Grades 3–5 Mathematics Item Specification Claim 4	
"Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making." (p.72, CCSSM)	
Primary Claim 4: Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 4 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or Claim 3 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade with strong emphasis on the major work of previous grades.	
Secondary Content Domain(s): While tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NBT domain in grades 3–5 can be used to construct higher difficulty items for the adaptive pool. The integration of the OA, G, and MD domains with NBT allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels	1, 2, 3, 4

Allowable Response Types	**Response Types:**
	Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Table (MA); Fill-in Table (TI)
	
	No more than five choices in MS and MA items.
	
	Short Text – Performance tasks only
	
	Scoring:
	Scoring rules and answer choices will focus on a student's ability to use the appropriate reasoning. For some problems, multiple correct responses and/or strategies are possible.
	• MC and MS will be scored as correct/incorrect (1 point)
	• If MA items require two skills, they will be scored as:
	  o All correct choices (2 points); at least ½ but less than all correct choices. (1 point)
	  o Justification[1] for more than 1 point **must be** clear in the scoring rules.

[1] For a CAT item to score multiple points; either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

	o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly. • EQ, GI, and TI items will be scored as: o Single requirement items will be scored as correct/incorrect. (1 point) o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct. (1 point) o Justification for more than 1 point **must be** clear in the scoring rules.
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen the reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine Construct-Relevant Vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT Items should take from 3 to 6 minutes to solve. Claim 4 items that are part of a performance task may take 5 to 15 minutes to solve.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	CAT items/tasks generating evidence for Claim 4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 4. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 4. Distinguishing between Claim 4 and Claims 1 and 2: • In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution. • In Claim 2 problems are well posed, while in Claim 4 they may have extraneous or missing information. • In Claims 1 and 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations. • In Claim 1, data analysis is straightforward procedural. In Claim 4, the analysis should be tied to some useful purpose in the real-world. At least 80% of the items written to Claim 4 should primarily assess the standards and clusters listed in the table that follows.

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Grade 3	Grade 4	Grade 5
3.OA.A	4.OA.A	5.NBT.B
3.OA.D	4.NF.B	5.NF.A
3.MD.A	4.MD.A*	5.NF.B
3.MD.C	4.MD.B*	5.MD.A*
3.MD.D*	4.MD.C*	5.MD.B*
		5.MD.C
		5.G.A*

* Denotes additional and supporting clusters

REMINDER: Claim 4 tasks may also ask students to apply content from prior grades in sophisticated applications.

Assessment Targets: Any given item/task should provide evidence for two or more Claim 4 assessment targets. Each of the following targets should not lead to a separate task. It is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.
Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3) Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates. **Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)** Items that require the student to make decisions about the solution path needed to solve a problem are aligned with this target. Target B is not intended to be the primary target for an item, but should be a secondary, tertiary, or quaternary target for an item with primary alignment to other targets. **Target C: State logical assumptions being used. (DOK 1, 2)** Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Interpret results in the context of a situation. (DOK 2, 3)** Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. (See Claim 2, Target C for further explication.) **Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)** Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data. Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future). Longer constructed-response items and extended performance tasks should be used to assess this target. **Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)** Unlike Claim 2 where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be

embedded in a larger context for items/tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4 Targets A, B, E, and G.

Target G*: Identify, analyze, and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)
Especially in extended performance tasks, students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed-response items should incorporate "hyperlinked" information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

*Measured in Performance Tasks only; functionality of linking to external resources is planned for future enhancements.

What sufficient evidence looks like for Claim 4[4]:

"A key feature of items and tasks in Claim 4 is that the student is confronted with a contextualized, or 'real world' situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, 'real world' situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical application context. In this way, items and tasks in Claim 4 differ from those in Claim 2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

"Items/tasks in Claim 4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim 4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim 1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower—normally met by content first taught in earlier grades, consistent with the emphases described under Claim 1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected-response and short constructed-response items will be appropriate to use.

"Modeling and data analysis in the Common Core State Standards trace a visible arc of growing prominence across the grades, showing low prominence in grades K–5, higher prominence in grades 6–8 (which is when the Statistics and Probability domain first appears), and highest prominence in high school (which is when Modeling appears as a content category with the full modeling cycle). Therefore to align to the Standards, Claim 4 will be more important on the assessment in high school, less important in grades 6–8, and the least important in grades 3–5. Again, to align to the Standards, Claim 4 tasks will be most sophisticated and complete in high school (cf. the modeling cycle in CCSSM pp. 72, 73), less sophisticated/more tied to specific content in middle school, and least sophisticated/most tied to specific content in grades 3–5."

[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 74-75).

Grade 3 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 3 Claim 4 items:** **Primary emphases for Claim 4 Items at Grade 3: Operations and Algebraic Thinking and Measurement and Data** **Operations and Algebraic Thinking (OA)** **3.OA.A: Represent and solve problems involving multiplication and division.** **3.OA.A.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* **3.OA.A.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **3.OA.A.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] **3.OA.A.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?.* **3.OA.D: Solve problems involving the four operations, and identify and explain patterns in arithmetic.** **3.OA.D.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] **3.OA.D.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Measurement and Data (MD)** **3.MD.A: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects**. **3.MD.A.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by

	representing the problem on a number line diagram. **3.MD.A.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7] **3.MD.C: Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **3.MD.C.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.C.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **3.MD.C.7** Relate area to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. **d.** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **3.MD.D: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **3.MD.D.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Grade 4 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 4 Claim 4 items:** **Primary emphases for Claim 4 Items at Grade 4: Operations and Algebraic Thinking, Number and Operations—Fractions, and Measurement and Data** **Operations and Algebraic Thinking (OA)** **4.OA.A: Use the four operations with whole numbers to solve problems.** **4.OA.A.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] **4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Number and Operations—Fractions (NF)** **4.NF.B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** **4.NF.B.3** Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

	a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* **b.** Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Measurement and Data (MD)** **4.MD.A: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* **4.MD.A.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **4.MD.A.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **4.MD.B: Represent and interpret data.** **4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

	4.MD.C: Geometric measurement: understand concepts of angle and measure angles. **4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: **a.** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **b.** An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Grade 5 Content Combinations:	**The following standards can be effectively used in various combinations in Grade 5 Claim 4 items:** **Primary emphases for Grade 5 Claim 4 Items: Number and Operations—Base Ten, Number and Operations—Fractions, Measurement and Data, and Geometry** **Number and Operations—Base Ten (NBT)** **5.NBT.B: Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.B.5** Fluently multiply multi-digit whole numbers using the standard algorithm. **5.NBT.B.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.B.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

	Number and Operations—Fractions (NF) **5.NF.A: Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** **5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.B.5** Interpret multiplication as scaling (resizing), by: **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a

	fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **5.NF.B.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* **c.** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* **Measurement and Data (MD)** **5.MD.A: Convert like measurement units within a given measurement system.** **5.MD.A.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **5.MD.B: Represent and interpret data.** **5.MD.B.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **5.MD.C: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

	a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.C.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.C.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **b.** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **Geometry (G)** **5.G.A: Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.A.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.A.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Range ALDs – Claim 4 Grades 3-5	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a non-scaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to apply mathematics to solve unfamiliar problems arising in everyday life, society, and the workplace by identifying important quantities and mapping, displaying, explaining, or applying their relationship and by locating missing information from relevant external resources. They should be able to construct chains of reasoning to justify a model used, produce justification of interpretations, state logical assumptions, and compare and contrast multiple plausible solutions.
	Level 4 Students should be able to apply mathematics to solve unfamiliar problems by constructing chains of reasoning to analyze a model, producing and analyzing justification of interpretations, stating logical assumptions, and constructing and comparing/contrasting multiple plausible solutions and approaches.

Target 4A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 4A

- The student is asked to solve a problem arising in everyday life, society, or the workplace.
- Information needed to solve the problem has a level of complexity that is not present in items within Claim 2 Target A. For example, the student must
  - distinguish between relevant and irrelevant information, or
  - identify information that is not given in the problem and request it, or
  - make a reasonable estimate for one or more quantities and use that estimate to solve the problem.
- The student must select a mathematical model independently and is not directly told what arithmetic operation or geometric structure to use to solve the problem.
- Tasks in this model often have secondary alignments to other Claim 4 targets, in particular Target 4B, constructing autonomous chains of reasoning, Target 4D, requiring the student to interpret results in the context of the problem, and Target 4F, requiring students to identify quantities and map relationships between them.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have Depth of Knowledge Level 2 or 3.

Task Model 4A.1

Task Expectations

- The student solves a multi-step problem involving one or more of the four operations.
- The student identifies needed information and chooses which operations to perform. The student may
  - ignore irrelevant information,
  - request missing information, and/or
  - make an estimate for one or more quantities and use that estimate to solve the problem.
- Problems in this model may have a tertiary or quaternary alignment to 4B or 4D.
- Problems in this model may have more than one possible solution.

Example Item 4A.1a (Grade 3)
Primary Target 4A (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 4D, Quaternary Target 4F

Eva has 2 quarters, 4 dimes, and 6 nickels. She wants to buy a different gift for each of her 3 friends.

Click on the gifts in the table to show 3 gifts that Eva could buy.

Gift	Cost
Balloon	60 ¢
Eraser	35 ¢
Gumball	25 ¢
Kazoo	75 ¢
Mood ring	50 ¢
Pencil	35 ¢
Sticker	20 ¢

Rubric: (1 point) The student is able to identify three items whose total cost is less than $1 and 20¢. (e.g., Mood ring, pencil, and sticker).

Response Type: Hot Spot

Commentary: The item aligns to 4F because it requires that students identify the total amount of money that Eva has as a key quantity in solving the problem, and relate it to the prices of different items. Complexity of this item can be decreased by directly giving the total amount of money. If this is done, the alignment to 4F should be removed. The item can be varied by specifying that she wants to give the same gift to each of her friends, turning it into a multiplication problem. Complexity and grade level can be increased by increasing the amount of money she has, the prices of the objects, or the number of friends, so that 3-digit addition or multiplication is required. For larger numbers, other contexts might make more sense.

Example Item 4A.1b (Grade 4)
Primary Target 4A (Content Domain NBT), Secondary Target 1A (CCSS 4.OA.B), Tertiary Target 4B, Quaternary Target 4D

A bag of 5 apples at the grocery store has a mass of 825 grams. The largest apple has a mass of 185 grams.

What is a reasonable estimate for the mass, in grams, of the smallest apple in the bag? Select Yes for each reasonable mass and No for each mass that is **not** reasonable.

	Yes	No
50 grams		
100 grams		
150 grams		
200 grams		

Rubric: (1 point) The student selects numbers that are reasonable estimates for the mass of the smallest apple. The student could select just 150 since an argument can be made that if the apples are fairly similar in size, then 150 is the only reasonable estimate, but if they vary a lot, then 100 would be reasonable as well. 200 would not be possible as that is larger than the largest apple, and 50 is not possible because that would require at least one other apple to be 197 grams. (There are three correct response patterns: {100}, {150}, or {100, 150}).

Response Type: Matching Table

Example Item 4A.1c (Grade 5)
Primary Target 4A (Content Domain MD, NBT), Secondary Target 1E (CCSS 4.MD.A, 4.NBT.B), Tertiary Target 4B, Quaternary Target 4F

How many minutes are in 1 day?	[Click here for more information if you need it]

Interaction: If the student clicks for more information, they get the following conversion data[5]:

- There are 60 seconds in 1 minute
- There are 60 minutes in 1 hour
- There are 24 hours in 1 day
- There are 7 days in 1 week
- There are 52 weeks in 1 year

Rubric: (1 point) The student enters the correct number of minutes (1440).

Response Type: Equation/Numeric (label the response box with minutes)

Commentary: This item requires students to recognize which quantities are of interest (minutes, hours, and days) and then identify the relationship between them. Identifying these different quantities and mapping their relationships draws on the skill set identified in Target 4F.

Example Item 4A.1d (Grade 5)
Primary Target 4A (Content Domain NBT), Secondary Target 1E (CCSS 5.NBT.B), Tertiary Target 4B, Quaternary Target 4D

> A parking meter accepts nickels, dimes, and quarters. It holds up to 1500 coins.
>
> Estimate the value of the coins, in dollars, in the meter when it is full.

Rubric: (1 point) The student enters a reasonable estimate (a multiple of 5 between 75 and 375).

Response Type: Equation/Numeric

[5] The ability to pull up information interactively is not currently available, but part of the plan for enhancements to the item-authoring system in 2017.

Example Item 4A.1e (Grade 5)
Primary Target 4A (Content Domain NF), Secondary Target 1I (CCSS 5.MD.C), Tertiary Target 4F

Gina is making cookies. The last three steps used to make the cookies are shown.

Step 5: Roll the dough into balls that are $\frac{1}{2}$-inch wide.

Step 6: Place the balls on a baking tray 2 inches apart.

Step 7: Bake for 12 minutes.

This recipe makes 18-24 cookies

Gina plans to
- give cookies to 9 people;
- give each person 3 cookies; and
- have no extra cookies remaining.

Which action will help Gina get closest to the exact number of cookies she needs?

A. Place the cookies 3 inches apart.
B. Bake the cookies for only 10 minutes.
C. Roll the cookies slightly larger than $\frac{1}{2}$-inch wide.
D. Roll the cookies slightly smaller than $\frac{1}{2}$-inch wide.

Rubric: (1 point) The student correctly determines which action will help Gina get closest to the exact number of cookies (D).

Response Type: Multiple Choice, single correct response

Example Item 4A.1f (Grade 3)
Primary Target 4A (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 4F, Quaternary Target 1D (CCSS 3.MD.A)

Jenny went to the store to buy 15 bottles of water.

- The bags at the store can each hold 6 kilograms.
- The bottles of water each weigh 2 kilograms.
- Jenny bought 15 bottles of water.

What is the fewest number of bags that Jenny needs to hold all 15 water bottles?

Rubric: (1 point) The student enters the smallest number of bags needed (5).

Response Type: Equation/Numeric

Example Item 4F.1a (Grade 3)
Primary Target 4A (Content Domain MD), Secondary Target 1G (CCSS 3.MD.1), Tertiary Target 4F

The table shows the start and end times for runners in a race.

Racing Times		
Runner	**Start Time**	**End Time**
Mike	12:03 p.m.	12:26 p.m.
Ann	12:10 p.m.	12:17 p.m.
John	12:13 p.m.	12:19 p.m.
Patty	12:16 p.m.	12:25 p.m.

What is the difference, in minutes, between Patty’s start time and Mike’s start time?

Rubric: (1 point) The student enters the correct difference (13).

Response Type: Equation/Numeric

Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

General Task Model Expectations for Target 4B

- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

Target 4B is assessed in conjunction with Target 4A, 4C, and 4E.

Target 4C: State logical assumptions being used.

Task Model 4C.1

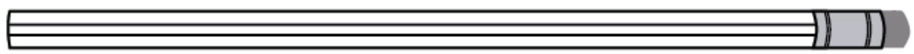
Task Expectations:

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
    - identifies information or assumptions needed to solve the problem or
    - provides a reasoned estimate of a quantity needed to solve the problem.

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model generally have either more information than is needed solve the problem (and students must choose) or not enough information (and students must make a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks for this target may also assess Target 4B or 4D.
- Tasks have DOK Level 1 or 2

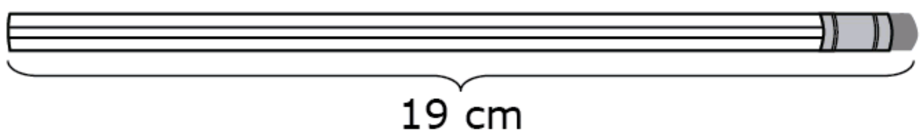
Example Item 4C.1a (Grade 3)
Primary Target 4C (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D, 2.MD.A), Tertiary Target 4D, Quaternary Target 4E

Part A
Estimate the length of this unsharpened pencil, in centimeters. [   ]



Enter your estimate in the response box.

Part B
The length of the pencil is about 19 cm.



How much longer or shorter is your estimate than the real length? [   ]

Enter your answer in the response box.

Interaction: The student must enter an estimate for the length of the pencil before seeing the actual length and cannot change it once the actual length is shown. The student's estimate does not factor into the score he or she receives.

Rubric: (1 point) The student finds the difference between their estimate, *a*, and the actual length of the pencil ($|19-a|$).

Response Type: Equation/Numeric

Note: Functionality for this item type does not currently exist, but is planned for future enhancements.

Commentary: This item type is new and may be unfamiliar to item writers and is designed to activate a particular practice which is important in mathematical modeling. Students are often required to make an estimate as one of the logical assumptions on which they will base a mathematical model. In grades 3-5, students are learning how to make reasoned estimates by first developing the habit of making their best estimate (without penalty) and then reflecting on the accuracy of their estimate. The difference between items in this task model and Task Model 4E.3 is that the emphasis here is on making and reflecting on the accuracy of the estimate and the emphasis in Task Model 4E.3 is on making and revising the estimate.

Example Item 4C.1b (Grade 5)
Primary Target 4C (Content Domain NF), Secondary Target 1H (CCSS 4.NBT.A), Tertiary Target 4D, Quaternary Target 4E

Part A
A liter is more than a cup. Estimate the number of liters in a cup. You can use the picture to help you make an estimate.

Enter your estimate, in liters, in the response box. [ ]

Part B
There are about 0.24 liters in one cup. How much greater or less than your estimate is the real amount?

Enter the difference in the response box. [ ]

Interaction: The student must select an estimate for the number of liters in a cup before seeing the actual value and cannot change it once the actual value is shown. The students' estimate does not factor into the score he or she receives.

Rubric: (1 point) The student finds the difference between their estimate, *a*, and the actual number of liters (|19-*a*|).

Response Type: Equation/Numeric

Example Item 4C.1c (Grade 4)
Primary Target 4C (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A), Tertiary Target 4B, Quaternary Target 4F

Sarah is helping her dad make cookies for her class using a recipe they found online. Her dad asks, “Do you think one batch of cookies will be enough?” Select **all** of the information they need to answer the question.

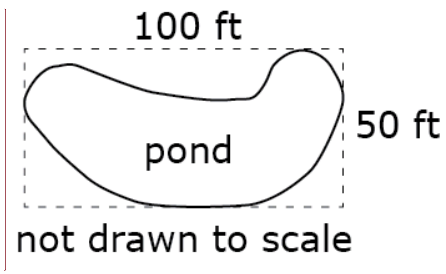
A. The amount of flour in the recipe.
B. The number of cookies in one batch.
C. The number of students in the class.
D. The temperature of the oven for baking the cookies.
E. The number of cookies you can fit onto a cookie sheet.

Rubric: (1 point) The student selects the correct pieces of information (B and C).

Response Type: Multiple Choice, multiple correct response

Example Item 4C.1d (Grade 4)
Primary Target 4C (Content Domain MD), Secondary Target 1I (CCSS 4.MD.3), Tertiary Target 4B, Quaternary Target 4D

Liam uses string to form a rectangle with length 100 feet and width 50 feet to estimate the area of a small pond.



Enter an estimate for the area of the pond in square feet in the response box. [ ]

Select a statement that supports your estimate:

A. The area of the rectangle is bigger than the area of the pond.
B. The area of the rectangle is smaller than the area of the pond.
C. The distance around the rectangle is bigger than the distance around the pond.
D. The distance around the rectangle is smaller than the distance around the pond.

Rubric: (1 point) The student enters a reasonable estimate and selects the supporting reason (a number between 2500 and 5000; A).

Response Type: Equation/Numeric; Multiple Choice, single correct response[6]

Note: Currently can be formatted as a Drag and Drop and Hot Spot.

[6] This combination of item types is currently not supported, but is planned for future enhancements to the item-authoring tool.

Target 4D: Interpret results in the context of a situation.

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer. Note that in Grades 3-5, Target 4D is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E. In later grades, students interpret more complex mathematical objects (like equations and graphs) in more sophisticated contexts.

General Task Model Expectations for Target 4D

- The student must solve a problem that results in a numerical answer and interpret the number in the context of the problem.

In Grades 3-5, Target 4D is assessed in conjunction with Target 4A, 4C, and 4E.

Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

General Task Model Expectations for Target 4E

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
    - chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem), or
    - evaluates a partial or complete (possibly incorrect) solution to the problem, or
    - constructs a mathematical model to solve the problem

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task (not just a single step). For example, it might be necessary for the student to construct a two-step arithmetic expression to evaluate a model or solution, or to try out a geometric shape and then perform a calculation to see if it satisfies the requirements.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2, 3, or 4

Task Model 4E.1

Task Expectations:

- Students construct a geometric figure, a numerical expression, or a numerical equation that models a given problem.
- Students may or may not perform a multi-step numerical calculation to verify that the model solves the problem.
- The operations to be performed should not be explicitly given, but should be inferred from the situation.
- Students are expected to reason autonomously from a context to the figure, expression, or equation.
- Difficulty and grade level maybe be varied by varying the types of numbers used (whole numbers, fractions, decimals), the complexity of the geometric figure (square, rectangle, triangle, polygon), the complexity of the numerical expression or equation (number of steps to build it up), whether or not it is required to perform a numerical calculation to complete the task.

Example Item 4E.1a (Grade 3)
Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 3.OA.A), Tertiary Target 4F

Tina has 4 packs of gum. Each pack has the same number of pieces of gum. Altogether there are 60 pieces of gum.

Part A
Make an equation to find the number of pieces of gum in each pack. Use *n* for the number of pieces in each pack.

Part B
How many pieces of gum are in each pack?

Rubric: (2 points) One point for a correct answer to each part. For Part A, the student enters a correct equation (e.g., $n=60\div4$, $4 \times n = 60$, $4 = 60 \div n$). For Part B, the student enters the correct number (15).

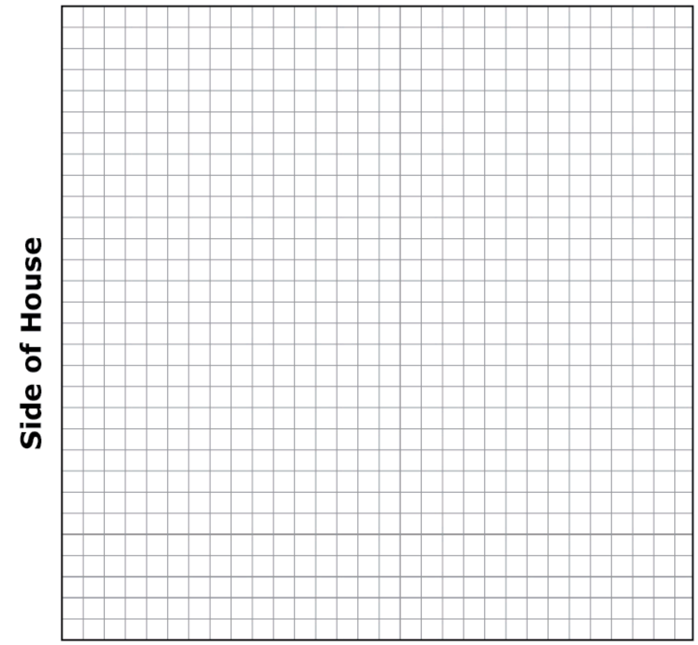
Response Type: Equation/Numeric (2 response boxes; label them *Part A* and *Part B*)

Example Item 4E.1b (Grade 4)
Primary Target 4E (Content Domain MD), Secondary Target 1I (CCSS 4.MD.3), Tertiary Target 4F

Tyra wants to enclose a section of her lawn for her dog to be able to have an outdoor play area. She knows that if she uses the side of her house as one side of the play area, her dog will have a larger outdoor play area. Tyra's plan for the play area includes the following:

- It will be in the shape of a rectangle.
- The side of the house will be used as one side of the rectangular area.
- She will use exactly 24 feet of fence material to enclose the play area.
- The length and width of the enclosure will be a whole number of feet.
- She wants the play area to be greater than 60 square feet.

Use the Connect Line tool to create a rectangular play area that meets Tyra's plan.



Key
☐ = 1 square foot

Rubric: (2 points) The student is able to construct a 4 by 16, 5 by 14, 6 by 12, 7 by 10, or 8 by 8 rectangle using the side of the house for the longer side.
(1 point) Partial credit is possible for constructing a rectangle that uses exactly 24 feet of fencing, but doesn't reflect using the side of the house as one of the sides, nor the area being greater than 60 square feet (e.g., 1 by 11, 2 by 10, 3 by 9, 4 by 8, 5 by 7, or 6 by 6).

Response Type: Graphing

Example Item 4E.1c (Grade 5)

Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 5.NBT.B), Tertiary Target 4F

A school spends $2.40 on every lunch it serves in the cafeteria and $0.30 for each carton of milk.

- 250 people at the school get a lunch each day
- 120 people take a carton of milk

Create an expression using this information that shows how much the school spends altogether on lunches and milk each day.

Rubric: (1 point). Student constructs a correct numerical expression (250 x 2.40 + 120 x 0.30 or its equivalent).

Response Type: Equation/Numeric

An alternate (easier) version of the problem above:

A school spends $2.40 on every lunch it serves in the cafeteria and $0.30 for each carton of milk.

- 250 people at the school get a lunch each day
- 120 people take a carton of milk

Which expression represents the amount of money the school spends altogether on lunches and milk each day?

A. 250 x 2.40 + 120 x 0.30
B. 250 x 0.30 + 120 x 2.40
C. 250 x (2.40 + 0.30)
D. 120 x (2.40 + 0.30)

Rubric: (1 point). Student selects the correct numerical expression (A).

Response Type: Multiple Choice, multiple correct response

Task Model 4E.2

Task Expectations:

- Students choose between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.

Example Item 4E.2a (Grade 3)

Primary Target 4E (Content Domain OA), Secondary Target 1D (CCSS 3.OA.D), Tertiary Target 4B

A large water jug holds 24 liters of water. Nan uses it for her animals.

- Nan fills her animals' water dish 2 times each day.
- She puts the same amount of water in the dish every time.
- She uses all of the water in 3 days.

Which equation can be solved to find the number of liters of water (n) she puts in the dish each time?

A. $3 \times 2 + n = 24$
B. $3 + 2 + n = 24$
C. $3 + 2 \times n = 24$
D. $3 \times 2 \times n = 24$

Rubric: (1 point) The student selects the correct equation (D).

Response Type: Multiple Choice, single correct response

Example Item 4E.2b (Grade 3)
Primary Target 4E (Content Domain OA), Secondary Target 1D (CCSS 3.OA.8), Tertiary Target 4B, Quaternary Target 4F

There are 123 girls and 135 boys in the third grade at a school. Today there are 9 third grade students absent.

Which equation can be used to find the total number of third grade students (s) in school today?

A. $123 + 135 = s$
B. $135 - 9 = s$
C. $123 + 135 + 9 = s$
D. $123 + 135 - 9 = s$

Rubric: (1 point) The student selects the correct equation (D).

Response Type: Multiple Choice, single correct response

Example Item 4E.2c (Grade 4)
Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 4.OA.1), Tertiary Target 4B, Quaternary Target 4D

Which situation is represented by the equation $4 \times 3 = \square$?

A. A kitten weighs 4 pounds. A puppy weighs 3 times as much as the kitten.
How much does the puppy weigh?

B. A kitten weighs 4 pounds. A puppy weighs 3 pounds more than the kitten.
How much do they weigh altogether?

C. A kitten weighs 4 pounds. A puppy weighs 3 pounds more than the kitten.
How much does the puppy weigh?

D. A kitten weighs 4 pounds. A puppy weighs 3 times as much as the kitten.
How much do they weigh altogether?

Rubric: (1 point) The student correctly identifies the context that represents the multiplication equation as a multiplicative comparison (A).

Response Type: Multiple Choice, single correct response

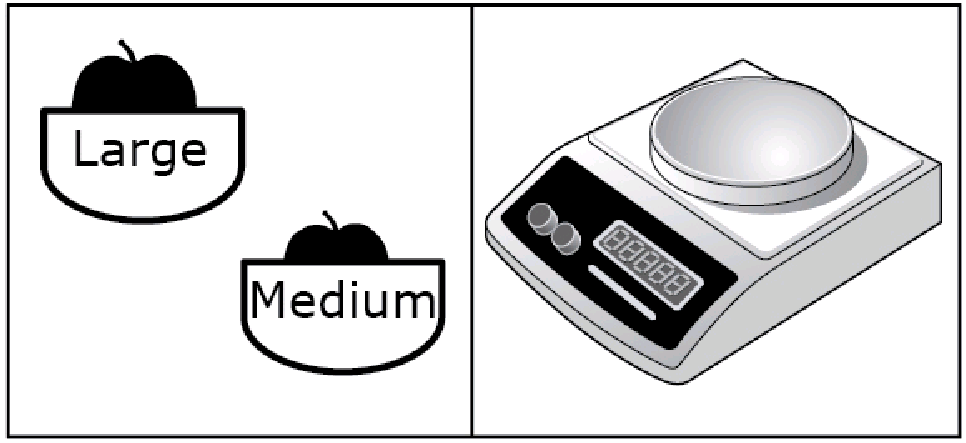
Example Item 4E.2d (Grade 5)
Primary Target 4E (Content Domain NBT), Secondary Target 1 (CCSS 5.NBT.B), Tertiary Target 4D, Quaternary Target 4F

Molly and Sam need about 2 pounds of apples for a pie. Medium apples cost $0.45 each. Large apples cost $0.65 each.

Molly says: “Let’s buy the medium apples, they are less expensive.”

Sam says: “I think it’s less expensive to buy large apples. They are more expensive but we won’t have to buy as many of them.”

Analyze both approaches. You can use the scale to weigh the apples.



Use the drop down menus to complete each statement.

Statement A:
Molly and Sam would need [1, 2, 3, 4, 5, 6, 7, 8] medium apples or [1, 2, 3, 4, 5, 6, 7, 8] large apples for the pie.

Statement B:
The number of medium apples that would be needed cost [more, less] than the number of large apples that would be needed. So [Molly, Sam] is correct.

Interaction: The student can drag apples one at a time onto the scale from bins labeled “Large” or “Medium” to get the weight in pounds, to the nearest $\frac{1}{8}$ pound. The scale should give weights as mixed numbers, in eighths of a pound. 6 medium apples should weight $2\frac{1}{8}$ pounds, 4 large apples should weigh $2\frac{1}{4}$ pounds. Reducing the number of apples by one should give a weight which is less than 2 pounds and not as not close to 2 pounds (e.g. $1\frac{3}{4}$ for 5 medium apples and $1\frac{5}{8}$ for 3 large apples).

Rubric: (2 points) The student selects the correct numbers and words in all of the drop-down menus (6, 4, more, Sam)

(1 point) Student identifies the correct number of each size of apple needed but does not compare their costs correctly or identify the right reasoning, or the numbers of apples are different but their cost is correctly compared and the correct conclusion is made about who is correct in their reasoning based on the numbers the student chose.

Response Type: Drop-Down Menu[7]

[7]\Drop-Down Menu response type is not currently available, but is a planned enhancement to the test-authoring tool by 2017.

Grades 3–5, Claim 4

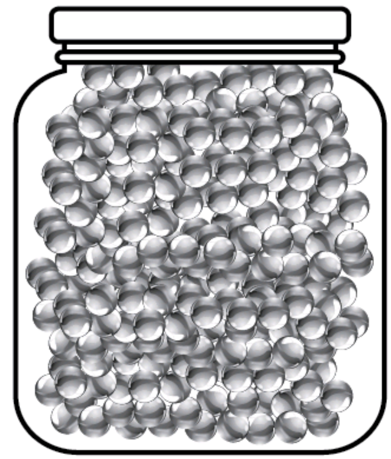
Task Model 4E.3

Task Expectations:

- The student makes estimates to solve a problem and then has a chance to improve the estimates.

Example Item 4E.3a (Grade 5)
Primary Target 4E (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A), Tertiary Target 4B, Quaternary Target 4D

Lilian wants to estimate the number of marbles in a glass jar that has a mass of 2.3 kilograms when it is full.



Part A:
Make an estimate for the mass of a single marble, in grams.
Enter your estimate in the response box. [ ]

Make an estimate for the mass of the jar, in grams.
Enter your estimate in the response box. [ ]

Estimate the number of marbles in the jar based on the assumptions you made.
Enter your estimate in the response box. [ ]

Part B:
The jar has a mass of about 500 grams and there are about 600 marbles in the jar. Which of the following estimates is closest to the actual mass of a single marble?

A. 2 grams
B. 20 grams
C. 200 grams
D. 1200 grams

Interaction: The student enters values for the mass of a single marble and the mass of the jar. The student's choices do not factor into the score he or she receives as long as the estimate for the number of marbles is consistent with those estimates. The student has to make those estimates before moving on to Part B.

Rubric: (2 points) The student estimates the mass of a single marble *m* and the mass of the jar *b*, and makes an estimate of the number of marbles in the jar that is consistent with the initial estimates [e.g., (2300-*b*)/*m* +/-50, rounded to a whole number] and then selects the best estimate from the choices given (A).

(1 point) The student makes an estimate for the number of marbles that is consistent with his/her estimated masses in Part A or selects the best estimate from the choices given in Part B.

Response Type: Equation/Numeric and Multiple Choice, single correct response

Note: Functionality for this item type does not currently exist.

Commentary: This item type is new and may be unfamiliar to item writers and is designed to activate a particular practice which is important in mathematical modeling. In grades 3-5, students are learning how to make reasoned estimates by first developing the habit of making their best estimate (without penalty) and then revising their estimate when more information is known. The difference between items in this task model and Task Model 4C.1 is that the emphasis here is on making and revising the estimate and the emphasis in Task Model 4C.1 is on making and reflecting on the accuracy of the estimate.

Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

Task Model 4F.1

Task Model Expectations

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Example Item 4F.1a (Grade 3)

Primary Target 4F (Content Domain MD), Secondary Target 1G (CCSS 3.MD.1), Tertiary Target 4A

The table shows the start and end times for runners in a race.

Racing Times		
Runner	**Start Time**	**End Time**
Mike	12:03 p.m.	12:26 p.m.
Ann	12:10 p.m.	12:17 p.m.
John	12:13 p.m.	12:19 p.m.
Patty	12:16 p.m.	12:25 p.m.

What is the difference, in minutes, between Patty's start time and Mike's start time?

Rubric: (1 point) The student enters the correct difference (13).

Response Type: Equation/Numeric (label the response box with minutes)