

HS Mathematics Item Specification C1 TD

Claim 1: Concepts and Procedures

Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Content Domain: **Algebra**

Target D [m]: Interpret the structure of expressions. (DOK 1,2)

Tasks for this target will require students to recognize equivalent forms of an expression as determined by the structure of the expression. Tasks for Claims 2 and 4 will ask students to interpret expressions or parts of expressions in the context of a problem.

Standards:	A-SSE.A Interpret the structure of expressions
A-SSE.A, A-SSE.A.2	A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i>
Related Below-Grade Standards for Purposes of Planning for Vertical Scaling:	Related Grade 8 Standards
8.EE.A, 8.EE.A.1	8.EE.A Work with radicals and integer exponents 8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</i>
DOK Level:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptors (Range ALD) Target D: Interpret the structure of expressions.	Level 1 Students should be able to identify parts of an expression, such as terms, factors, coefficients, exponents, etc.
	Level 2 Students should be able to interpret parts of an expression, such as terms, factors, coefficients, exponents, etc. They should also be able to recognize equivalent forms of linear expressions.
	Level 3 Students should be able to recognize equivalent forms of expressions and use the structure of an expression to identify ways to rewrite it.
	Level 4 Students should be able to look for and use structure and repeated reasoning to make generalizations about the possible equivalent forms expressions can have.
Evidence Required:	1. The student uses the structure of an expression to identify ways of rewriting it.
Allowable Response Types:	Multiple Choice, single correct response; Matching Tables, Drag and Drop
Allowable Stimulus Materials:	algebraic expressions, words or symbols for common structures for algebraic expressions
Construct-Relevant Vocabulary:	expression, factor, difference of squares, difference of cubes, sum of cubes, quadratic expression
Allowable Tools:	None
Target-Specific Attributes:	The expressions given should be ones that fit common structures such that recognition of the structure then allows for application of a factoring strategy to be applied.
Non-Targeted Constructs:	

HS Mathematics Item Specification C1 TD

<p>Accessibility Guidance:</p>	<p>Item writers should consider the following Language and Visual Element/Design guidelines¹ when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics <p>Items are selected for a student’s test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.²</p>
<p>Development Notes:</p>	<p>Tasks for Claims 2 and 4 will ask students to interpret expressions or parts of expressions in the context of a problem.</p>

¹ For more information, refer to the General Accessibility Guidelines at: <http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

² For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

<p>Task Model 1</p> <p>Response Type: Multiple Choice, single correct response</p> <p>DOK Level 1</p> <p>A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i></p> <p>Evidence Required: 1. The student uses the structure of an expression to identify ways of rewriting it.</p> <p>Tools: None</p>	<p>Prompt Features: The student is prompted to use the structure of an expression to select another expression that is equivalent to the given expression.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> Expressions may be: <ul style="list-style-type: none"> difference of two squares sum/difference of two cubes the product of two or three expressions sum/difference of expressions that have a common factor rational exponential Difficulty level can be altered by varying the type of expression and/or the order of factors in a compound expression, and by using different variables and coefficients. <p>TM1a Stimulus: The student is presented with an expression that is a difference of two squares.</p> <p>Example Stem: Select the expression that is equivalent to $x^2 - 4$.</p> <p>A. $(x - 2)^2$ B. $(x - 2)(x + 2)$ C. $x^2 + 2x + 4$ D. $x^2 - 2x + 4$</p> <p>Rubric: (1 point) The student selects the correct option (e.g., B).</p> <p>Response Type: Multiple Choice, single correct response</p> <p>TM1b Stimulus: The student is presented with an expression that is the sum/difference of expressions that have a common factor.</p> <p>Example Stem: Select the expression that is equivalent to $(x + 4)^2 - (x - 2)(x + 4)$.</p> <p>A. $4(x + 4)$ B. $2(x + 1)(x + 4)$ C. $(x + 4) - (x - 2)$ D. $(x + 4)[(x + 4) - (x - 2)]$</p> <p>Rubric: (1 point) The student selects the correct option (e.g., D).</p> <p>Response Type: Multiple Choice, single correct response</p>
---	---

Task Model 1

Response Type:
Matching Tables

DOK Level 1

A-SSE.A.2

Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Evidence Required:

1. The student uses the structure of an expression to identify ways of rewriting it.

Tools: None

TM1c

Stimulus: The student is presented with an expression that is a sum/difference of two cubes.

Example Stem 1: Determine whether each expression is equivalent to $(x^3 + 8)$. Select Yes or No for each expression.

	Yes	No
$(x + 2)^3$		
$(x - 2)(x^2 + 2x + 4)$		
$(x + 2)(x^2 - 2x + 4)$		

Example Stem 2: Determine whether each expression is equivalent to $(8x^3 - 64)$. Select Yes or No for each expression.

	Yes	No
$(2x - 4)^3$		
$8(x - 8)^3$		
$8(x - 2)(x^2 + 2x + 4)$		
$(2x - 4)(4x^2 + 8x + 16)$		

Rubric: (1 point) The student selects the correct options (e.g., NNY; NNYY).

Response Type: Matching Tables

Task Model 1
Response Type:
Matching Tables
DOK Level 2
A-SSE.A.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Evidence Required:

1. The student uses the structure of an expression to identify ways of rewriting it.

Tools: None

Prompt Features: The student is prompted to use the structure of expressions to determine if two expressions are equivalent.

Stimulus Guidelines:

- Equivalences consist of equations of expressions, which may involve:
 - difference of two squares
 - sum/difference of two cubes
 - the product of two or three expressions
 - sum/difference of expressions that have a common factor
 - rational expressions
 - exponential expressions
- Difficulty level can be altered by varying the type of expression and/or the order of factors in an equation, and by using different variables and coefficients.

TM1d

Stimulus: The student is presented with four equations.

Example Stem 1: Determine if each equation is true for all values of x . Select Yes or No for each equation.

	Yes	No
$x^2 + 4 = (x + 2)^2$		
$(2x + 6)^2 = 4(x + 3)^2$		
$(x - 3)(x - 3) = (x - 9)^2$		
$x^2 - 10x + 25 = (x - 5)(x + 5)$		

Example Stem 2: Determine if each equation is true for all values of x . Select Yes or No for each equation.

	Yes	No
$2^{3x} = 6^x$		
$100^x = 10^{2x}$		
$e^x \cdot e^x = e^{2x}$		
$2^{10x} = 10^{2x}$		

Rubric: (1 point) The student selects the correct options (e.g., NYNN; NYYN).

Response Type: Matching Tables

Task Model 1

Response Type: Drag and Drop

DOK Level 2

A-SSE.A.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Evidence Required:

1. The student uses the structure of an expression to identify ways of rewriting it.

Tools: None

Accessibility Note:

Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to use the structure of a rational expression to create an expression that is equivalent to the given expression.

Stimulus Guidelines:

- Equivalences consist of equations of rational expressions.
- Difficulty level can be altered by varying the complexity of the equations, the type of rational expressions, and by using different variables and coefficients.

TM1e

Stimulus: The student is presented with two equivalent rational expressions with missing components that may be found using structure without carrying out the calculation.

Example Stem 1: Drag one or more expressions into each box to create an equation that is true for all values of x . (Assume no denominator equals zero.)

$$\frac{3}{x+2} + \frac{4}{x} = \frac{3\boxed{} + 4\boxed{}}{\boxed{}}$$

Preset Choices:

x x^2 $(x+2)$ $x(x+2)$ (x^2+2)

Example Stem 2: Drag one or more expressions into each box to create an equation that is true for all values of x . (Assume no denominator equals zero.)

$$\frac{3}{x+2} + \frac{4}{x} + \frac{2}{x^2} = \frac{3\boxed{} + 4\boxed{} + 2\boxed{}}{\boxed{}}$$

Preset Choices:

x x^2 $(x+2)$ $x(x+2)$ $x^2(x+2)$

Rubric: (1 point) The student drags the correct options.

Example Stem 1: x , $(x+2)$, $x(x+2)$;

Example Stem 2: x^2 , $x(x+2)$, $(x+2)$, $x^2(x+2)$

Response Type: Drag and Drop

Task Model 1
Response Type:
Drag and Drop
DOK Level 2
A-SSE.A.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Evidence Required:

1. The student uses the structure of an expression to identify ways of rewriting it.

Tools: None

Accessibility Note:

Drag and Drop items are not currently able to be Brailled.

Minimize the number of items developed to this TM.

Prompt Features: The student is prompted to use the structure of an expression to create an expression that is equivalent to the given expression.

Stimulus Guidelines:

- Equivalences consist of equations of expressions.
- Expressions may be:
 - difference of two squares
 - sum/difference of two cubes
 - the product of two or three expressions
 - sum/difference of expressions that have a common factor
 - rational
 - exponential
- Difficulty level can be altered by varying the type of expression and/or the order of factors in a compound expression, and by using different variables and coefficients.

TM1f

Stimulus: The student is presented with two equivalent expressions with missing numbers that may be found using structure without carrying out the calculation.

Example Stem 1: Drag a number into each box to create an equation that is true for all values of x .

$$2(4x + 3)(3x + 5) = \square x^2 + 58x + \square$$

Palette Choices: 6 8 12 15 24 29 30 58

Example Stem 2: Drag a number into each box to create an equation that is true for all values of x .

$$\frac{3(n + 2)(4n + 1)}{6} = \square n^2 + \frac{9}{2}n + \square$$

Palette Choices: $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{2}$ 1 2 4 6 12

Example Stem 3: Drag a number into the box to create an equation that is true for all values of x .

$$(x + 2)^2 - 5 = x^2 + 4x + \square$$

Palette Choices: -4 -1 4 9

Task Model 1

Response Type:
Drag and Drop

DOK Level 2

A-SSE.A.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Evidence Required:

1. The student uses the structure of an expression to identify ways of rewriting it.

Tools: None

Accessibility Note:

Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.

Example Stem 4: Drag a number into the box to create an equation that is true for all values of x .

$$(x - 7)^2 + 51 = x^2 + \square x + 100$$

Palette Choices: -14 0 2 14 49

Example Stem 5: Drag a number into the box to create an equation that is true for all values of x .

$$\square(x - 10)(x + 12) = 3(x + 1)^2 - 363$$

Palette Choices: -12 -1 1 3 10

Rubric: (1 point) The student places the correct number in the box(es).

Example Stem 1: 24, 30;

Example Stem 2: 2, 1;

Example Stem 3: -1;

Example Stem 4: -14;

Example Stem 5: 3.

Response Type: Drag and Drop