

Grade 8 Mathematics Item Specification C1 TH

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| <p><b>Claim 1:</b> Concepts and Procedures</p> <p>Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p>   |   |
| <p>Content Domain: <b>Geometry</b></p>   |   |
| <p><b>Target H [m]:</b> Understand and apply the Pythagorean Theorem. (DOK Level 1, 2)</p> <p>Tasks associated with this target will ask students to use the Pythagorean Theorem to solve real-world and mathematical problems in two and three dimensions, including problems that ask students to find the distance between two points in a coordinate system.</p> <p>Some applications of the Pythagorean Theorem will be assessed at deeper levels in Claims 2 and 4. Understanding of the proofs of the Pythagorean Theorem would contribute evidence to Claim 3.</p> |   |
| <p>Standards:<br/>8.G.B, 8.G.B.6,<br/>8.G.B.7, 8.G.B.8</p>   | <p><b>8.G.B Understand and apply the Pythagorean Theorem.</b></p> <p><b>8.G.B.6</b> Explain a proof of the Pythagorean Theorem and its converse.</p> <p><b>8.G.B.7</b> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p> <p><b>8.G.B.8</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>  |
| <p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>7.G.B, 7.G.B.4,<br/>7.G.B.5, 7.G.B.6</p> <p>G-CO.C, G-CO.C.9,<br/>G-CO.C.10,<br/>G-SRT.B,<br/>G-SRT.B.4,<br/>G-SRT.B.5,<br/>G-SRT.C,<br/>G-SRT.C.8</p>  | <p><b>Related Grade 7 standards</b></p> <p><b>7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</b></p> <p><b>7.G.B.4</b> Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p> <p><b>7.G.B.5</b> Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p> <p><b>7.G.B.6</b> Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p> <p><b>Related High School Standards</b></p> <p><b>G-CO.C Prove geometric theorems.</b></p> <p><b>G-CO.C.9</b> Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i></p> <p><b>G-CO.C.10</b> Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i></p> <p><b>G-SRT.B Prove theorems involving similarity.</b></p> |

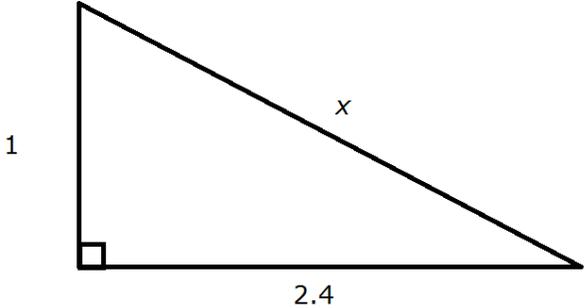
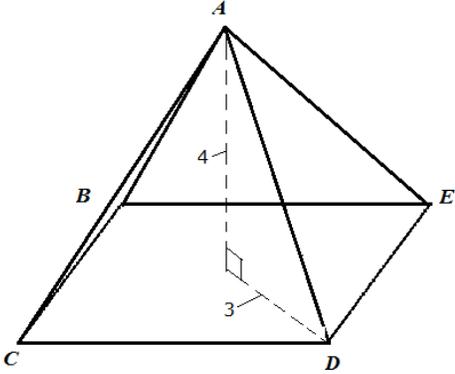
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|  | <p><b>G-SRT.B.4</b> Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i></p> <p><b>G-SRT.B.5</b> Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</p> <p><b>G-SRT.C Define trigonometric ratios and solve problems involving right triangles.</b></p> <p><b>G-SRT.C.8</b> Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p> |
| DOK Levels   | 1, 2  |
| <b>Achievement Level Descriptors:</b>  |   |
| <p><b>RANGE Achievement Level Descriptor (Range ALD)</b><br/>Target H:<br/>Understand and apply the Pythagorean Theorem.</p> | <p><b>Level 1</b> Students should be able to identify the hypotenuse and the legs of a right triangle given the side lengths or an image of a right triangle.</p>   |
|  | <p><b>Level 2</b> Students should be able to apply the converse of Pythagorean Theorem to determine whether or not a given triangle is a right triangle, given its side lengths. They should be able to find the distance between two points on a horizontal or vertical line in a two-dimensional coordinate system.</p>   |
|  | <p><b>Level 3</b> Students should be able to apply the Pythagorean Theorem to determine the unknown side lengths of right triangles and to find the distance between two points in a coordinate system in two dimensions.</p>   |
|  | <p><b>Level 4</b> Students should be able to apply the Pythagorean Theorem to find the distance between two points in a coordinate system in three dimensions.</p>  |
| Evidence Required:   | <ol style="list-style-type: none"> <li>1. The student solves real-world and mathematical problems of right triangles in two and three dimensions by applying the Pythagorean Theorem and its converse.</li> <li>2. The student finds the distance between two points in a coordinate system by applying the Pythagorean Theorem.</li> </ol>   |
| Allowable Response Types:  | Equation/Numeric; Multiple Choice, single correct response  |
| Allowable Stimulus Materials:  | Two-dimensional representations of triangles, three-dimensional models that contain right triangles, and coordinate systems   |
| Construct-Relevant Vocabulary:   | Pythagorean Theorem, leg, hypotenuse, right triangle, base  |
| Allowable Tools:   | Calculator  |
| Target-Specific Attributes:  | The size of the coordinate system must be considered for items that involve the use of coordinate systems.  |
| Non-Targeted Constructs:   |   |
| Accessibility Guidance:  | Item writers should consider the following Language and Visual Element/Design guidelines <sup>1</sup> when developing items.  |

<sup>1</sup> For more information, refer to the General Accessibility Guidelines at:  
<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

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|                           | <p>Language Key Considerations:</p> <ul style="list-style-type: none"> <li>• Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context</li> <li>• Avoid sentences with multiple clauses</li> <li>• Use vocabulary that is at or below grade level</li> <li>• Avoid ambiguous or obscure words, idioms, jargon, unusual names and references</li> </ul> <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> <li>• Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context</li> <li>• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary</li> <li>• Avoid crowding of details and graphics</li> </ul> <p>Items are selected for a student’s test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.<sup>2</sup></p> |
| <p>Development Notes:</p> | <p><b>8.G.B.6</b> Explaining a proof of the Pythagorean Theorem will be assessed in connection with targets from Claim 3. Some applications of the Pythagorean Theorem will be assessed at deeper levels in Claims 2 and 4. Understanding of the proofs of the Pythagorean Theorem would contribute evidence to Claim 3.</p>  |

<sup>2</sup> For more information about student accessibility resources and policies, refer to [http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced\\_Guidelines.pdf](http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf)

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| <p><b>Task Model 1</b></p> <p><b>Response Type:</b><br/>Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>8.G.B.7</b><br/>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p> <p><b>Evidence Required:</b><br/>1. The student solves real-world and mathematical problems of right triangles in two and three dimensions by applying the Pythagorean Theorem and its converse.</p> <p><b>Tools:</b> Calculator</p> <p><b>Version 3 Update:</b><br/>Added new TM1b to address the converse of the Pythagorean Theorem, so original TM1 was changed to TM1a.</p> | <p><b>Prompt Features:</b> The student is prompted to apply the Pythagorean Theorem to identify an unknown side length of a right triangle.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>Context should be familiar to 13–15 year olds.</li> <li>Item difficulty can be adjusted via these methods: <ul style="list-style-type: none"> <li>Finding the hypotenuse or legs</li> <li>Measurements are whole numbers, rational numbers, or irrational numbers</li> <li>Right triangle is in a 2D or 3D figure</li> <li>Pythagorean triplets, such as a 3-4-5 right triangle</li> </ul> </li> </ul> <p><b>TM1a</b><br/><b>Stimulus:</b> The student is presented with a situation in two or three dimensions where the Pythagorean Theorem must be used to determine the missing sides of the right triangle.</p> <p><b>Example Stem 1:</b> A right triangle is shown.</p>  <p>Enter the value of <math>x</math>.</p> <p><b>Example Stem 2:</b> A right square pyramid is shown. The height of the pyramid is 4 units. The distance from the center of the base of the pyramid to vertex <math>D</math> is 3 units, as shown.</p>  <p>Enter the length of segment <math>AD</math>, in units.</p> |
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**Task Model 1**

**Response Type:**  
Equation/Numeric

**DOK Level 2**

**8.G.B.7**

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

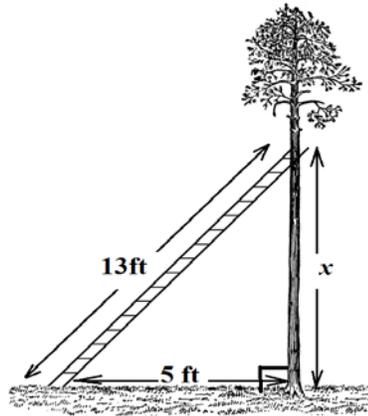
**Evidence Required:**

1. The student solves real-world and mathematical problems of right triangles in two and three dimensions by applying the Pythagorean Theorem and its converse.

**Tools:** Calculator

**Version 3 Update:** Added new TM1b to address the converse of the Pythagorean Theorem.

**Example Stem 3:** A 13-foot ladder is leaning on a tree. The bottom of the ladder is on the ground at a distance of 5 feet from the base of the tree. The base of the tree and the ground form a right angle as shown.



Enter the distance between the ground and the top of the ladder,  $x$ , in feet.

**Rubric:** (1 point) Student enters correct value (e.g., 2.6; 5; 12).

**Response Type:** Equation/Numeric

**TM1b**

**Prompt Features:** The student is prompted to determine whether a triangle meets the definition of a right triangle.

**Stimulus Guidelines:** Same as TM1a

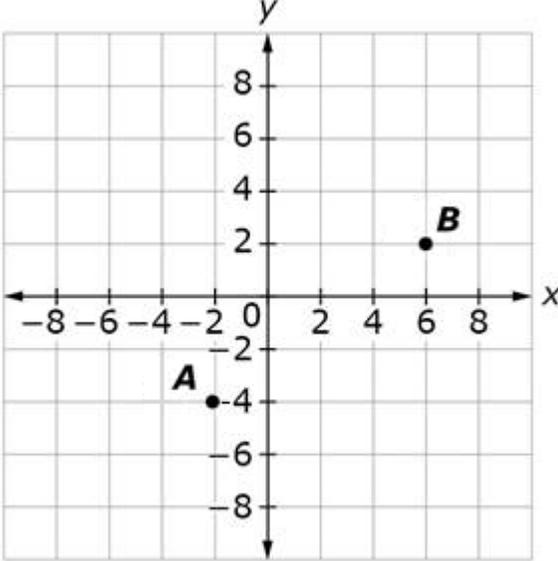
**Example Stem:** The table shows the side lengths for some triangles. Determine whether the side lengths define a right triangle.

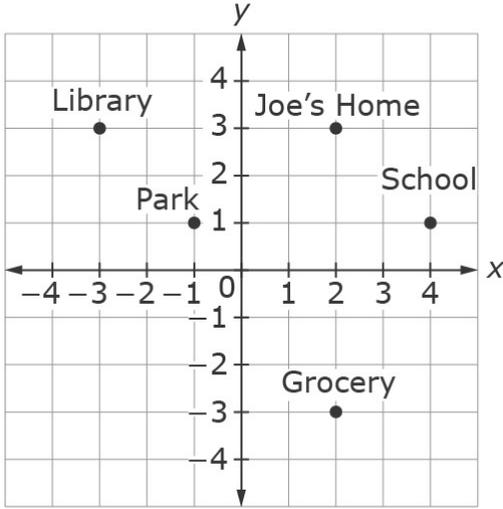
Select Yes if it is a right triangle. Select No if it cannot be a right triangle.

| Triangle Side Lengths | Yes | No |
|-----------------------|-----|----|
| 4 cm, 5 cm, 8 cm      |     |    |
| 8 ft, 10 ft, 16 ft    |     |    |
| 21 in, 28 in, 35 in   |     |    |

**Rubric:** (1 point) Student correctly classifies triangles (e.g., N,N,Y).

**Response Type:** Matching tables

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| <p><b>Task Model 2</b></p> <p><b>Response Type:</b><br/>Multiple Choice,<br/>single correct<br/>response;<br/>Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>8.G.B.8</b><br/>Apply the<br/>Pythagorean<br/>Theorem to find the<br/>distance between<br/>two points in a<br/>coordinate system.</p> <p><b>Evidence<br/>Required:</b><br/>2. The student finds<br/>the distance<br/>between two points<br/>in a coordinate<br/>system by applying<br/>the Pythagorean<br/>Theorem.</p> <p><b>Tools:</b> Calculator</p> | <p><b>Prompt Features:</b> The student is prompted to identify the distance between two points in a coordinate system by applying the Pythagorean Theorem.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• Context should be familiar to 13–15 year olds.</li> <li>• Item difficulty can be adjusted via these methods:             <ul style="list-style-type: none"> <li>○ Calculations require whole numbers, integers, rational numbers, or irrational numbers.</li> <li>○ The points may be given verbally or with a coordinate grid.</li> <li>○ Pythagorean triplets, such as a 3-4-5 right triangle</li> <li>○ Finding the hypotenuse or legs.</li> <li>○ Measurements are whole numbers, rational numbers, or irrational numbers.</li> <li>○ Right triangle is in a 2D or 3D figure</li> <li>○ Levels of scaffolding.</li> </ul> </li> </ul> <p><b>TM2a</b><br/><b>Stimulus:</b> The student is presented with two or more points in a coordinate system where the Pythagorean Theorem must be used to determine the distance between the points.</p> <p><b>Example Stem 1:</b> A coordinate plane is shown with labeled points.</p> <div style="text-align: center;">  </div> <p>What is the distance between point A and point B on the coordinate plane?</p> <p>A. 5<br/>B. 6<br/>C. 10<br/>D. 14</p> |
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| <p><b>Task Model 2</b></p> <p><b>Response Type:</b><br/>Multiple Choice,<br/>single correct<br/>response;<br/>Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>8.G.B.8</b><br/>Apply the<br/>Pythagorean<br/>Theorem to find the<br/>distance between<br/>two points in a<br/>coordinate system.</p> <p><b>Evidence<br/>Required:</b><br/>2. The student finds<br/>the distance<br/>between two points<br/>in a coordinate<br/>system by applying<br/>the Pythagorean<br/>Theorem.</p> <p><b>Tools:</b> Calculator</p> | <p><b>Example Stem 2:</b> What is the distance between points (5, 2) and (−3, −4) on the coordinate plane?</p> <p>A. 5<br/>B. 6<br/>C. 10<br/>D. 14</p> <p><b>Rubric:</b> (1 point) Student selects the distance between point <i>A</i> and point <i>B</i> (e.g., C; C).</p> <p><b>Response Type:</b> Multiple Choice, single correct response</p> <p><b>TM2b</b><br/><b>Stimulus:</b> The student is presented with two or more points in a coordinate system where the Pythagorean Theorem must be used to determine the distance between the points.</p> <p><b>Example Stem:</b> The points show different locations in Joe’s town. Each unit represents 1 mile.</p> <p style="text-align: center;"><b>Places in Joe’s Town</b></p>  <p>What is the distance, in miles, between Joe’s Home and the Park?<br/>Round your answer to the nearest tenth.</p> <p><b>Rubric:</b> (1 point) The student finds the distance (e.g., 3.6).</p> <p><b>Response Type:</b> Equation/Numeric</p> |
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