discovers that while the majority of her students understand and are able to describe these place value relationships, a small group of students are struggling to express their thoughts in writing. This small group contains 2 Emerging ELs, 1 Expanding EL, 1 student with a disability, and 2 other students that she has noticed are struggling with place value concepts. She decides that she will work with these students in small groups the following day to determine if they are having trouble with the concept or if they are having difficulty using writing to explain their thinking. Mrs. Verners sees that students were able to deepen their understanding of place value relationships
through the use of this task and decides that she would like to give the students the opportunity to engage in another task to further develop these concepts before the end of the place value unit.

**Sources**
Task: “Thousands and Millions of Fourth Graders,” Illustrative Mathematics, [https://www.illustrativemathematics.org/content-standards/tasks/1808](https://www.illustrativemathematics.org/content-standards/tasks/1808)

Resources created by Dinuba Unified School District 4th Grade Teachers for “Thousands and Millions of Fourth Graders”

**Resources**

**Companion Documents**
DUSD Launch - Explore - Summarize Lesson Plan for “Thousands and Millions of Fourth Graders” created by Dinuba Unified School District 4th Grade Teachers
DUSD Student Handout for “Thousands and Millions of Fourth Graders” created by Dinuba Unified School District 4th Grade Teachers

**Additional Information**
This Integrated ELD and Mathematics Instruction Vignette was created by the Tulare County Office of Education under the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License, [http://creativecommons.org/licenses/by-nc-sa/4.0/deed.en_US](http://creativecommons.org/licenses/by-nc-sa/4.0/deed.en_US).
Math Task Planning Template: Thousands and Millions of 4th Graders

Grade Level: 4th

Task Name: Thousands and Millions of 4th Graders
Unit 1: Place Value and Operations

Source: Illustrative Mathematics, https://www.illustrativemathematics.org/content-standards/tasks/1808
Lesson Created by: Dinuba Unified School District 4th Grade Teachers

Part One: Goals and Objectives

Mathematical Goals
• What are the big mathematical ideas of this task?
• What do I want students to know and be able to do when this task is completed?
• Identify place value
• Understand that a digit to the left is ten times greater than the same digit on its right (For example, in the number 5,500, the 5 in the thousands place is ten times greater than the 5 in the hundreds place.)
• Create a table to organize information/data
• Write numbers from word form to standard form
• Compare multi-digit numbers (context: populations of 4th grade students)
• Describe the relationship between a digit in different locations when comparing numbers

Content and Practice Standards
• Which grade level content standards are being addressed?
• Which Standards for Mathematical Practice will you foster during the task? How will students become more proficient with this Standard for Mathematical Practice?

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700/70 = 10 by applying concepts of place value and division.

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

SMP 1 Make sense of problems and persevere in solving them – Students will make sense of the data they are given in the task to write the population of 4th graders in standard form. They will compare and describe the relationships using their knowledge of place value.

SMP 7 Look for and make use of structure – Students understand the structure of our base ten place value system to explain the relationship between digits in different locations.

Also, SMP 6 Attend to Precision – Students will use precise language to describe the relationship between digits and as they choose their labels for the data table.

### Part Two: Task Delivery

**Launch** (5–10 minutes)
- How will I present this task to the students?
- What prior knowledge do my students need?

Teacher introduces the task by saying that today we are going to explore populations of 4th grade students across the United States.

Students receive the task handout and teacher guides the geography portion of the lesson.
- First, we are going to outline the United States in blue.
- Next, let’s shade California yellow.
- Mississippi – shade orange
- Texas – shade green
- Washington D.C. – draw a star (see the small circle on the east coast)
- Dinuba – draw a black dot
- Predict how many 4th graders are in our school, in our school district, and in California.
- Discuss the predictions and how they came up with them.

Now, we are going to explore the populations of the locations, the places, we identified on our map.

Optional:
- You may want to complete the Launch the day prior during the afternoon to finish the geography connection ahead of time. This will give you more time to focus on the mathematics the next day.
- Make a map key.
- Show the locations on Google Earth.

**Explore** (33–40 minutes)
- How will I organize the students to explore this task? (Individuals? Groups? Pairs?)
- How will students be actively engaged in this part of the lesson?
- What are the different strategies I anticipate students using?
- What kinds of questions can I ask? (Assessing questions – scaffold instruction for students who are stuck, Advancing questions – further learning for students who are ready to deepen their understanding)

**Whole class (3-5 minutes):**
- Teacher introduces the task by having students read the task silently.
- After reading, have the groups complete a RoundRobin where each student will share one fact about what they read.
- Next, teacher reads the task aloud and students annotate however they would like.
- Groups discuss what they annotated and why.
- Teacher asks 2-3 groups to share something they underlined/circled/etc. and why.

**Individually (5 minutes):**
- Teacher talk: Now create a table to organize the population data from our task. Remember to label the parts of the table.
- Students create a table to organize the data. Teacher circulates asking questions as
needed.

• Students may begin reading and working on questions a, b, and c if they finish early.

Table Group (25-30 minutes):

• Teacher brings the class back together and describes how they will work with their table groups on the next portion of the task.
• Groups share their data tables using a RoundRobin. If a student created their table in the same way as another student, they may simply say “I did it the same way as ______ because I was thinking ______.”
• Teacher talk:
  o After sharing your tables with your group, create a group data table on your chart/construction paper. Decide what you think is the best way to organize the data table and be prepared to explain why.
  o Work with your group to complete questions a, b, and c. Show your work on your own paper and discuss your answers as a group.
  o Once you have finished answering the questions, work with your group to complete your task chart/construction paper. Have a different person write the responses to questions a, b, and c (Pass the Pen: 1 – Table, 2 – Question A, 3 – Question B, 4 – Question – C). Make sure that everyone in your group is prepared to share.

Possible teacher questions:

• How did reorganizing your table as a group help you to see the relationship between these numbers?
• Do you notice a pattern in the place value of the digit 4?
• What tools might help you as you’re trying to represent the place value of the 4 in each of these numbers? (WMP? WMV?, Place Value chart)
• How would you describe the relationship between the digit 4 in these numbers?
• You noticed that each place value is x 10 from the place before it. How might you group those to find the relationship between 4,000 and 4,000,000?

Summarize (10–22 minutes)

• How will student questions and reflections be elicited during the summary of the lessons?
• How will I focus the conversation and student sharing on the big mathematical ideas and connections that can be made within the task?
• How will student understanding of the mathematical goals be determined?

• Teacher talk: Now we are going to discuss what we learned from today’s task.
  o Teacher selects 2 – 3 students to share their group's written explanation to question a. Note: When selecting student work for this task, find 2 – 3 group responses that are all correct, but explain the idea differently.
  o After students share, discuss how these ideas are similar and what they tell us about the meaning of the digits.
  o Now look at question b and think about how you would explain the relationship between the digit 4 in these two numbers.
  o Pair Share your explanation with your partner.
  o To end today’s lesson, we’re going to focus on part c. Please take the last few minutes to write an explanation of this relationship independently so I can see what you have learned.

**Part Three: Reflection**

- What strategies did students use?
- What questions did students ask?
- What questions did I pose that received the most interesting or varied responses?
- What do students understand and not understand based on evidence from the lesson?

**Part Four: Task Scoring**

**Scoring Guidelines**

- How will each section of the task be scored and why will it be scored this way?
- What does a full credit response for each section look like?

Not scored individually, teacher uses this task to formatively assess student understanding of place value concepts. Teacher will collect the student handout to read the final explanation to assess individual student understanding.

Can also be scored for effort and group participation.

**Students will be able to:**

- Identify place value
- Understand that a digit to the left is ten times greater than the same digit on its right
- Create a table to organize information/data
- Write numbers from word form to standard form
- Compare multi-digit numbers
- Describe the relationship between a digit in different locations when comparing numbers

**Analyzing Student Work**
- What elements of the task did students understand? What elements of the task did they not understand?
- How will I respond to the students that did not understand the task?
- How will I respond to the students that did understand the task?

**Part Five: Planning for Next Steps**
- What are my next steps as a teacher and why? What evidence supports these next steps?
- When will these next steps occur?
- Who will the next steps include?
1. There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States. We write 4 million as 4,000,000. There are about 4 thousand fourth graders in Washington, D.C.

Complete the t-table to organize the data for the population of 4th graders in different locations. Write each population in standard form.

<table>
<thead>
<tr>
<th>Location</th>
<th>4th grade Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mississippi</td>
<td></td>
</tr>
<tr>
<td>Texas</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td></td>
</tr>
<tr>
<td>Washington D.C.</td>
<td></td>
</tr>
</tbody>
</table>

Note: Do not copy back to back. Students need to be able to use the table as they answer the questions.
Use the population t-table to complete parts a, b, and c.

a) How many times more fourth graders are there in Texas than in Mississippi? Explain your thinking.

b) How many times more fourth graders are there in the United States than in Texas? Explain your thinking.

c) How many times more fourth graders are there in the United States than in Washington, D.C.? Explain your thinking.
**Background**

Mr. Garcia’s 6th grade class has recently started their unit on expressions and equations. The class has explored the difference between equations and expressions. They have also been using the properties of operations to generate equivalent expressions and determine if two expressions are equivalent. Mr. Garcia’s class of 32 students has four students with an Individualized Education Plan and eight English learners. Of his English learners, one is at the Bridging level, five are at the Expanding level, and two are at the Emerging level. One of his students at the Emerging level joined the class several weeks ago after moving to the United States from Mexico. Each of the four 6th grade classes are similar in their composition of English learners, with 8 - 10 EL students per class. Mr. Garcia meets weekly with the other three self-contained sixth grade teachers to collaborate. During this time, they discuss recent data, upcoming units of instruction, and areas of focus for Designated ELD instruction when they deploy their students to receive specialized instruction (see the additional Designated ELD resources below).

**Lesson Context**

The sixth graders are several lessons into their unit on expressions and equations. Mr. Garcia has been working with his students to create equivalent expressions and to determine whether or not two expressions are equivalent. He wants to use a formative assessment lesson to gauge his students current level of understanding with this concept and determine areas of need to guide his next steps. To do this, he has selected an Illustrative Mathematics task where students will have to determine which student expressions are equivalent and justify their thinking. He hopes that this lesson will serve to deepen student understanding about equivalent expressions by connecting them to a familiar context, the perimeter of a rectangle. He also believes that this context will be useful for guiding conversations about why expressions are equivalent based on the structure of the rectangle and the parts of the expressions. Mr. Garcia plans to ask students to justify the equivalence of the expressions by connecting the expression to the labeled picture of the rectangle.

**Lesson Excerpts**

Mr. Garcia’s lesson for today engages students in analyzing given expressions to determine if they are equivalent. The task also includes a context with a visual support to encourage students to connect the expressions to the corresponding elements in the visual representation. He is curious about whether or not students understand that different equivalent expressions can illustrate different aspects of the same situation. He wants to determine which students have internalized the academic language and use it naturally to explain their thinking.

**Learning Target:** The students will analyze different student expressions for the perimeter of a rectangle to determine if the expressions are equivalent and they will justify the equivalence in conversations and in writing.

**CA CCSS for Mathematics:** 6.EE.4 - Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For ex., the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number $y$ stands for; SMP 7 - Look for and make use of structure; SMP 3 - Construct viable arguments and critique the reasoning of others.

**CA ELD Standards:** ELD.PI.6.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics; ELD.PI.6.11 - Justifying own arguments and evaluating others’ arguments in writing.
Mr. Garcia begins the lesson by showing students the image below and asking them to write an expression for the perimeter of this rectangle using the given variables. He begins in this way in order to connect to what students have learned about creating expressions since the beginning of the unit. He believes that by having students create their own expressions first, they will have a foundation for forming their arguments about whether or not the other expressions in the task represent the perimeter of the rectangle.

![Rectangle Image]

After students have created an expression for the image, Mr. Garcia asks them to share their expressions with their table groups. He asks them to briefly discuss whether their expressions are the same or different, and if they are different, if the group believes that they are equivalent or not.

Mr. Garcia: I want you to think about the expression you wrote and the other expressions that were shared at your table. Using what you have learned about equivalent expressions, expressions that mean the same thing and have the same value, I want you to explore this task.

Next, Mr. Garcia reads the task aloud as students read along on their own copies of the task. As Mr. Garcia reads, students mark the text to indicate important information, ideas, and questions they may have.

The students in Mr. Nolan's class are writing expressions for the perimeter of a rectangle of side length \( \ell \) and width \( w \). After they share their answers, the following expressions are on the board:

- Sam: \( 2(\ell + w) \)
- Joanna: \( \ell + w + \ell + w \)
- Kiyo: \( 2\ell + w \)
- Erica: \( 2w + 2\ell \)

![Rectangle Image]

Which of the expressions are correct and how might the students have been thinking about finding the perimeter of the rectangle?

After posing the task, Mr. Garcia provides the students several minutes of independent time to think about and work on the task to determine which expressions correctly represent the situation and why. Students are given several minutes to work on the task independently. Next, Mr. Garcia asks the groups to discuss which of the expressions are correct and justify their thinking. He circulates around the room while groups are discussing their ideas and makes notes about what he is hearing and which students he would like to ask to share.
Mr. Garcia: As I walked around the classroom, I heard students using the word equation and expression interchangeably to mean the same thing. Before we share ideas about the task, I want your groups to discuss whether or not equation and expression mean the same thing, and if not, how are they different?

Mr. Garcia walks around the classroom, visiting several groups to hear their discussions. He stops at one of the tables to listen to their discussion. He tells the table group that he would like them to share their conversation with the class and he asks Cecily, an English learner at the Expanding level, if she would be willing to share for the group. She agrees and he asks her to practice what she will say with her group before sharing with the whole class.

Mr. Garcia: As I listened to table groups, I heard conversations explaining the difference between expressions and equations. I have asked Cecily from Table 4 to share her group’s ideas with the class.

Cecily: My group discussed how equations and expressions are different. We think that equations have equal signs and expressions do not.

Mr. Garcia: Can anyone add on to what Cecily said? Alex.

Alex: My group agreed with Cecily’s group and we also said that an equation shows two expressions that are equal to each other. The expression on one side equals the expression on the other side.

Mr. Garcia: Okay, so Alex, you’re saying that if $5x$ is an expression (Mr. Garcia writes this on the whiteboard and labels it expression) then $5x = 4x + 2$ is an equation (Mr. Garcia writes this on the whiteboard and labels it equation), correct?

Alex: Yes, an equation is made up of two expressions.

Mr. Garcia: Now that you’ve heard some ideas about the difference between expressions and equations, please tell your partner what you have learned.

Students discuss the difference between expressions and equations with their partner as Mr. Garcia walks around the classroom listening to partner discussions. He intentionally visits two partner groups where one of the partners is an English learner to see if these students are understanding the conceptual difference behind these two math terms. Next, Mr. Garcia brings the class back together to have a class conversation about the task. He asks students to share a correct expression and explain how the parts of the expression relate to the picture. Mr. Garcia has also been using talk moves with his class to strengthen their classroom discussions and makes a conscious effort to model and use these moves throughout the discussion. Recently, he has been focusing on supporting the talk moves of reasoning and turn and talk.

Mr. Garcia: Looking at today’s task, can you share an expression that is correct and explain why you believe that it’s correct? (Mr. Garcia, gives the students some time to think and refer to their work.) Okay, who would like to share? Gabby.

Gabby: (Referring to her work.) I think that Erica is correct because $2w + 2l$ means that there are 2 widths and 2 lengths.
Mr. Garcia: When you say that there are 2 widths and 2 lengths, can you show us what you mean using this picture of the rectangle? (Mr. Garcia points to where the task is displayed by the projector.)

Gabby: Sure. (Gabby walks to the front of the room and points.) The two widths are the sides on the left and right. The two lengths are the top and the bottom.

Eduardo: Well, then why doesn’t the equation say \( w + w + l + l \)?

Mr. Garcia: Class, is there an expression that has it written the way Eduardo suggested? (Note: When Mr. Garcia asks his question, he correctly uses the term expression instead of equation as Eduardo did. He decides to make this gentle correction by restating with the correct term and makes a note to listen to Eduardo’s partner conversation to see if he truly understands the concept and term expression.)

Gabby: Yes, Joanna’s way shows it like that. It’s just in a different order.

Mr. Garcia: So if Joanna’s way, her expression, shows what Eduardo mentioned, turn and talk to your partner about which property you could use to rewrite \( l + w + l + w \) as \( w + w + l + l \) and how you know this property would work?

Students discuss the property they would use to demonstrate that \( l + w + l + w \) and \( w + w + l + l \) are equivalent expressions. As they are discussing, Mr. Garcia walks to Eduardo’s group to listen to how Eduardo explains his thinking. He hears Eduardo use the term expression correctly in his explanation and makes a note to continue to reinforce this concept with students during the duration of the unit as he notices that some students are continuing to struggle accurately use these math terms. Mr. Garcia has pre-selected two groups to share their ideas about which property can be used to rewrite the expression. One of these groups includes a student that has struggled in mathematics recently, so Mr. Garcia wants him to be able to share his ideas with the class to demonstrate his success with this idea. He also asks a pair of girls to share that have not shared a math idea with the class during the last several lessons. Mr. Garcia wants to create opportunities where all student voices are heard and valued, so he carefully selects and records which students share their ideas during math class. As the two pairs share with the class, he asks each group to justify their reasoning by explaining how they know that the commutative property allows them to change the order of an addition expression.

Mr. Garcia: Now that we’ve talked about two of the equivalent expressions, I’d like to see if there are any expressions from the list that are not equivalent.

Jordan: I think that Kiyo’s expression is wrong.

Mr. Garcia: Who agrees with Jordan that Kiyo’s expression is incorrect? (Students show their agree or disagree silent signal.) I see that the majority of the class agrees with Jordan. Please turn and talk with your partner about why you agree or disagree with Jordan. (Mr. Garcia provides time for students to talk with their partners.) Is there anyone who would like to share why you agree or disagree? Sara.

Sara: I agree with Jordan that Kiyo is incorrect because she has \( 2l \), but she only has \( 1w \), so I think that she forgot one of the widths.
Mr. Garcia: Can you show us what you mean on screen?

Sara: Sure. These are her two lengths and she only wrote $w$, so she has 1 width included, but she forgot this one (pointing to the other side).

Mr. Garcia: Please repeat what Sara said to your partner. (Students turn and talk to repeat Sara’s idea.)

After students have repeated Sara’s idea, Mr. Garcia shares several ideas and key points that he has heard from students during the lesson. He refers to the examples on the board from earlier in the lesson illustrating the difference between an expression and an equation. He also elaborates on several of the student ideas to connect to the mathematical goal of today’s lesson. Next, he draws the class’ attention to two sentence frames that he has written on the board and tells students that they may choose to use these frames or they can create their own sentences to begin their writing today.

Sentence Frames:
- _______ and _______ are equivalent expressions because ____________.
- The expressions _______ and _______ are equivalent because ____________.

Mr. Garcia: On the back of your task, I would like you to select two of the expressions that are equivalent and explain how you know they are equivalent. Please include numbers, words, and pictures to strengthen your explanation.

Students know that the expectation is to write several sentences as needed to completely explain their thinking and that these frames serve as an optional starting point for their writing. Mr. Garcia provides several minutes for students to complete their writing. They also know that in mathematics, their writing is supported through the use of expressions and/or visuals. He wraps up class by having students read their writing to their partner, provide feedback, and revise their writing as needed. Students turn in their writing to end the class session.

Next Steps

Mr. Garcia reads through the student explanations and sorts them into two piles: Got It and Not Yet (Van de Walle, 2005). He looks at the responses in the Not Yet pile to determine common errors or areas of difficulty for students. He discovers that a group of his students are having difficulty justifying equivalence through use of the distributive property, making errors while distributing. He decides to support this small group of students by working with them at the back table over the next several days. Mr. Garcia also decides to recheck the Got It pile and finds that students were less likely to choose to explain the expressions that were equivalent through the use of the distributive property, making him think that this may be an area of weakness for the class overall. Based on this, he decides that the whole class would benefit from further work on the distributive property. He looks for a task that will focus student attention on creating and identifying equivalent expressions using the distributive property. He also selects a task to serve as a warm up where students will analyze a worked sample showing distribution to find and explain an error within the work. As Mr. Garcia continues to teach the lessons in the expressions and equations unit, he uses what he learned about his students from this lesson to connect ideas and deepen student understanding of equivalent expressions.

Source

Resources

Companion Documents
Equivalent Expressions Designated ELD Connected to Mathematics in Grade Six
Equivalent Expressions Designated ELD: Math & ELD 5-Day Lesson Plan D-ELD 6th

Additional Information
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In mathematics, Mr. Garcia is teaching his sixth graders to create and support their thinking about equivalent expressions. He wants them to recognize that equivalent expressions show different forms of the same expression. He also wants his students to use precise language to express their understanding of the math content. Listening to his students talk about math provides Mr. Garcia with a glimpse into their understandings of content as well as their current levels of language production. He uses what he hears from students to guide the feedback he provides and the integrated and designated lesson he prepares for his students.

During his Designated ELD time, Mr. Garcia works with a group of EL students at the Expanding level of English language proficiency. He has familiarized himself with the ELD standards and recognizes that it can sometimes be difficult to articulate explanations of equivalent expressions when using precise mathematical language. To this end, he invites students to use a bank of new and familiar words to help them apply content vocabulary as part of their mathematical discussions. He also teaches, models, and provides opportunities for students to use temporary scaffolds like extended sentence frames. Understanding the value of oral language practice before constructing written responses, Mr. Garcia structures many opportunities for students to use mathematical language such as perimeter, length, width, equivalent, expression, representation, variable, etc., and invites justification and reasoning using language such as because and although. Mr. Garcia also encourages the use of academic phrases such as, I believe _____ and _____ are (not) equivalent expressions because _____; Another way to mathematically express _____ is _____; and _____ and _____ are equivalent expressions because ______; However, _____ and _____ are not equivalent expressions because ___.” He has worked to build a classroom culture where students are encouraged to take language risks. Mr. Garcia promotes the use of content vocabulary and academic structures in tandem in partner conversations, small group discussions, and large group discussions on a daily basis.

As part of his effort to improve student collaboration, use of productive language, and extend academic discourse, Mr. Garcia has worked with his ELD students on two talk moves to help them add on to or challenge one another’s ideas (reasoning) as they apply their learning of equivalent expressions and explain their mathematical thinking.

**Learning Target:** Students will use math language to identify equivalent expressions and justify why they are equivalent.

**CA ELD Standards:** ELD.PI.6.1,3,6a,12a; ELD.PII.6.2b,6

**CCSS for Mathematics:** 6.EE.4 - **Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).** For ex., the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for; SMP 7 - Look for and make use of structure; SMP 3 - Construct viable arguments and critique the reasoning of others.

**Additional Information**
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## ELD Lesson Plan: Designated ELD

**Grade:** 6  
**Date:** (1 week-dates TBD)  
**Proficiency Level(s)?** Em | Ex | Br

**Reference Material/Content:** Math: Equivalent Expressions (Vignette)

<table>
<thead>
<tr>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
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</thead>
<tbody>
<tr>
<td><strong>CONTENT:</strong> Math 6.EE.4</td>
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<tr>
<td><strong>LANGUAGE OBJECTIVE:</strong> Students will use key math language to orally describe expressions that are equivalent and that are not equivalent.</td>
<td><strong>LANGUAGE OBJECTIVE:</strong> Students will orally support their positions why two representations are equivalent expressions using a complex sentence structure.</td>
<td><strong>LANGUAGE OBJECTIVE:</strong> Students will use connecting words to orally contrast ideas (expressions that are/are not equivalent) before writing.</td>
<td><strong>LANGUAGE OBJECTIVE:</strong> Students will use talk moves (building/adding on or challenge an idea/reasoning) to support accuracy or refute misconceptions about equivalent expressions.</td>
<td><strong>LANGUAGE OBJECTIVE:</strong> Students will orally express and explain their understanding of equivalent expressions in partner conversations using formalities of greeting and closing a conversation.</td>
</tr>
</tbody>
</table>
| **ELD STANDARD(S):** ELD.PI.6.12a-Ex (Use acad. words while speaking) | **ELD STANDARD(S):** ELD.PI.6.6a-Ex (Explain ideas)  
ELD.PI.6.3-Ex (Support opinions)  
ELD.PI.6.6-Ex (Connecting ideas) | **ELD STANDARD(S):** ELD.PI.6.2b-Ex (Link ideas through connecting word or phrases)  
ELD.PI.6.6a-Ex (Explain ideas) | **ELD STANDARD(S):** ELD.PI.6.3-Ex (Support opinions and persuade others) | **ELD STANDARD(S):** ELD.PI.6.1-Ex (Exchange info & ideas)  
ELD.PI.6.3-Ex (Support opinions and persuade others) |

### Instructional Day 1
- Teacher will refamiliarize students with known content vocabulary, i.e., perimeter, equal, illustrate, variables, and introduce new terms critical to content learning, i.e. equivalent

### Instructional Day 2
- Students will identify pairs of equivalent expressions on cards.
- Teacher models (orally and in writing on sentence strips):

### Instructional Day 3
- Using math expression cards, students determine if two expressions are equivalent or not. (Use cards in set of three that reflect two equivalent expressions and one

### Instructional Day 4
- Teacher shows several slides that reflect either examples or nonexamples of equivalent expressions, along with a statement of thinking. Slides may say things like:

### Instructional Day 5
- Teacher will inform students that they will be using all the language skills they’ve learned throughout the week in a collaborative conversation structure. Before beginning, students discuss ways...
expression, representation, etc.
New words will be defined in writing, along with visual support (math example) using Define-Example-Ask routine.

Hook: Teacher uses the examples of money (.01 = one cent = 1¢) and fractions/decimals/visual ($\frac{1}{2} = 50\% = 0.5 = \Box\Box\Box\Box\Box$) as different ways to represent or express the same things before introducing practice examples of equivalent expressions.

Students will be shown example and nonexample cards to check for understanding of equivalent expressions.

Teacher will point out the change in sentence structure (word change and order change) between question (interrogative) and response statement (declarative) sentences.

<table>
<thead>
<tr>
<th>2w and w + w are equivalent expressions.</th>
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<tbody>
<tr>
<td>“_____ and _____ are equivalent expressions.”</td>
</tr>
<tr>
<td>Students use the same frame to practice articulating equivalent expressions from expression cards (manipulatives) on their tables.</td>
</tr>
<tr>
<td>Teacher models using a statement of support to explain/elaborate: 2w and w + w are different representations of w multiplied by 2.</td>
</tr>
<tr>
<td>“_____ and _____ are different representations of __________.”</td>
</tr>
<tr>
<td>Students offer a statement of support using the frame to explain/elaborate on their pairs of equivalent expressions.</td>
</tr>
<tr>
<td>Teacher models how to connect ideas using “although” to create a outlier not equivalent expression.)</td>
</tr>
</tbody>
</table>

“John believes 5w is equivalent to 5+5+5+5+5” [where the value of w is a number other than 5]

Do you agree or disagree with John? Explain why.

or

Lisa says that 3y is equivalent to y + y + y. Do you agree or disagree with Lisa? Explain why.

“I agree with _____ that _____ because ______.”

“I understand _____’s thinking that ____. However____.”

Teacher models one slide using think aloud and sentence frame, leads a guided process for the next slide to support students and listen for language, then releases students to talk in partnerships to negotiate accuracy or to greet a person they encounter, as well as ways they can close a conversation when exiting. Teacher may chart “Greetings and Closings” as students share ideas.

Using the familiar Quiz-Quiz-Trade strategy, students will “quiz” one another using equivalent expression and non-equivalent expression cards with the same expression printed on both sides of the card so the holder of the card and the viewer of the card can see the same thing.

Students join a just-for-now partner and partners greet one another.

Partner A will read the expression card aloud to a Partner B before asking, “Is this an equivalent expression or a non-equivalent expression?”

I agree with _____ that _____ because ______.

I understand _____’s thinking that ____. However____.

The expression _____ and _____ are equivalent. However,
Teacher invites students to write another expression that is equivalent to an example expression provided. Group orally confirms or refutes and revises shared examples.

Students engage in oral language practice with peers using frame(s).

Extend the frames by adding “because” to include details for each example, i.e., “The expression 3y and y+y+y are equivalent expressions because…” [provided that x and y represent different numbers in the example]

Students are encouraged to extend and use the “talk move” of building on or challenging another’s ideas as they engage in partner discussions and also as they share out in whole group.

Students will use the frame to orally justify their thinking.

Students engage in oral language practice before recording their samples in their math notebooks.

misconception before using a sentence frame. Partnership responses are shared aloud within the group.

expression? Explain why.”

Partner B will respond to express their thinking and offering support.

Students engage in oral language practice with peers using frame(s).

Teacher manipulates sentence strips to show how sentences change when combined. Teacher points out how, in this structure, the dependent clause (support/explanation) appears followed by a comma before the independent clause; the pronoun “they” is included: Although 2w and w + w are different representations of w multiplied by 2, they are equivalent expressions.

“Although _____ and _____ are different representations of _____, they are equivalent expressions.

Although _____ and _____ are different representations of equivalent expressions.”

Students engage in oral language practice with peers using frame(s).

Students are encouraged to extend and use the “talk move” of building on or challenging another’s ideas as they engage in partner discussions and also as they share out in whole group.

“Although _____ and _____ are different representations of equivalent expressions.”

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“Although _____ and _____ are different representations of equivalent expressions.”
Note: The Equivalent Expressions, Grade 6 Integrated ELD and Mathematics Instruction Vignette lesson would be well supported after Day 3 of Designated ELD instruction. Students would have had the opportunity to develop the skills necessary to support their thinking and develop an argument to justify whether or not expressions were equivalent.

swap cards, close and exit the conversation by thanking one another. Both students move on to a new partner using the new card and repeat the process.

Routine continues until teacher calls for students to regroup as a whole and debrief their learnings orally.

Additional Information
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