

Grade 8 Mathematics Item Specification C1 TF

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| <p>Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p> | |
| <p>Content Domain: Functions</p> | |
| <p>Target F [s]: Use functions to model relationships between quantities. (DOK Levels 1, 2)</p> <p>Tasks for this target will ask students to construct a function to model a linear relationship between two quantities and determine the rate of change or initial value of a linear function from given information.</p> <p>Other tasks will ask students to identify parts of a graph that fit a particular qualitative description (e.g., increasing or decreasing) or sketch a graph based on a qualitative description.</p> | |
| <p>Standards: 8.F.B, 8.F.B.4, 8.F.B.5</p> | <p>8.F.B Use functions to model relationships between quantities.</p> <p>8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> |
| <p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>7.RP.A, 7.RP.A.2, 7.RP.A.2a, 7.RP.A.2b, 7.RP.A.2c, 7.RP.A.2d</p> <p>F-IF.B, F-IF.B.4, F-IF.B.5, F-IF.B.6</p> | <p>Related Grade 7 Standards</p> <p>7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>7.RP.A.2 Recognize and represent proportional relationships between quantities.</p> <ol style="list-style-type: none"> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane, and observing whether the graph is a straight line through the origin. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Represent proportional relationships by equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</i> Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. <p>Related High School Standards</p> |

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| | <p>F–IF.B Interpret functions that arise in applications in terms of the context</p> <p>F–IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> <p>F–IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person–hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> <p>F–IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p> |
| DOK Levels: | 1, 2 |
| Achievement Level Descriptors: | |
| <p>RANGE Achievement Level Descriptor (Range ALD) Target F: Use functions to model relationships between quantities.</p> | <p>Level 1 Students should be able to identify a function that models a linear relationship between two quantities.</p> <p>Level 2 Students should be able to construct a graph or table to represent a linear relationship between two quantities, and should be able to find the rate of change in a linear relationship represented by a graph or table. They should be able to analyze and describe a graph of a linear function.</p> <p>Level 3 Students should be able to construct a function to represent a linear relationship between two quantities and a graph to represent verbally-described qualitative features, and determine the rate of change and initial value of a function from a graph, a verbal description of a relationship, or from two sets of x, y values given as coordinate pairs or displayed in a table. They should be able to analyze a graph of a linear or nonlinear function to qualitatively describe it.</p> <p>Level 4 Students should be able to interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.</p> |
| Evidence Required: | <ol style="list-style-type: none"> 1. The student constructs a function to model a linear relationship between two quantities. 2. The student determines the rate of change and initial value of a function, either from a description of a relationship or from two (x, y) values, including reading the rate of change and/or the value of the function from a table or a graph. 3. The student interprets features of a linear function, such as rate of change and initial value, in terms of the situation it models, its graph, or a table of values. 4. The student qualitatively describes the functional relationship between two quantities by analyzing a graph (e.g., whether the function is increasing or decreasing, or whether the graph is linear or nonlinear). |

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| | 5. The student draws a graph that exhibits the qualitative features of a function that has been described in writing. |
| Allowable Response Types: | Equation/Numeric; Matching Tables; Multiple Choice, single correct response; Graphing |
| Allowable Stimulus Materials: | Graphs, equations, tables, written descriptions |
| Construct-Relevant Vocabulary: | Function, slope, y–intercept, linear, nonlinear, rate of change, increasing, decreasing, constant, interval, relation |
| Allowable Tools: | Calculator |
| Target–Specific Attributes | |
| Non–Targeted Constructs: | |
| Accessibility Guidance: | <p>Item writers should consider the following Language and Visual Element/Design guidelines¹ when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics <p>Items are selected for a student’s test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.²</p> |
| Development Notes: | Function notation is not required in Grade 8. Items and tasks developed for the Grade 8 Function domain should not use formal function notation [i.e., $f(x)$]. ³ |

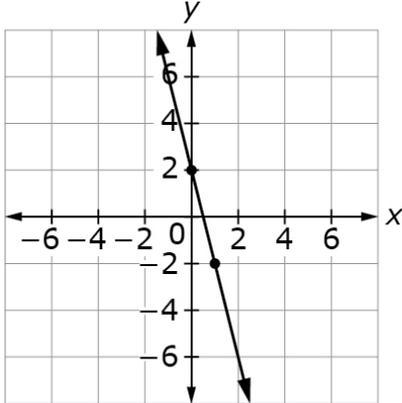
¹ For more information, refer to the General Accessibility Guidelines at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

² For more information about student accessibility resources and policies, refer to

http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

³ Items that use function notation may appear on the tests of high performing students as the adaptive algorithm allows for higher grade-level items to be selected.

| <p>Task Model 1</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 2</p> <p>8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>Evidence Required: 1. The student constructs a function to model a linear relationship between two quantities.</p> <p>Tools: Calculator</p> | <p>Prompt Features: The student is prompted to construct a linear function given a linear relationship between two quantities.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Tables should be labeled. • Graph scale should contain only integers. • Context should be familiar to students 13 to 15 years old. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ◦ Slopes can be positive or negative. ◦ Axes scales can be varied, including having different scales on each axis. ◦ Table values can be integers, fractions, or decimals. ◦ Graphs can include intercepts clearly plotted or any other points that are clearly marked. ◦ Proportional relationship with positive rate of change given. ◦ Linear relationship with positive or negative rate of change given. ◦ Linear relationship where student must find the rate of change. <p>TM1 Stimulus: The student is presented with a table of input and output values, a graph, or a verbal statement that represents a linear function.</p> <p>Example Stem 1: This table of values represents a linear function.</p> <table border="1" data-bbox="505 1163 695 1297"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>-6</td> </tr> <tr> <td>3</td> <td>-6.5</td> </tr> <tr> <td>8</td> <td>-9</td> </tr> </tbody> </table> <p>Enter an equation in the form $y = mx + b$ that represents the function.</p> <p>Example Stem 2: This graph represents a linear function.</p>  <p>Enter an equation in the form $y = mx + b$ that represents the function.</p> | x | y | 2 | -6 | 3 | -6.5 | 8 | -9 |
|--|--|-----|-----|---|----|---|------|---|----|
| x | y | | | | | | | | |
| 2 | -6 | | | | | | | | |
| 3 | -6.5 | | | | | | | | |
| 8 | -9 | | | | | | | | |

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| <p>Task Model 1</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 2</p> <p>8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>Evidence Required: 1. The student constructs a function to model a linear relationship between two quantities.</p> <p>Tools: Calculator</p> | <p>Example Stem 3: A swimming pool with 1600 gallons of water is emptied at a constant rate of 300 gallons every 2 hours.</p> <p>Enter an equation in the form $y = mx + b$ that represents the amount of water y, in gallons, remaining in the pool after x hours.</p> <p>Rubric: (1 point) Student enters the correct equation (e.g., $y = -0.5x - 5$; $y = -4x + 2$; $y = -150x + 1600$).</p> <p>Response Type: Equation/Numeric</p> |
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| <p>Task Model 2</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 1</p> <p>8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>Evidence Required: 2. The student determines the rate of change and initial value of a function, either from a description of a relationship or from two (x, y) values, including reading the rate of change and/or the value of the function from a table or a graph.</p> <p>Tools: Calculator</p> | <p>Prompt Features: The student is prompted to determine the rate of change or the initial value given a representation of a linear function as a table or graph.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Tables and graphs must be labeled. • Graph scale should contain only integers. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> Rate of change: <ul style="list-style-type: none"> ○ Input values increase by 1 unit on table or graph. Table values are whole numbers or integers. ○ Input values increase by multiples of 2, 3, or 5 on table or graph. ○ Table values are rational numbers. Values in decimal form should have only one digit after the decimal point. ○ Input values do not increase by the same number of units each time. Table values are integers. Initial value: <ul style="list-style-type: none"> ○ Table starts with an input value of 0. ○ Table does not show an input value of 0 Input values increase by 1 or by multiples of 2, 3, or 5 units on table or graph. Table values are rational numbers. ○ Values in decimals form should have only one digit after the decimal place. ○ Table does not show an input value of 0. Input values do not increase by the same number of units each time. Table values are integers. <p>TM2</p> <p>Stimulus: The student is presented with a table of values or a graph and asked to determine the rate of change or initial value.</p> <p>Example Stem 1: In this table, y is a linear function of x.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>50</td> </tr> <tr> <td>2</td> <td>40</td> </tr> <tr> <td>4</td> <td>30</td> </tr> <tr> <td>6</td> <td>20</td> </tr> </tbody> </table> <p>Enter the rate of change of this function.</p> <p>Rubric: (1 point) The student enters the correct rate of change (e.g., -5).</p> <p>Response Type: Equation/Numeric</p> | x | y | 0 | 50 | 2 | 40 | 4 | 30 | 6 | 20 |
|---|--|-----|-----|---|----|---|----|---|----|---|----|
| x | y | | | | | | | | | | |
| 0 | 50 | | | | | | | | | | |
| 2 | 40 | | | | | | | | | | |
| 4 | 30 | | | | | | | | | | |
| 6 | 20 | | | | | | | | | | |

Task Model 2

Response Type:
Equation/Numeric

DOK Level 1

8.F.B.4

Construct a function to model a linear relationship between two quantities.

Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Evidence

Required:

2. The student determines the rate of change and initial value of a function, either from a description of a relationship or from two (x, y) values, including reading the rate of change and/or the value of the function from a table or a graph.

Tools: Calculator

Example Stem 2: This table shows water level in a tank as a linear function of time.

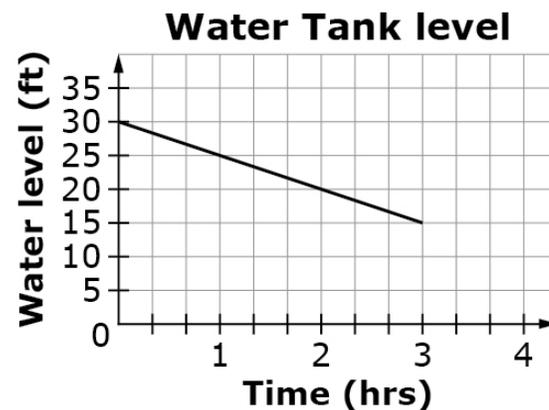
| Time (hr) | Water Level (ft) |
|-----------|------------------|
| 0 | 50 |
| 2 | 40 |
| 4 | 30 |
| 6 | 20 |

Enter the rate of change of the water level, in feet per hour.

Rubric: (1 point) The student enters the correct rate of change (e.g., -5).

Response Type: Equation/Numeric

Example Stem 3: This graph shows water level in a tank as a linear function of time.



Enter the initial water level, in feet, of the water tank.

Rubric: (1 point) The student enters the correct initial value (e.g., 30).

Note: Without other information that points to a specific value on the graph, allow for a range of responses (e.g., 29.5-30.5).

Response Type: Equation/Numeric

| <p>Task Model 3</p> <p>Response Type: Matching Tables</p> <p>DOK Level 2</p> <p>8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>Evidence Required: 3. The student interprets the rate of change and the initial value of a linear function in terms of the situation it models, its graph, or a table of values.</p> <p>Tools: Calculator</p> | <p>Prompt Features: The student is prompted to determine true statements regarding the rate of change and the initial value from a representation of the function using a table, graph, or verbal statement.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Tables and graphs must be labeled. • Graph scale should contain only integers. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> Rate of change: <ul style="list-style-type: none"> ○ Input values increase by 1 unit on table or graph. Table values are whole numbers or integers. ○ Input values increase by multiples of 2, 3, or 5 on table or graph. ○ Table values are rational numbers. Values in decimal form should have only one digit after the decimal point. ○ Input values do not increase by the same number of units each time. Table values are integers. Initial value: <ul style="list-style-type: none"> ○ Table starts with an input value of 0. ○ Table does not show an input value of 0. Input values increase by 1 or by multiples of 2, 3, or 5 units on table or graph. Table values are rational numbers. ○ Values in decimals form should have only one digit after the decimal place. ○ Table does not show an input value of 0. Input values do not increase by the same number of units each time. Table values are integers. <p>TM3</p> <p>Stimulus: The student is presented with a graph, table, or verbal description of a linear function.</p> <p>Example Stem: A swimming pool containing 1600 gallons of water is emptied at a constant rate of 300 gallons every 2 hours.</p> <p>Determine whether each statement about the amount of water in the pool is true. Select True or False for each statement.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="text-align: left;">Statement</th> <th>True</th> <th>False</th> </tr> </thead> <tbody> <tr> <td style="text-align: left;">The initial amount of water in the pool is 1600 gallons.</td> <td></td> <td></td> </tr> <tr> <td style="text-align: left;">The amount of water in the pool decreases by 150 gallons every 1 hour.</td> <td></td> <td></td> </tr> <tr> <td style="text-align: left;">The amount of water in the pool at 3 hours is 450 gallons.</td> <td></td> <td></td> </tr> </tbody> </table> <p>Rubric: (1 point) Student determines each statement as being either true or false (e.g., T, T, F).</p> <p>Response Type: Matching Tables</p> | Statement | True | False | The initial amount of water in the pool is 1600 gallons. | | | The amount of water in the pool decreases by 150 gallons every 1 hour. | | | The amount of water in the pool at 3 hours is 450 gallons. | | |
|--|---|-----------|------|-------|--|--|--|--|--|--|--|--|--|
| Statement | True | False | | | | | | | | | | | |
| The initial amount of water in the pool is 1600 gallons. | | | | | | | | | | | | | |
| The amount of water in the pool decreases by 150 gallons every 1 hour. | | | | | | | | | | | | | |
| The amount of water in the pool at 3 hours is 450 gallons. | | | | | | | | | | | | | |

Task Model 4

Response Type:
Matching Tables

DOK Level 2

8.F.B.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Evidence Required:
4. The student qualitatively describes the functional relationship between two quantities by analyzing a graph (e.g., whether the function is increasing or decreasing, whether the graph is linear or nonlinear).

Tools: Calculator

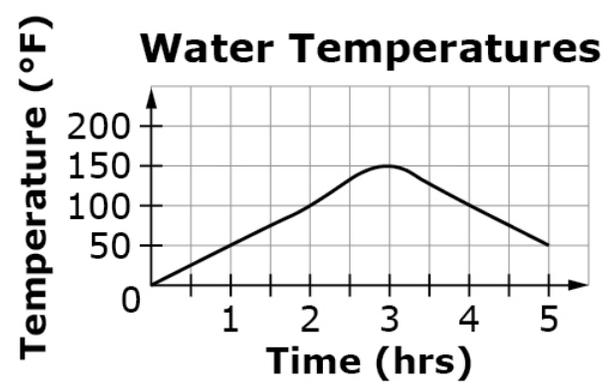
Prompt Features: The student is prompted to determine true statements that qualitatively describe the functional relationship between two quantities.

Stimulus Guidelines:

- Tables and graphs must be labeled.
- Functions should have constant, decreasing, and/or increasing segments.
- Context should be familiar to students 13 to 15 years old.

TM4
Stimulus: The student is presented with a graph of a function which contains linear and/or nonlinear segments.

Example Stem: This graph shows the water temperature as a function of time.



Based on the graph, determine whether each statement is true. Select True or False for each statement.

| Statement | True | False |
|--|------|-------|
| The water temperature is increasing between hour 1 and hour 2. | | |
| The water temperature is increasing between hour 3 and hour 4. | | |
| The water temperature is constant between hour 0 and hour 1. | | |

Rubric: (1 point) Student determines each statement as being either true or false (e.g., T, F, F). Each statement is a sentence describing the behavior of the function.

Response Type: Matching Tables

Task Model 5

Response Type:
Multiple Choice,
single correct
response

DOK Level 2

8.F.B.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Evidence

Required:
5. The student draws a graph that exhibits the qualitative features of a function that has been described in writing.

Tools: Calculator

Prompt Features: The student is prompted to identify a graph or statements about a graph that match a given qualitative description.

Stimulus Guidelines:

- Graph can contain increasing, decreasing, and constant linear and nonlinear segments.
- Context should be familiar to students 13 to 15 years old.
- Item difficulty can be adjusted via these example methods:
 - Student verbally describes the qualitative features exhibited on a graph.
 - Student identifies a graph that exhibits the features of a function that has been described verbally with three or more qualitative statements.

TM5a

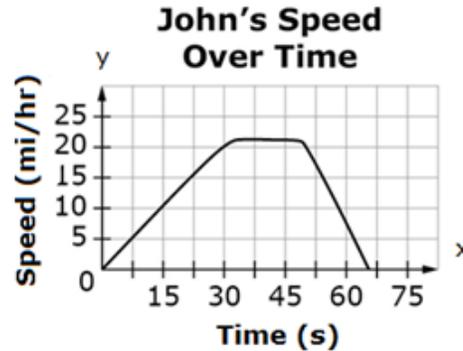
Stimulus: The student is presented with a description of a context or a graph of a function.

Example Stem: John is riding his bike.

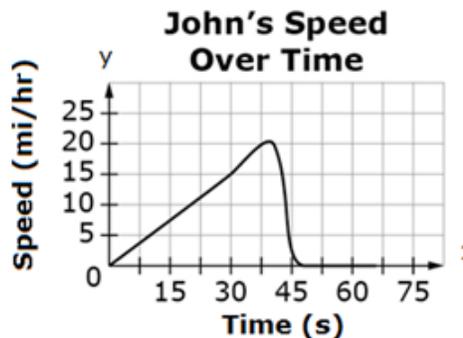
- He increases his speed for 30 seconds.
- He stays approximately the same speed for the next 20 seconds.
- He slows down to a stop during the last 15 seconds.

Select the graph that best represents John's speed over time.

A.



B.



Task Model 5

Response Type:
Multiple Choice,
single correct
response

DOK Level 2

8.F.B.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Evidence

Required:

5. The student draws a graph that exhibits the qualitative features of a function that has been described in writing.

Tools: Calculator

C.



D.



Answer Choices: Each answer choice should be a graph that matches the verbal description. Distractors should be incorrect graphs that do not match the verbal description.

Rubric: (1 point) The student selects the correct graph that represents John's speed over time (e.g., A).

Response Type: Multiple Choice, single correct response

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| <p>Task Model 5</p> <p>Response Type: Graphing</p> <p>DOK Level 1</p> <p>8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> <p>Evidence Statement: 5. The student draws a graph that exhibits the qualitative features of a function that has been described in writing.</p> <p>Tools: Calculator</p> <p>Accessibility Note: Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.</p> | <p>Prompt Features: The student is prompted to create a graph that has given characteristics such as increasing, decreasing, or constant segments.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Context can contain increasing, decreasing, and constant linear and nonlinear segments and should be familiar to students 13 to 15 years old. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ○ Three or more qualitative statements ○ Requires particular ordered pairs to be plotted. <p>TM5b Stimulus: The student is presented with a blank grid with x- and y-axes labeled and the description of the context.</p> <p>Example Stem: John is riding his bike.</p> <ul style="list-style-type: none"> • He increases his speed for 30 seconds. • He stays at the same speed for the next 20 seconds. • He slows down to a stop during the last 15 seconds. <p>Use the Connect Line tool to draw a graph that represents John’s speed over time.</p> <p>Interaction: The student uses Add Point and Connect Line tools to plot points in the coordinate plane and connects the points with lines. Delete tool will also be provided.</p> <p>Rubric: (1 point) The student correctly draws a graph that represents the relationship (e.g., see below).</p> <div style="text-align: center;"> <p>John’s Speed Over Time</p>  </div> <p>Response Type: Graphing</p> |
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