Grades 6-8 Mathematics Item Specification Claim 4	
"Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making." (p.72, CCSSM)	
Primary Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 4 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or Claim 3 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade with strong emphasis on the major work of previous grades.	
Secondary Content Domain(s): While items/tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NS domain in grades 6-8 can be used to construct higher difficulty items for the adaptive pool. The integration of the RP, EE, SP, F, and G domains with NS allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels	1, 2, 3, 4
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA), Fill-in Table (TI) No more than six choices in MS and MA items. Short Text –CAT items for Targets B, E and Performance Tasks **Scoring:** Scoring rules and answer choices will focus students' ability to use the appropriate reasoning. For some problems, multiple correct responses and/or strategies are possible. • MC and MS items will be scored as correct/incorrect (1 point). • If MA items require two skills, they will be scored as:

	o All correct choices (2 points); at least but less than all correct choices (1 point) o Justification[1] for more than 1 point **must be** clear in the scoring rules o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered correctly. • EQ, GI, and TI items will be scored as: o Single requirement items: will be scored as correct/incorrect (1 point) o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) o Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes:	CAT Items should take from 3 to 8 minutes to solve; Claim 4 items that are part of a performance task may take 5 to 15 minutes.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	CAT items/tasks generating evidence for Claim 4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 4. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 4. Distinguishing between Claim 4 and Claims 1 and 2: • In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution. • In Claim 2 problems are well posed, while in Claim 4 they may have extraneous or missing information. • In Claims 1 and 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations. • In Claim 1, data analysis is straightforward procedural. In Claim 4, the analysis should be tied to some useful purpose in the real-world. At least 80% of the items written to Claim 4 should primarily assess the standards and clusters listed in the table that follows.

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Grade 6	Grade 7	Grade 8
6.RP.A	7.RP.A	8.EE.A.3
6.NS.A	7.NS.A	8.EE.A.4
6.NS.C	7.EE.B	8.EE.B
6.EE.B	7.G.A*	8.EE.C
6.EE.C	7.G.B*	8.F.B*
6.G.A*	7.SP.A*	8.G.B
6.SP.A*	7.SP.B*	8.G.C*
6.SP.B*	7.SP.C*	8.SP.A*

* Denotes additional and supporting clusters

REMINDER: Claim 4 tasks may also ask students to apply content from prior grades in sophisticated applications.

Assessment Targets: Any given item/task should provide evidence for two or more Claim 4 assessment targets. Each of the following targets should not lead to a separate task. It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.
Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3) Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates. **Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)** Items that require the student to make decisions about the solution path needed to solve a problem are aligned with this target. Target B is not intended to be the primary target for an item, but should be a secondary, tertiary, or quaternary target for an item with primary alignment to other targets. **Target C: State logical assumptions being used. (DOK 1, 2)** Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Interpret results in the context of a situation. (DOK 2, 3)** Tasks used to assess this target should ask students to link their answer(s) back to the problem's context (See Claim 2, Target C, for further explication.) **Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)** Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data. Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future). Longer constructed-response items and extended performance tasks should be used to assess this target. **Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)**

Unlike Claim 2, where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be embedded in a larger context for items/tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4, Targets A, B, E, and G. **Target G*: Identify, analyze, and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)** Especially in extended performance tasks, students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed-response items should incorporate “hyperlinked” information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

*Measured in Performance Tasks only; functionality of linking to external resources is planned for future enhancements.

What sufficient evidence looks like for Claim 4 (Modeling and Data Analysis)[4]:

"A key feature of items and tasks in Claim 4 is that the student is confronted with a contextualized, or 'real world' situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, 'real world' situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical application context. In this way, items and tasks in Claim 4 differ from those in Claim 2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

"Items/tasks in Claim 4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim 4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim 1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower—normally met by content first taught in earlier grades, consistent with the emphases described under Claim 1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected-response and short constructed-response items will be appropriate to use.

"Modeling and data analysis in the Common Core State Standards trace a visible arc of growing prominence across the grades, showing low prominence in grades K-5, higher prominence in grades 6-8 (which is when the Statistics and Probability domain first appears), and highest prominence in High School (which is when Modeling appears as a content category with the full modeling cycle). Therefore to align to the Standards, Claim 4 will be more important on the assessment in high school, less important in grades 6-8, and the least important in grades 3-5. Again, to align to the Standards, Claim 4 tasks will be most sophisticated and complete in high school (cf. the modeling cycle in CCSSM pp. 72, 73), less sophisticated/more tied to specific content in middle school, and least sophisticated/most tied to specific content in grades 3-5."

[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 72-73).

Grade 6 Content Combinations:	**Primary emphases for Claim 4 Items: Ratios and Proportional Relationships, The Number System, and Expressions and Equations** **The following standards can be effectively used in various combinations in Grade 6 Claim 4 items:** **Ratios and Proportional Relationships (RP)** **6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, "If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?"* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities **The Number System (NS)** **6.NS.A: Apply and extend previous understanding of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ac/bd.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide*

	is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? **6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.C.7** Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $\|-30\| = 30$ to describe the size of the debt in dollars.* **d.** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **Expressions and Equations (EE)** **6.EE.B: Reason about and solve one-variable equations and inequalities.** **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine

	whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **6.EE.C: Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.* **Standards to integrate with the primary emphases:** **Geometry (G)** **6.G.A: Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

	Statistics and Probability (SP) **6.SP.A: Develop understanding of statistical variability.** **6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **6.SP.B Summarize and describe distributions.** **6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.B.5** Summarize numerical data sets in relation to their context, such as by: **a.** Reporting the number of observations. **b.** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **d.** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Grade 7 Content Combinations:	**Primary emphases for Claim 4 Items at Grade 7: Ratios and Proportional Relationships, The Number System, and Expressions and Equations** **The following standards can be effectively used in various combinations in Grade 7 Claim 4 items:** **Ratios and Proportional Relationships (RP)** **7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent

	ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate. **7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **The Number System (NS)** **7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.A.2** Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with a non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers.

	d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers. **Expressions and Equations (EE)** **7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.B.3** Solve multi-step, real-life, and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problems, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Give an inequality for the number of sales you need to make, and describe the solutions.* **Standards to integrate with the primary emphases:** **Geometry (G)** **7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the

	conditions determine a unique triangle, more than one triangle, or no triangle. **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right-rectangular prisms and right-rectangular pyramids. **7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Statistics and Probability (SP)** **7.SP.A Use random sampling to draw inferences about a population.** **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **7.SP.B Draw informal comparative inferences about two populations.** **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **7.SP.C Investigate chance processes and develop, use, and evaluate probability models.** **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

	7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. ***a.*** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* ***b.*** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.C.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. ***b.*** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. ***c.*** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Grade 8 Content Combinations:	**Primary emphases for Grade 8 Claim 4 Items: Expressions and Equations and Geometry** **The following standards can be effectively used in various combinations in Grade 8 Claim 4 items:** **Expressions and Equations (EE)** **8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.* **8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where

	both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.C.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.C.8** Analyze and solve pairs of simultaneous linear equations. **a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **b.** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line though the first pair of points intersects the line through and second pair.* **Geometry (G)** **8.G.B: Understand and apply the Pythagorean Theorem.** **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real–world and mathematical problems in two and three dimensions. **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

	Standards to integrate with the primary emphases **Functions (F)** **8.F.B: Use functions to model relationships between quantities.** **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **Geometry (G)** **8.G.C: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **Statistics and Probability (SP)** **8.SP.A Investigate patterns of association in bivariate data.** **8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Range ALDs – Claim 4 Grades 6 - 8	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a nonscaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to map, display, and identify relationships, use appropriate tools strategically, and apply mathematics accurately in everyday life, society, and the workplace. They should be able to interpret information and results in the context of an unfamiliar situation.
	Level 4 Students should be able to analyze and interpret the context of an unfamiliar situation for problems of increasing complexity and solve problems with optimal solutions.

Target 4A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 4A

- The student is asked to solve a problem arising in everyday life, society, or the workplace.
- Information needed to solve the problem has a level of complexity that is not present in items within Claim 2 Target A. For example, the student must
  - distinguish between relevant and irrelevant information, or
  - identify information that is not given in the problem and request it, or
  - make a reasonable estimate for one or more quantities and use that estimate to solve the problem.
- The student must select a mathematical model independently and is not directly told what arithmetic operation or geometric structure to use to solve the problem.
- Tasks in this model often have secondary alignments to other Claim 4 targets, in particular Target 4B, constructing autonomous chains of reasoning, Target 4D, requiring the student to interpret results in the context of the problem, and Target 4F, requiring students to identify quantities and map relationships between them.
- Problems in this model may have more than one possible solution.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have Depth of Knowledge (DOK) Level 2 or 3.

Task Model 4A.1

Task Expectations

- The student solves a multi-step problem involving the four operations with rational numbers or solving equations.
- The student identifies needed information and chooses which operations to perform or which equation to solve. The student may
  - ignore irrelevant information,
  - request or conduct research to find missing information,
  - identify constraints that are not explicitly stated, and/or
  - Make an estimate for one or more quantities and use that estimate to solve the problem.
- Example items from Task Model 4A.1 for Grades 3–5 may be adapted to this task model by increasing the complexity of the numbers involved and introducing rational numbers and decimal fractions.

Grades 6-8, Claim 4

Example Item 4A.1a (Grade 6)
Primary Target 4A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 4B, Quaternary Target 1A (CCSS 6.RP.A)

Juan has $7\frac{3}{4}$ cups of nuts. He wants to make either banana nut muffins or carrot muffins. The table shows how many cups of nuts are needed for each batch.

Amount of Nuts Needed Per Batch of Muffins

Muffin Type	**Amount of Nuts per Batch**
Banana nut	$\frac{1}{2}$ cup
Carrot	$\frac{5}{8}$ cup

Juan decided to make only carrot muffins. What is the maximum number of whole batches of carrot muffins Juan can make with $7\frac{3}{4}$ cups of nuts?

Enter your answer in the response box.

Rubric: (1 point) Student enters the correct number (12).

Response Type: Equation/Numeric

Commentary: The task could also ask about banana nut muffins, or about both for a 2-point item. A more cognitively demanding version of the task could ask how many whole batches can be made if he wants to make half banana nut and half carrot.

Example Item 4A.1a (Grade 6)
Primary Target 4A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 4B, Quaternary Target 4F

Hummingbirds drink nectar from flowers and sugar water from bird feeders.

- Sugar water is made by mixing 50 grams of sugar with 200 grams of water.
- A hummingbird's favorite flower nectar is 21% sugar by mass.

The amount of food a hummingbird eats at one time is always the same whether it eats sugar water or flower nectar.

Part A
Will the hummingbird get more sugar from a meal of sugar water made according to the recipe, or from an equal-sized meal of flower nectar? [Drop down choices: sugar water, flower nectar]

Part B
How much more sugar, in grams, would a hummingbird get from 4 grams of the [fills in with student's choice for the more sugary food type from part A] than from 4 grams of the [fills in with student's choice for the less sugary food type from part A]?

Interaction: Once the student selects the more sugary food type in part A, part B populates with the student's choice. The student can go back and change the choice in part A, in which case the statement of part B changes as well. Title the response box in Part B "Grams of sugar."

Rubric: (2 points) The student selects the more sugary food item (flower nectar) and identifies the additional amount of sugar correctly (0.04).
(1 point) The student identifies the food made by the recipe and enters the difference as 0.16, which corresponds to assuming the recipe is 25% sugar by weight (a likely mistake) but then correctly computing the difference.

Response Type: Drop Down Menu[5] and Equation/Numeric

Note: Functionality for this item type does not currently exist, although the item could be modified to work with current technology by making Part A a hot Spot (choose between "Recipe" and "Flower Nectar") and by wording Part B, "How much more sugar, in grams, would a hummingbird get from 4 grams of the option you chose in Part A than from 4 grams of the other option?"

[5] Drop-Down Menu response type is not currently available, but is a planned enhancement to the test-authoring tool by 2017.

Example Item 4A1.b (Grade 7)
Primary Target 4A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 4B, Quaternary Target 4D
[Adapted from Illustrative Mathematics task 50]

Alice, Raul, and Maria are baking cookies together.

They need $\frac{3}{4}$ cup of flour and $\frac{1}{3}$ cup of butter to make one batch of cookies.

They each brought the ingredients they had at home.

- Alice brought 2 cups of flour and $\frac{1}{4}$ cup of butter
- Raul brought 1 cup of flour and $\frac{1}{2}$ cup of butter
- Maria brought $1\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cups of butter.

Assume the students have plenty of the other ingredients (sugar, salt, baking soda, etc.) they need to make the cookies.

What is the maximum number of whole batches of cookies they can make with the ingredients they brought from home?

Enter your answer in the second response box.

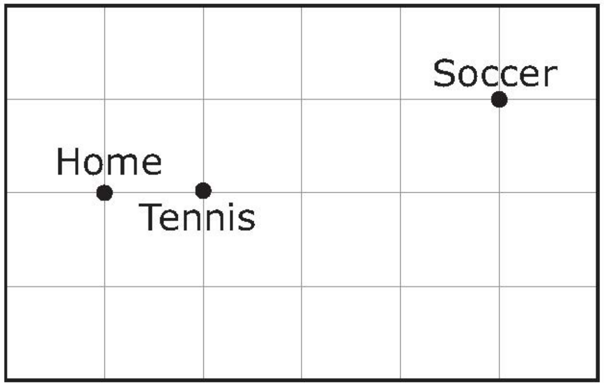
Response Type: Equation/Numeric

Commentary: Difficulty and grade level can be varied by varying the complexity of the numbers used. Item aligns with 4D because students must choose which fraction division limits the number of batches that can be made.

Grades 6-8, Claim 4

Example Item 4A.1c (Grade 6)
Primary Target 4A (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 4B, Quaternary Target 4F
Adapted from https://www.illustrativemathematics.org/illustrations/985

- Mrs. Jonas, her son Cody, and her daughter Laura drove from home to Cody's tennis practice.
- Mrs. Jonas then drove Laura to her soccer game and stayed to watch.
- After the game, mother and daughter picked up Cody from the tennis courts on the way home.
- Once home, Mrs. Jonas saw that they had driven 15 miles that day.



Mrs. Jonas took the shortest routes to and from each destination.

The figure shows the location of the Jonas family home, the tennis courts, and the soccer field. The gridlines in the figure represent the streets, and all distances between cross streets are approximately the same.

Part A:
Write an equation that can be used to find the distance, d, between the tennis courts and home.
Enter your answer in the first response box.

Part B:
What is the distance, in miles, between home and the tennis courts?
Enter your answer in the second response box.

Rubric: (2 points) Student correctly answers both parts ($10d = 15$, or $d + 4d + 4d + d = 15$ or equivalent equation for Part A; 1.5 or 1 ½ for Part B)

(1 point) Student correctly answers only one part.

Response Type: Equation/Numeric (Note: Label the two response boxes "Part A" and "Part B.")

Task Model 4A.2

Task Expectations

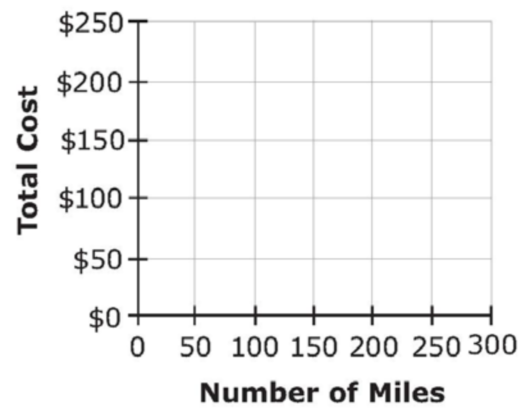
- The student solves a problem involving ratios, proportional relationships, or linear functions.
- The student identifies needed information and chooses the ratio, proportional relationship, or linear function required to complete the problem. The problem should require the student to do one of the following:
  - ignore irrelevant information,
  - request or conduct research to find missing information,
  - identify constraints that are not explicitly stated, or
  - make an estimate for one or more quantities and use that estimate to solve the problem.

Grades 6-8, Claim 4

Example Item 4A.2a (Grade 8)
Primary Target 4A (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 4D, Quaternary Target 4F

This table represents the cost of renting a truck from Moving Company X and Moving Company Y. Each company charges a one-time rental fee plus a charge for each mile driven.

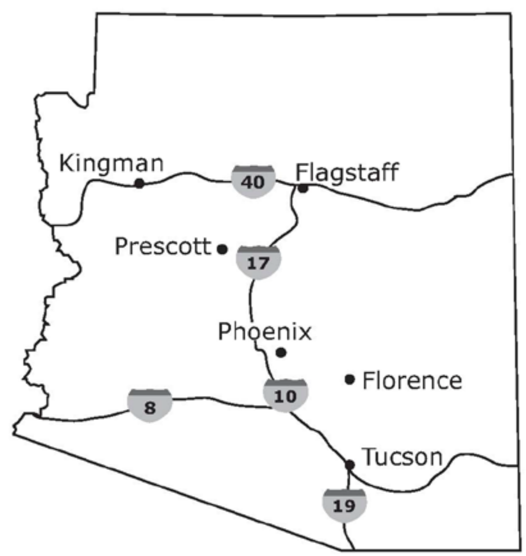
Moving Company	One-time Rental Fee	Charge per Mile
X	\$150	\$0.25
Y	\$ 50	\$0.75



Part A
Use the Add Arrow tool to graph two linear equations that represent the cost of using each moving company given a number of miles driven.

Part B
Select the moving company that will be the **least** expensive to move between the given cities. Refer to the map shown to determine the distances.

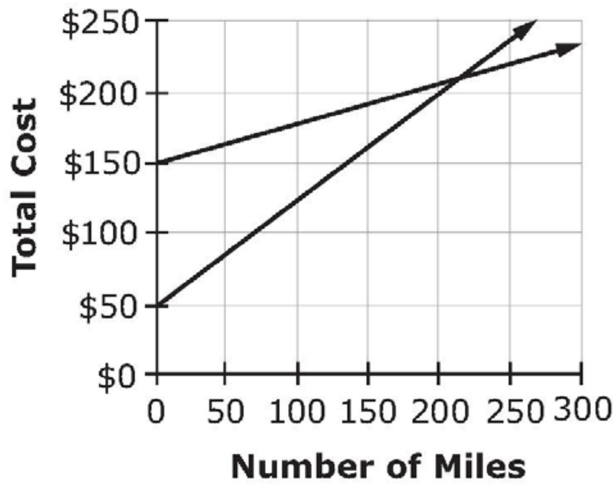
Cities	Company A	Company B
Tucson to Phoenix		
Phoenix to Flagstaff		
Tucson to Flagstaff		



Interaction: The student can use the ruler tool to measure distances on the map.

Rubric: Each part of this item is scored independently for a total of 2 points.
Part A (1 point) The student correctly graphs both functions.
Part B (1 point) The student selects the correct cells in the table.

Exemplar:



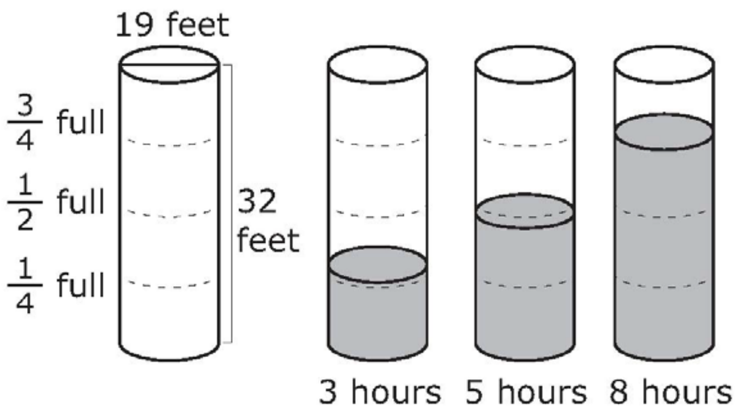
Cities	Company A	Company B
Tucson to Phoenix		(shaded)
Phoenix to Flagstaff		(shaded)
Tucson to Flagstaff	(shaded)	

Interaction: The Add Arrow tool will be available (with one arrow) to graph the lines, as well as Hot Spot to select the correct cells in the table. Also, the ruler tool needs to be active.

Response Type: Graphing and Hot Spot

Example Item 4A.2b (Grade 8)
Primary Target 4A (Content Domain G), Secondary Target 1I (CCSS 8.G.C), Tertiary Target 1A (CCSS 7.RP.A), Quaternary Target 4B

An empty tank in the shape of a cylinder is being filled with water. The tank is filled at a constant rate for a total of 10 hours. The figure shows the height of water in the tank at the given number of hours after filling started.



Enter the **percent** of the tank that is filled with water at 10 hours.

Rubric: (2 points) The student enters the correct numerical value for the percent (93.75–94).
(1 point) The student gives the height of water in the tank after 10 hours (30-30.1) OR the volume of water in the tank 10 hours (8500–8532), but forgets to find the percentage.

Response Type: Equation/Numeric (label the response box with %)

Commentary: The task can be done knowing only the information from the third picture (the height is 24 feet after 8 hours), so students who ignore extraneous information are rewarded. Notice that it is not necessary to compute the volume to find the percent, since it can be found by computing the ratio of the heights. Although it is not expected that many students will notice this, the task thus also rewards students with good modeling sense and geometric insight.

Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

General Task Model Expectations for Target 4B

- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

Target 4C: State logical assumptions being used.

General Task Model Expectations for Target 4C

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - identifies information or assumptions needed to solve the problem,
  - researches to provide information needed to solve the problem, or
  - provides a reasoned estimate of a quantity needed to solve the problem.

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model generally have either more information than is needed solve the problem (and students must choose) or not enough information (and students must make a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks for this target may also assess Target 4F.
- Tasks have DOK Level 1 or 2

Task Model 4C.1

Task Expectations:

- Student chooses from a list of possible assumptions, or makes an estimate, and then solves a problem using the assumption or estimate.

Example Item 4C.1a (Grade 7)
Primary Target 4C (Content Domain SP), Secondary Target 1I (CCSS 7.SP.C), Tertiary Target 4B, Quaternary Target 4D

Ramos flips a coin 100 times and records the results in a table.

Results of 100 Coin Flips

Outcome of Flip	Number of Times
Heads	74
Tails	26

Part A
Select an assumption about the outcome of a single flip of this coin [heads and tails are equally likely; heads are 3 times as likely as tails]

Part B
Based on your assumption, which would be the most likely outcome for the next **2** flips?

A.	two heads
B.	two tails
C.	one head and one tail

Interaction: The student must first select from the drop-down menu to make an assumption, and then select a correct option based on that assumption.

Rubric: (1 point) Student makes correct choice based on the assumption they choose (C for the first assumption, A for the second assumption).

Response Type: Drop-down Menu; Hotspot

Grades 6-8, Claim 4

Task Model 4C.2

Task Expectations:

- The student is given a problem with insufficient information and must indicate what information is needed to complete the solution to a problem.

Example Item 4C.2a (Grade 7)
Primary Target 4C (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 4F
[Adapted from Illustrative Mathematics task 1564.]

Chichén Itzá was a Mayan city in what is now Mexico. The picture shows El Castillo, also known as the pyramid of Kukulcán, which is located in the ruins of Chichén Itzá.

The pyramid is approximately 30 meters tall, and there are 91 steps leading up to a temple at the top.

What additional information do you need to know to estimate the height above the ground, in meters, of the 50th step? Select **all** that apply.

A. Each of the steps has approximately the same height.
B. The base of the pyramid is about 55 meters wide.
C. The height of the temple is about 6 meters.
D. The base of the pyramid is a square.

Rubric: (1 point) The student selects the correct options (A and C).

Response Type: Multiple Choice, multiple correct response

Target 4D: Interpret results in the context of a situation.

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer as well as many problems where students construct an equation or a graph.

General Task Model Expectations for Target 4D

- The student is presented with a problem situation in everyday life, society, or the workplace or a mathematical model of such a situation. The student interprets the solution to the problem in terms of the context, in terms of the model, or compares the results of the model with the real-world data it represents.
  - Item types with a primary alignment to 4D focus on interpreting results in terms of the model or comparing the results of the model with the real-world data it represents.
  - It is not necessary for a student to generate a complete solution for problems with a primary alignment to this target.
- Tasks in Targets 4A, 4C, 4E, and 4F frequently have this target as a tertiary or quaternary alignment because students must interpret their results in terms of the context.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2 or 3.

Task Model 4D.1

- The student is presented with a mathematical model of real-world data.
- The student interprets the solution to the problem in terms of the model or compares the results of the model with the real-world data it represents.

Example Item 4D.1a (Grade 8)
Primary Target 4D (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4C

This graph shows the average number of words a child can say from birth to 36 months.

Which statement is the **most accurate** description of the growth in the number of words a child speaks based on the graph shown?

A. Children learn to say new words at a steady rate starting about 12 months of age.
B. Children are constantly learning to say new words from the moment they are born.
C. Children learn to say new words more slowly during their second year than during their third year.
D. Children begin learning to say words around 24 months and stop learning to say new words at 36 months.

Rubric: (1 point) The student chooses the best interpretation of the graph (C).

Note: To distinguish from Claim 1 items, interpretations should extend beyond simply looking at the graph and should help to evaluate whether students understand which interpretations are defensible. Item authors should be careful with language not to "overstate" a particular conclusion since all data based interpretations are subject to some error.

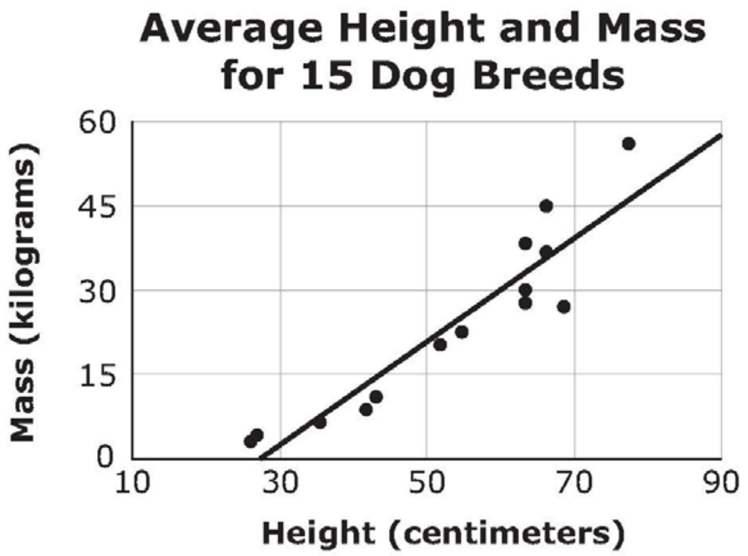
Response Type: Multiple Choice, single correct response

Example Item 4D.1b (Grade 8)
Primary Target 4D (Content Domain SP), Secondary Target 1J (CCSS 8.SP.A), Tertiary Target 4E

This scatter plot and line of best fit show the relationship between the height and mass of 15 different dog breeds.

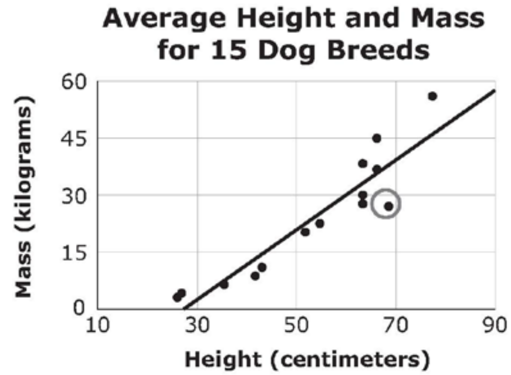
The mass of the Afghan Hound is less than would be predicted by the line of best fit, and the difference between the predicted mass and the actual mass is greater than for any other breed.

Click on the point in the scatterplot that corresponds to the Afghan Hound.



Rubric: (1 point) The student clicks the point that below and farthest away from the graph (see figure).

Response Type: Hot Spot



Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

General Task Model Expectations for Target 4E

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - Chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem)
  - Evaluates a partial or complete (possibly incorrect) solution to the problem
  - Constructs a mathematical model to solve the problem

  It is not necessary that a student to generate a complete solution for problems in this target.
- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task (not just a single step). For example, it might be necessary for the student to construct a two-step arithmetic expression to evaluate a model or solution, or to try out a geometric shape and then perform a calculation to see if it satisfies the requirements.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 3 or 4

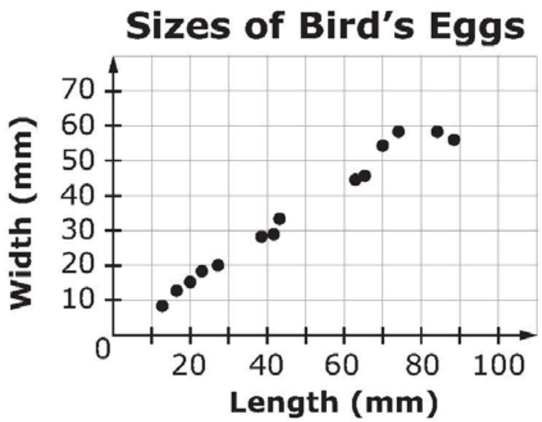
Task Model 4E.1

Task Expectations:

- Students construct an expression, equation, proportional relationship, linear function, or geometric figure that models a given problem.
- Models can be represented in symbolic or graphical form.
- The model is not explicitly given, but should be inferred from the situation.
- Students are expected to reason autonomously from a context to the model.

Example Item 4E.1a (Grade 8)
Primary Target 4E (Content Domain SP), Secondary Target 1J (CCSS 8.SP.A), Tertiary Target 4D, Quaternary 4B

This scatter plot shows the lengths and the widths (in millimetres) of the eggs of some American birds.



Use the information in the scatter plot to support each answer.

Part A
The scatter plot shows an association between the length of a bird egg and its width. Describe that association.

Part B
Fossils show that dinosaur eggs closely resemble the shape of bird eggs. One type of dinosaur (sauropods) grew from eggs that were 180 millimeters in length.

Assume that sauropod eggs were the same shape as bird eggs. What is the approximate width, in millimeters, of sauropod eggs? Explain how you determined your answer.

Rubric: (2 points) The student is able to answer both parts correctly and provide sufficient explanation/support for the answer to *Part B*.
(1 point) The student only answers one part correctly.

Exemplar[6]:
Part A: Typically, the greater the length of the egg, the greater the width.
Part B: The width is approximately 126 mm (accept values between 115 and 135 mm).
"I multiplied the length by about 0.7" or "The width is a little less than the length" or "I doubled the width of the egg that is 90 mm long."

Response Type: Short Text (handscored)

Example Item 4E.1b (Grade 8)
Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4F, Quaternary Target 4D

Cory is buying copper for a construction project. He pays $1.85 per pound of copper for the first 100 pounds. He pays $1.75 per pound of copper for every pound over 100 pounds. Cory calculated that it would cost $228.75 to purchase 125 pounds of copper. He wrote an equation that allows him to determine the cost of copper for any number of pounds of copper over 100 pounds.

His equation is in the form $y = n(x - 100) + p$ where y is the amount of money, in dollars, Cory pays for x total pounds of copper when x is greater than 100. What are his values for n and p?

Enter the value of n in the first response box.

Enter the value of p in the second response box.

Rubric: (1 point) The student enters the correct values for n and p (1.75 and 185).

Response Type: Equation/Numeric (Note: Label each response box n = [box], p = [box])

[6] An exemplar response represents only one possible solution. Typically, many other solutions/responses may receive full credit. The full range of acceptable responses is determined during rangefinding and/or scoring validation.

Grades 6-8, Claim 4

Task Model 4E.2

Task Expectations:

- The student chooses between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.
- The student assesses the fit of a particular model being used.

Example Item 4E.2a (Grade 8)
Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4F, Quaternary Target 4D
(Source: Adapted from Illustrative Mathematics 8-F Modeling with a Linear Function)

Select **all** situations that can be modeled by the linear equation $y = 2x + 5$.

A. There are initially 5 rabbits on a farm. Each month thereafter the number of rabbits is 2 times the number in the month before. How many rabbits are there after *x* months?

B. Joe earns $2 for each magazine sale. He also earns $5 for each hour he spends trying to sell magazines. How much money will he earn after selling magazines for *x* hours?

C. Sandy charges $2 an hour for babysitting. Parents are charged $5 if they arrive home later than scheduled. Assuming the parents arrived home late, how much money does she earn for *x* hours?

D. The Reader's Club is a members-only audio book rental store. There is a $2 sign-up fee and a $5 per audio book rental fee. How much would Laney owe on her first visit if she becomes a member and rents *x* audio books?

E. Andre is saving money for a new CD player. He began saving with a $5 gift and will continue to save $2 each week. How much money will he have saved at the end of *x* weeks?

Rubric: (1 point) The student identifies all situations modeled by the equation (C and E).

Response Type: Multiple Choice, multiple correct response

Example Item 4E.2b (Grade 8)
Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The table shows the relationship between the average number of hours students studied for a mathematics test and their average grade.

Hours Studied	Average Grade
0	62
1	78
2	85
5	74

Which type of function is most likely to model these data?

A. linear function with positive rate of change
B. linear function with negative rate of change
C. non-linear function that decreases then increases
D. non-linear function that increases then decreases

Rubric: (1 point) The student recognized the function most likely to model the data (D).

Response Type: Multiple Choice, single correct response

Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

Task Model 4F.1

Task Model Expectations

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Example Item 4F.1a (Grade 7)

Primary Target 4E (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 4F, Quaternary Target 4D

Megan has $2500. She spends money on the following:

- $800 on rent
- $400 on food
- $200 on utility services
- $250 on loan payments
- $$x$ on other expenses

Let y represent the amount of money in dollars Megan has left. Write an equation that represents the relationship between the amount of money Megan spends on other expenses and the amount of money Megan has left.

Rubric: (1 point) The student computes Megan's spending and represents the remaining money with an equation ($y = 850 - x$, or equivalent).

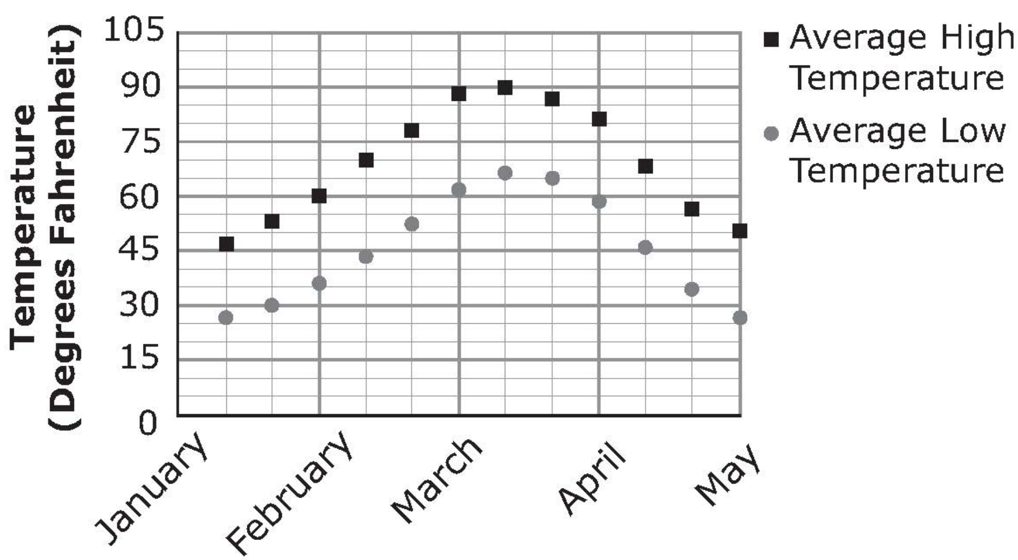
Response Type: Equation/Numeric

Grades 6-8, Claim 4

Example Item 4F.1b (Grade 6)
Primary Target 4F (Content Domain EE), Secondary Target 1G (CCSS 6.EE.C), Tertiary Target 4D

Part A
If you were going to plan a picnic, what temperature would you hope to have for the picnic?
Enter the temperature, in degrees Fahrenheit, you think would be best in the first response box. You may change your answer later if you wish.

Part B
The average monthly high and low temperatures for a town are shown in the graph below.



Select a month from the drop down menu where the temperature you chose would fall between the high and low temperatures for that month. [January, February,... December, no month will work]

Interaction: The student enters a temperature for a theoretical picnic in the first response box, then answers Part B with a drop down menu. The student can change his or her preferred temperature. The temperature a student chooses does not affect his or her score for the item except that the next choice must be consistent with it. When the student mouses over the points in the graph, the corresponding value appears (alternatively, there is a table of values as well).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average high in °F:	47	53	60	69	79	88	90	87	81	69	56	46
Average low in °F:	26	30	36	43	53	62	66	65	58	46	34	26

Rubric: (1 point) The student selects a month where the temperature he or she chose falls between the high and low temperatures for that (e.g., if the student selects 80, then they choose either June, July, August, or September).

Response Type: Equation/Numeric and Drop-down

Note: Functionality for this item type does not currently exist, but is planned for future enhancements to the item authoring tool in 2017.

Example Item 4F.1c (Grade 8)
Primary Target 4F (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The relationship between Jack's distance from home and the time since he left home is linear, as shown in the table.

Time (hrs)	Distance (mi)
0	7.5
2	17.5
4	27.5

Based on the values in the table, determine whether each statement is true. Select True or False for each statement.

Statement	True	False
Jack's initial distance from home is 7.5 miles.		
Jack's distance increases by 5 miles every 1 hour.		
Jack's distance from home at 3 hours is 23.5 miles.		

Rubric: (1 point) Student determines each statement as being either true or false (TTF).

Response Type: Matching Table