BUILDING
MULTIPLICATION
FACT FLUENCY

Presented by Julie Joseph
Tulare County Office of Education
November 7, 2015
GOALS

• Develop all students’ fluency with multiplication and division facts through well orchestrated instructional strategies.

• Learn how students develop understanding and fluency through activities and games to use in your classroom throughout the year.
AGENDA

• Research
• Games and Activities
• Assessment
WHAT IS FLUENCY?

IN WHAT WAYS DO YOU DEVELOP AND ASSESS STUDENT FLUENCY?
STANDARD 3.OA.7

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
WHAT IS FLUENCY IN MATHEMATICS?

“According to CCSSM, fluency is “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO 2010, p. 6). Thus, far from just being a measure of speed, fluency with multiplication facts involves flexibly and accurately using an appropriate strategy to find the answer efficiently.”

Procedural Fluency

Knowing from Memory ≠ Memorization
CCSSM Standards for Fluency

K
- K.OA.5: Add/subtract within 5

1st
- 1.OA.6: Add/subtract within 10

2nd
- 2.OA.2: Single-digit sums and differences (sums by memory by end of grade)
- 2.NBT.5: Add/subtract within 100

3rd
- 3.NBT.2: Add/subtract within 1000

4th
- 4.NBT.4: Add/subtract within 1,000,000

3rd
- 3.OA.7: Single-digit products and quotients (products by memory by end of grade)

5th
- 5.NBT.5: Multi-digit multiplication

6th
- 6.NS.2: Multi-digit decimal operations
Fluency Without Fear: Research Evidence on the Best Way I Learn Math Facts

By Edie Szekely
Professor of Mathematics Education, co-founder youcubed, & Assistant Professor of Mathematics Education, Stanford University

Introduction

A few years ago, a British politician, Stephen Twigg, made a bold statement in an interview with the BBC: “We need to give the children in this country the papers to do 7×8.” And he was right. The UK government has taken steps to improve math education, and we need to do the same. The UK government’s National Curriculum states that all students should be able to multiply and divide by the age of 10. Yet, in the US, the emphasis on math facts is not as strong.

Mathematics facts are important, but the memorization of math facts through rote practice and timed testing is unnecessary and damaging. The US government has recently linked 7×8 to linked 5×6 to linked 3×4 to linked 1×2, and so on. This is the result of a flawed understanding of how children learn math.

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Two Ways to Learn Math Facts

Strategies

Memorization

RESEARCH

“...research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.”

– Boaler, Page 3
RESEARCH FINDINGS

Study of students learning math facts in two ways - through strategies or memorization.

“Importantly the study...found that those who learned through strategies achieved ‘superior performance’ over those who memorized, they solved problems at the same speed, and showed better transfer to new problems.” (Delazer et al, 2005)

From “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts” by Jo Boaler, 2015, page 4
RESEARCH FINDINGS

Data from 13 million 15-year olds on International PISA mathematics test.

“...the lowest achieving students are those who focus on memorization and who believe that memorizing is important when studying mathematics. The highest achievers in the world are those who focus on big ideas in mathematics and connections between ideas.

From “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts” by Jo Boaler, 2015, page 5
DEVELOPING FLUENCY

IN WHAT WAYS DO YOU DEVELOP STUDENT FLUENCY?
Number Talk

• If your friend was having trouble remembering this fact, what strategy would you suggest to him or her?

• $8 \times 7$
PHASES OF BASIC FACT MASTERY

Phase 1: Modeling and/or Counting
(Counts with objects mentally)

Phase 2: Deriving answers using reasoning strategies based on known facts

Phase 3; Mastery
(efficient production of answers)

Adapted from Baroody, 2006
NUMBER TALKS

• Intentional focus on specific strategies
• Develop connections between strategies
• Compare the effectiveness of strategies for given problems
• Opportunities to hear about and utilize new strategies
**Directions:**

1. Roll a dice twice and draw pizzas.
   a. The first roll tells how many pizzas to draw.
   b. The second roll tells how many pepperonis to put on EACH pizza.

1. Write the number sentence that matches your picture.

1. How many pepperonis in all?

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From “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts” by Jo Boaler, 2015, page 13
MASTERING BASIC FACTS

Phase 1: Counting
(Counts with objects mentally)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery
(efficient production of answers)

Adapted from Baroody, 2006
DEVELOPING STRATEGIES

**Foundational Facts**
- 2, 5, 10
- 0, 1 (use contexts)

**Derived Fact Strategies**
- Adding or subtracting a group
- Halving and doubling
- Using a square product
- Decomposing a factor
• How many peeps?
• How does $3 \times 16$ appear in this picture?
• How does $4 \times 12$ appear in this picture?
• How does $4 \times (3 \times 4)$ appear in this picture?
Directions:

1. Each player takes a turn rolling the dice to get two factors.
2. The player outlines and colors a rectangle on the game board to match the pair of factors. Example: a roll of 6 and 3 is colored as a 6 x 3 rectangle or a 3 x 6 rectangle.
3. The player writes the equation (area) inside the rectangle.
4. A player loses a turn when the rectangle cannot be drawn on the game board.
5. The winner is the player with the most area colored.
VARIATIONS

Split a factor
• As above, but on each roll the player is allowed to split one of their factors and fill in two arrays. For example, if $5 \times 6$ would not fit on the board, they could split it into $2 \times 6$ and $3 \times 6$. They would then outline these two arrays and two products, claiming both areas.

Change the numbers
• Use cards instead of dice. Remove face cards.
• Practice one factor at a time and roll the other. For example, if the 6 times tables are being focused on, one factor is always 6.
• Use various sided dice (10 sided, 12 sided, 20 sided)
Math Cards

1. Lay all of the cards down on a table.
2. Have students take turns picking them. They can pick as many as they can find with the same answer (shown through any representation.)
3. Students explain how they know that the different cards are equivalent.
MASTERING BASIC FACTS

Phase 1: Counting
(Counts with objects mentally)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery
(efficient production of answers)

Adapted from Baroody, 2006
TOP IT

- Place students in pairs and give each pair a deck of cards (omitting face cards and using aces as 1).
- Have each student take half of the deck.
- Both players turn over two cards and say the product of the two cards.
- Whoever has the larger product wins the cards.
- Play continues until time is called. Whoever has the most cards wins.
- Differentiation: Use only specific numbers for the deck rather than using all factors 0-10.

SALUTE!

- Place students in groups of 3, and give each group a deck of cards (omitting face cards and using aces = 1).
- Two students draw a card without looking at it and place it on their forehead facing outward (so others can see it).
- The student with no card tell the product. The other 2 players determine the value of their cards.
- Once both players have done so, they look at their cards and then students rotate roles.
CAROLINA CLIP-IT

Directions:

1. Player one places paper clips on two numbers at the bottom of the page.

2. They multiply the two numbers and place a marker on the correct product.

3. Player two can move only one of the paper clips at the bottom of the page.

4. They multiply the two numbers and place a marker on the correct product.

5. Both paper clips may be placed on the same number.

6. Play continues until one player has 4 markers in a row, horizontally, vertically or diagonally.

Phase 2 Games
  • Breakapart

Phase 3 Games
  • Kakooma
  • Missing
WHY USE GAMES?

Games:
• Are engaging.
• Provide opportunities for strategy discussion and assessment.
• Should be sequenced developmentally.
• Can be targeted practice or general practice.
• Lend to differentiation.

Developing and Assessing Fact Fluency, Amanda Ruch and Gina Kling, NCTM 2015
ASSESSING FLUENCY

WHAT CAN WE LEARN FROM THIS ASSESSMENT IN REGARDS TO STUDENT FLEXIBILITY, ACCURACY, EFFICIENCY, AND APPROPRIATE STRATEGY USE?
My kids don’t know their basic facts . . .

Instead ask: Which kids? And Which facts?

In order to know which kids and which facts, we need to assess and monitor.
ASSESSING FLUENCY

• Observation
• Interviews
• Writing prompts
• Strategy quizzes
• Self-assessment
OBSERVATION

Use questions such as the following to encourage good mathematical thinking during game play:

- How did you figure it out?
- Can you say out loud how you thought about it in your head?
- Is there another way you could figure it out?
- Can you think of another fact that strategy would work well for?
- If someone didn’t know the answer to____, how would you tell them to figure it out?
- What are you hoping for next?
- What are all the possibilities?
# Multiplication Observation Log

**3.OA.7, Grades 3 - 6, Quarter:** ___

As you observe students learning their multiplication facts during games, practice, and Number Talks, ask them how they knew a particular fact. Continue to use your log throughout the quarter to track student progress, guide inquiry instruction, and work with small groups. If you do not have information on particular facts, use your observation log to guide your use of formative assessments based on particular facts.

**Key:**
- Blank - Phase 1: Counting or counting on
- / - One slash - Phase 2: Deriving Facts
- X - X - Phase 3: Mastery

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Foundational Facts</th>
<th>Other Facts</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex. Student A</td>
<td>0 1 2 5 10</td>
<td>5 4 6 7 8 9</td>
<td>Student A is counting by 2’s for the 2 facts</td>
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<td>1</td>
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</table>
INTERVIEWS

1. Write 7 x 8 on a card. (point at card) What does 7 x 8 mean?

2. What is your solution to 7 x 8?

3. How did you find your solution? Can you find it another way?

4. If your friend was having trouble remembering this fact, what strategy might you suggest to him/her?
STRATEGY QUIZZES

• Students solve problems and indicate how they solved them.
• Add the information to your observation log.
• Connect to student self assessment.

Quizzes that focus on fluency are alternatives to timed tests.

(a) Quiz questions can be used to see if students “just know” foundational facts.

Solve these problems and tell how you solved them.

\[
\begin{align*}
4 \times 5 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
10 \times 6 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
6 \times 2 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
5 \times 3 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
2 \times 9 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
3 \times 10 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
5 \times 7 &= \quad \text{Check one: } \quad \text{I used this strategy: } \\
8 \times 10 &= \quad \text{Check one: } \quad \text{I used this strategy: }
\end{align*}
\]

(b) A quiz assesses if students recognize the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies.

On completion, say to class, “Circle the row that was easier for you to solve. If they were both the same, write ‘same’.”

Solve these addition problems.

\[
\begin{align*}
\text{ROW A:} & \quad 9 + 1 = \quad 8 \quad \pm 1 \quad 5 + 1 = \quad 3 + 1 = \quad 6 \\
\text{ROW B:} & \quad 1 + 8 = \quad 1 + 7 = \quad 1 + 9 = \quad 1 \quad 1 \quad \pm 4 \quad \pm 2
\end{align*}
\]
FACT ASSESSMENTS

• Use Traditional Fact Assessments as strategy quizzes.
• For example,
  – Solve all the facts you know in your mind. Skip the others. 😊
  – Solve only the facts you need to work out.
  – Solve only the products that are greater than 36.
DISCUSS

• How will you support and develop fluency in the classroom?

• What ideas or activities are you most excited to try with your students? Why?
RESOURCES

• Developing and Assessing Fact Fluency, Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, NCTM 2015


• Kling, Gina and Jennifer M. Bay-Williams. 2014. Assessing Basic Fact Fluency. Teaching Children Mathematics, Volume 20, Number 8, 488-497.

CONTACT INFORMATION


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**poll code for this session**

___ 16706 ___  

Speaker was engaging and an effective presenter (0-3)

(1 space)

Speaker was well-prepared and knowledgeable (0-3)

(no spaces)

Session matched title and description in program book (0-3)

(1 space)

Other comments, suggestions, or feedback (words)

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**Example:** 16706 323  Inspiring, good content

**Non-Example:** 16706 3 2 3  Inspiring, good content

**Non-Example:** 16706 3-2-3 Inspiring, good content