



Mathematics Bookmarks

*Standards Reference to Support
Planning and Instruction*
<http://commoncore.tcoe.org>



5th Grade

**Tulare County
Office of Education**

Tim A. Hire, County Superintendent of Schools



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Grade-Level Introduction

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

- (1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
- (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

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- (2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

- (3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

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FLUENCY
<p>In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., “fluently” multiply multi-digit whole numbers using the standard algorithm (5.NBT.5 ▲). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.</p> <p>The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.</p>

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Explanations of Major, Additional and Supporting Cluster-Level Emphases
<p>Major3 [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. The ▲ symbol will indicate standards in a Major Cluster in the narrative.</p>
<p>Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade</p>
<p>Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis.</p>
<p>*A Note of Caution: Neglecting material will leave gaps in students’ skills and understanding and will leave students unprepared for the challenges of a later grade.</p>

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices

1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”.

Students:	Teachers:
<ul style="list-style-type: none"> • Analyze and explain the meaning of the problem • Actively engage in problem solving (Develop, carry out, and refine a plan) • Show patience and positive attitudes • Ask if their answers make sense • Check their answers with a different method 	<ul style="list-style-type: none"> • Pose rich problems and/or ask open ended questions • Provide wait-time for processing/finding solutions • Circulate to pose probing questions and monitor student progress • Provide opportunities and time for cooperative problem solving and reciprocal teaching

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2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

Students:	Teachers:
<ul style="list-style-type: none"> • Represent a problem with symbols • Explain their thinking • Use numbers flexibly by applying properties of operations and place value • Examine the reasonableness of their answers/calculations 	<ul style="list-style-type: none"> • Ask students to explain their thinking regardless of accuracy • Highlight flexible use of numbers • Facilitate discussion through guided questions and representations • Accept varied solutions/representations

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3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

Students:	Teachers:
<ul style="list-style-type: none"> • Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying and probing questions 	<ul style="list-style-type: none"> • Provide opportunities for students to listen to or read the conclusions and arguments of others • Establish and facilitate a safe environment for discussion • Ask clarifying and probing questions • Avoid giving too much assistance (e.g., providing answers or procedures)

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5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

Students:	Teachers:
<ul style="list-style-type: none"> Select and use tools strategically (and flexibly) to visualize, explore, and compare information Use technological tools and resources to solve problems and deepen understanding 	<ul style="list-style-type: none"> Make appropriate tools available for learning (calculators, concrete models, digital resources, pencil/paper, compass, protractor, etc.) Use tools with their instruction

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6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.

Students:	Teachers:
<ul style="list-style-type: none"> • Calculate accurately and efficiently • Explain their thinking using mathematics vocabulary • Use appropriate symbols and specify units of measure 	<ul style="list-style-type: none"> • Recognize and model efficient strategies for computation • Use (and challenging students to use) mathematics vocabulary precisely and consistently

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7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

Students:	Teachers:
<ul style="list-style-type: none"> Look for, develop, and generalize relationships and patterns Apply reasonable thoughts about patterns and properties to new situations 	<ul style="list-style-type: none"> Provide time for applying and discussing properties Ask questions about the application of patterns Highlight different approaches for solving problems

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8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Students:	Teachers:
<ul style="list-style-type: none"> Look for methods and shortcuts in patterns and repeated calculations Evaluate the reasonableness of results and solutions 	<ul style="list-style-type: none"> Provide tasks and problems with patterns Ask about possible answers before, and reasonableness after computations

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Grade 5 Overview

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

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CCSS Where to Focus Grade 5 Mathematics

Not all of the content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have a greater emphasis is not to say that anything in the standards can be safely neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

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MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 5
 Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
 Key: ■ Major Clusters □ Supporting Clusters ● Additional Clusters

- 5.OA.A ● Write and interpret numerical expressions.
- 5.OA.B ● Analyze patterns and relationships.
- 5.NBT.A ■ Understand the place value system.
- 5.NBT.B ■ Perform operations with multi-digit whole numbers and with decimals to hundredths.
- 5.NFA ■ Use equivalent fractions as a strategy to add and subtract fractions.
- 5.NFB ■ Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
- 5.MD.A □ Convert like measurement units within a given measurement system.
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- 5.MD.C ■ Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
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REQUIRED FLUENCIES FOR GRADE 5

5.NBT.B.5	Multi-digit multiplication
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Student Achievement Partners, Achieve the Core <http://achievethecore.org/>, Focus by Grade Level, <http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level>

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5.OA.A Write and interpret numerical expressions.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Essential Skills and Concepts:

- Add, subtract, multiply and divide fluently
- Evaluate expressions within brackets, braces or parenthesis as a set
- Compare expressions using grouping symbols

Question Stems and Prompts:

- ✓ When could you use parenthesis in an equation?
- ✓ How does adding parenthesis change the outcome of a problem?

Vocabulary

Tier 2

- parenthesis
- brackets
- braces
- evaluate
- expression

Tier 3

- numerical expression

Spanish Cognates

parentésis

evaluar

expresión

expresión numérica

Standards Connections

5.OA.1 → 5.OA.2

5.OA.1 Examples:

Examples:
$15 - 7 - 2 = 10$ with correct grouping symbols is $15 - (7 - 2) = 10$
$3 \times 125 + 25 + 7 = 22$ with correct grouping symbols is $3 \times (125 + 25) + 7 = 22$
Compare $3 \times 2 + 5$ and $3 \times (2 + 5)$
Compare $15 - 6 + 7$ and $15 - (6 + 7)$

5.OA.1 Illustrative Task:

- Watch Out for Parentheses 1, <https://www.illustrativemathematics.org/illustrations/555>

Evaluate the following numerical expressions.

- a. $2 \times 5 + 3 \times 2 + 4$
- b. $2 \times (5 + 3 \times 2 + 4)$
- c. $2 \times 5 + 3 \times (2 + 4)$
- d. $2 \times (5 + 3) \times 2 + 4$
- e. $(2 \times 5) + (3 \times 2) + 4$
- f. $2 \times (5 + 3) \times (2 + 4)$

Can the parentheses in any of these expressions be removed without changing the value the expression?

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Standard Explanation

In preparation for the progression of expressions and equations in the middle grades, students in grade five begin working more formally with expressions. Previously in third grade, students began to use the conventional order or operations (e.g., multiplication and division are done before addition and subtraction). In fifth grade, students build on this work to write, interpret and evaluate simple numerical expressions, including those that contain parentheses, brackets, or braces (ordering symbols) (5.OA.1-2).

Students need experiences with multiple expressions to understand when and how to use ordering symbols. Instruction in the order of operations should be carefully sequenced from simple to more complex problems. (Source: Progressions K-5 OA) In grade five, this work should be viewed as exploratory rather than for attaining mastery; for example, expressions should not contain nested grouping symbols, and they should be no more complex than the expressions one finds in an application of the associative or distributive property, such as $(8 + 27) + 2$ or $(6 \times 30) + (6 \times 7)$. (Adapted from The University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA 2011).

To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently. Students can begin by using these symbols with whole numbers and then expand the use to decimals and fractions. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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5.OA.A Write and interpret numerical expressions.

5.OA.2 Write simplify expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.*

Essential Skills and Concepts:

- Understand word order and language
- Write expressions using mathematical symbols

Question Stems and Prompts:

- ✓ How is _____ different from _____?
- ✓ Does order matter?

Vocabulary

Tier 2

- parenthesis
- brackets
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Tier 3

- numerical expression

Spanish Cognates

parentésis

evaluar
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Standards Connections

5.OA.2 ← 5.OA.1, 5.NF.5

5.OA.2 Illustrative Tasks:

- Words to Expressions 1,
<https://www.illustrativemathematics.org/illustrations/556>

Write an expression that records the calculations described below, but do not evaluate.

Add 2 and 4 and multiply the sum by 3. Next, add 5 to that product and then double the result.

- Comparing Products,
<https://www.illustrativemathematics.org/illustrations/139>

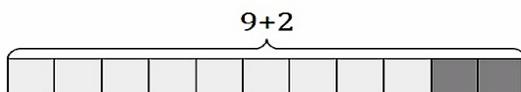
Leo and Silvia are looking at the following problem:

How does the product of 60×225 compare to the product of 30×225 ?

Silvia says she can compare these products without multiplying the numbers out. Explain how she might do it. Draw pictures to illustrate your explanation.

- Seeing is Believing,
<https://www.illustrativemathematics.org/illustrations/1222>

Below is a picture that represents $9 + 2$.



- a. Draw a picture that represents $4 \times (9 + 2)$.
- b. How many times bigger is the value of $4 \times (9 + 2)$ than $9 + 2$? Explain your reasoning.

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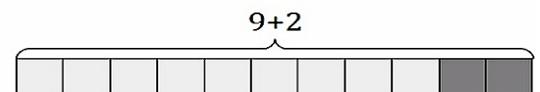
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Standard Explanation

Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression.

Students need opportunities to describe numerical expressions without evaluating them. For example, they express the calculation “add 8 and 7, then multiply by 2” as $(8 + 7) \times 2$. They recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without calculating the sum or product. Students begin to think about numerical expressions in anticipation for their later work with variable expressions (e.g., three times an unknown length is $3 \times L$). (Adapted from Arizona 2012 and Kansas Association of Teachers of Mathematics [KATM] 5th 123 FlipBook 2012).

OA Progression Information

As preparation for the Expressions and Equations Progression in the middle grades, students in Grade 5 begin working more formally with expressions.^{5.OA.1, 5.OA.2} They write expressions to express a calculation, e.g. writing $2 \times (8+7)$ to express the calculation “add 8 and 7, then multiply by 2.” They also evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret $3 \times (18932 + 921)$ as being three times as large as $18932 + 921$.

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5.OA.A Write and interpret numerical expressions.

5.OA.2.1 Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$. CA

Essential Skills and Concepts:

- Use knowledge of multiplication and division to factor
- Multiply and divide fluently
- Understand prime numbers
- Write expressions using prime factors

Question Stems and Prompts:

- ✓ How can you decompose this number using multiplication?
- ✓ How do you know when you have completed your prime factorization?
- ✓ What will happen if two students choose to begin their factorization with different numbers?

Vocabulary

Tier 2

- product
- factor(s)

Tier 3

- prime number
- composite number
- prime factorization

Spanish Cognates

producto
factor, los factores

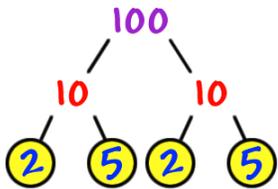
número primo
número compuesto

Standards Connections

5.OA.1 → 5.OA.2

5.OA.2.1 Examples:

Factor Tree Method

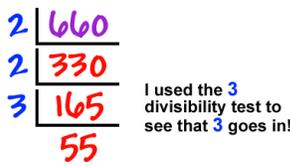


$100 = 2 \times 2 \times 5 \times 5$

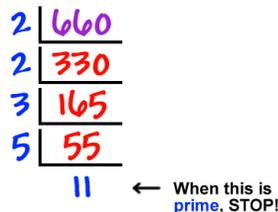
Source:

<http://www.coolmath.com/prealgebra/00-factors-primes/04-prime-factorizations-01.htm>

Ladder Method



3 is used up... Go to 5 next:



Now, just go down the stack to get your answer!

$660 = 2 \times 2 \times 3 \times 5 \times 11$

5.OA.A Write and interpret numerical expressions.

5.OA.2.1 Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$. CA

Essential Skills and Concepts:

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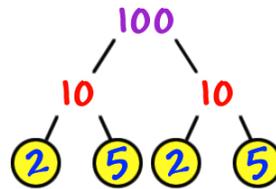
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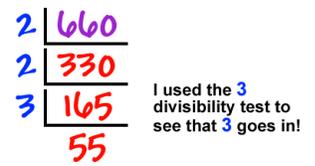


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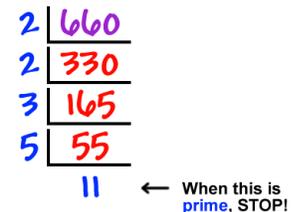
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Standard Explanation

Building on work that students did in 4th grade with factors and multiples (4.OA.4), students in 5th grade will learn how to factor numbers completely. They will use their knowledge of whether numbers are prime or composite as they learn how decompose composite whole numbers into their prime factors. Decomposing whole numbers into their prime factors continues the work that students have done to find all the factors of a given number and the decomposing of whole numbers into addends in previous grades.

Students will write the prime factorization of a number by writing an expression for the product of its primes. Students are not expected to rewrite the prime factorization of a number using exponents at this grade level as they have only worked with powers of 10. Two methods for finding the prime factorization of a number are using a factor tree or the ladder method. Both are shown below. (CA Mathematics Framework, adopted Nov. 6, 2013)

<p>How do I write the prime factorization of 24 using the FACTOR TREE method?</p>	<p>Method 1: use a factor tree.</p> <ul style="list-style-type: none"> Choose any two factors of 24 to begin. Keep finding factors until each branch ends at a prime factor! <p style="text-align: right;">**It should look like a tree!</p> <p>$24 = 3 \times 2 \times 2 \times 2$ $= 3 \times 2^3$ <<<< exponent form</p> <p style="text-align: center;">OR</p> <p>Method 2: use a ladder diagram</p> <ul style="list-style-type: none"> Choose a prime factor of 216 to begin. Keep dividing by PRIME numbers until the last number is prime. <table style="border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">216</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">108</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">54</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">27</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">9</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">3</td><td></td></tr> </table> <p>$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 3^3$ <<<< exponent form</p>	2	216		2	108		2	54		3	27		3	9		3	3	
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<p>How do I write the Prime factorization of 216 using the LADDER method?</p>	<p>Method 1: use a factor tree.</p> <ul style="list-style-type: none"> Choose any two factors of 24 to begin. Keep finding factors until each branch ends at a prime factor! <p style="text-align: right;">**It should look like a tree!</p> <p>$24 = 3 \times 2 \times 2 \times 2$ $= 3 \times 2^3$ <<<< exponent form</p> <p style="text-align: center;">OR</p> <p>Method 2: use a ladder diagram</p> <ul style="list-style-type: none"> Choose a prime factor of 216 to begin. Keep dividing by PRIME numbers until the last number is prime. <table style="border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">216</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">108</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">54</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">27</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">9</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">3</td><td></td></tr> </table> <p>$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 3^3$ <<<< exponent form</p>	2	216		2	108		2	54		3	27		3	9		3	3	
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(Source: <http://ipamillermath.pbworks.com/f/GN1-+Prime+Factorization.pdf>)

5.OA.A Write and interpret numerical expressions.

5.OA.2.1 Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$. CA

Standard Explanation

Building on work that students did in 4th grade with factors and multiples (4.OA.4), students in 5th grade will learn how to factor numbers completely. They will use their knowledge of whether numbers are prime or composite as they learn how decompose composite whole numbers into their prime factors. Decomposing whole numbers into their prime factors continues the work that students have done to find all the factors of a given number and the decomposing of whole numbers into addends in previous grades.

Students will write the prime factorization of a number by writing an expression for the product of its primes. Students are not expected to rewrite the prime factorization of a number using exponents at this grade level as they have only worked with powers of 10. Two methods for finding the prime factorization of a number are using a factor tree or the ladder method. Both are shown below. (CA Mathematics Framework, adopted Nov. 6, 2013)

<p>How do I write the prime factorization of 24 using the FACTOR TREE method?</p>	<p>Method 1: use a factor tree.</p> <ul style="list-style-type: none"> Choose any two factors of 24 to begin. Keep finding factors until each branch ends at a prime factor! <p style="text-align: right;">**It should look like a tree!</p> <p>$24 = 3 \times 2 \times 2 \times 2$ $= 3 \times 2^3$ <<<< exponent form</p> <p style="text-align: center;">OR</p> <p>Method 2: use a ladder diagram</p> <ul style="list-style-type: none"> Choose a prime factor of 216 to begin. Keep dividing by PRIME numbers until the last number is prime. <table style="border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">216</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">108</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">2</td><td style="padding-right: 5px;">54</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">27</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">9</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">3</td><td style="padding-right: 5px;">3</td><td></td></tr> </table> <p>$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 3^3$ <<<< exponent form</p>	2	216		2	108		2	54		3	27		3	9		3	3	
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5.OA.B Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Essential Skills and Concepts:

- Continue a given pattern using the four operations
- Create a table of values for the pattern
- Plot the coordinate pairs from the table on a coordinate plane

Question Stems and Prompts:

- ✓ What number comes next? How do you know?
- ✓ Describe the pattern.
- ✓ What do you notice about each sequence?
- ✓ How are the sequences related?

Vocabulary

Tier 2

- pattern

Tier 3

- function table/t-table
- ordered pairs/coordinates
- coordinate plane
- independent variable
- dependent variable

Spanish Cognates

- tabla de funciones
- par ordenado/coordenadas
- plano de coordenada
- variable independiente
- variable dependiente

Standards Connections

5.OA.3 ← 4.OA.5

5.OA.3 Examples:

Example: Create two sequences of numbers, both starting from 0, but one generated with a “+3” pattern, and the other with a “+ 6” pattern.							
a. How are the sequences related to each other?							
b. Graph the sequences together as ordered pairs, the first sequence being the x-coordinate and the second sequence being the y-coordinate.							
c. How can you see how the sequences are related together based on the graph?							
Solution:							
Starting with 0, students create two sequences of numbers:							
Sequence A:	0,	3,	6,	9,	12,	15,	...
Sequence B:	0,	6,	12,	18,	24,	30,	...

Common Misconception.

Students reverse the points when plotting them on a coordinate plane. They count up first on the y-axis and then count over on the x-axis

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Standard Explanation

Understanding patterns is fundamental to algebraic thinking. Students extend their grade four pattern work to include two numerical patterns that can be related and examine these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane (5.OA.3). This work prepares students for studying proportional relationships and functions in middle school, and is closely related to graphing points in the coordinate plane (5.G.1-2).

This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function, which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

OA Progression Information

Students extend their Grade 4 pattern work by working briefly with two numerical patterns that can be related and examining these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane.^{5.OA.3} This work prepares students for studying proportional relationships and functions in middle school.

5.OA.3 Example:

Howard County Public School System,

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.OA.3>

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5.NBT.A Understand the place value system.

5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

Essential Skills and Concepts:

- Understand the place value of digits within a number
- Understand the symmetry and directionality of the place value system
- Describe and compare the size of digits within a number (10 times larger, 10 times smaller, $\frac{1}{10}$ of the size)

Question Stems and Prompts:

- ✓ Explain the relationship between the two 7's in the number 657,782.
- ✓ What do you notice when you move to the digit to the right? What if you move to the digit on the left?
- ✓ What is a number that is 10 times larger than ____?
- ✓ What is a number that is 10 times smaller than ____?
- ✓ What patterns do you see within the place value system?
- ✓ How do two numbers compare that are the same digit, but are in different place value locations?

Vocabulary

Tier 3

- digit
- place value

Spanish Cognates

dígito

Standards Connections

5.NBT.2 → 5.NBT.5, 5.NBT.7

5.NBT.1 Example:**Example:**

Through exploration with base-ten blocks or attaching cubes, students can tangibly explore the relationship between place values. They may be able to name place values, but this is not an indication that they understand the relationship between them. For example, a student may know that the difference between the digits 5 in the number 4554, represent 500 and 50, but the further relationship that $500 = 50 \times 10$ and $50 = 500 \times (\frac{1}{10})$ needs to be explored and made explicit.

To extend this understanding of place value to their work with decimals, students could use a model of one unit; cut it into 10 equal pieces, shade in, or describe $\frac{1}{10}$ of that model using fractional language ("This is 1 out of 10 equal parts. So it is $\frac{1}{10}$ ". I can write this using $\frac{1}{10}$ or 0.1"). Students repeat the process by finding $\frac{1}{10}$ of a $\frac{1}{10}$ (e.g., dividing $\frac{1}{10}$ into 10 equal parts to arrive at $\frac{1}{100}$ or 0.01) and explain their reasoning, "0.01 is $\frac{1}{10}$ of $\frac{1}{10}$ thus is $\frac{1}{100}$ of the whole unit." Simple 10×10 grids can be very useful for exploring these ideas. Also, since the metric system is a base-10 system of measurement, working with simple metric length measurements and rulers can support this understanding (see standard **5.MD.1**).

In general, students are led to the following pattern: Students recognize that in a multi-digit number:

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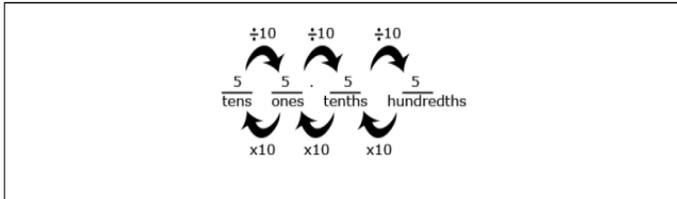
5.NBT.A.1

Standard Explanation

In grade five, a critical area of instruction is for students to integrate decimal fractions into the place value system, develop an understanding of operations with decimals to hundredths, and work towards fluency with whole number and decimal operations.

Students extend their understanding of the base-ten system from whole numbers to decimals focusing on the relationship between adjacent place values, how numbers compare, and how numbers round for decimals to thousandths. Before considering the relationship of decimal fractions, students reason that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and what it represents in the place to its left (5.NBT.1 ▲). (Adapted from Progressions K-5 NBT 2011). Students use place value to understand that multiplying a decimal by 10 results in the decimal point appearing one place to the right (e.g., $10 \times 4.2 = 42$) since the result is ten times larger than the original number; similarly, multiplying a decimal by 100 results in the decimal point appearing two places to the right since the number is 100 times bigger. Students also make the connection that dividing by 10 results in the decimal point appearing one place to the left (e.g. $4 \div 10 = .4$) since the number is 10 times smaller (or $\frac{1}{10}$ of the original), and dividing a number by 100 results in the decimal point appearing two places to the left since the number is 100 times smaller (or $\frac{1}{100}$ of the original).

(CA Mathematics Framework, adopted Nov. 6, 2013)



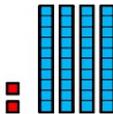
(Adapted from Arizona 2012 and KATM 5th 171 FlipBook 2012).

5.NBT.1 Illustrative Tasks:

- Tenths and Hundredths

<https://www.illustrativemathematics.org/illustrations/1800>

Jossie drew a picture to represent 0.24:



She said,

The little squares represent tenths and the rectangles represent hundredths, which makes sense because ten little squares makes one rectangle, and ten times ten is one hundred.

- Explain what is wrong with Jossie's reasoning.
- Name three numbers that Jossie's picture could represent. In each case, What does a little square represent? What does a rectangle represent?

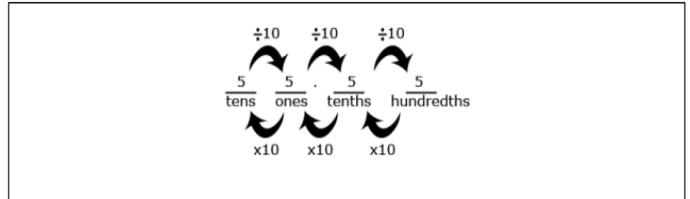
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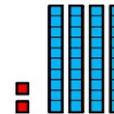
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5.NBT.A Understand the place value system

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Essential Skills and Concepts:

- Use exponents to represent powers of 10
- Explain the pattern for the number of zeros
- Understand and explain the shifting of the decimal point when multiplying decimals by a power of ten

Question Stems and Prompts:

- ✓ What patterns do you see when you multiply by different powers of ten?
- ✓ What do you notice when you multiply decimals by powers of ten?
- ✓ What pattern do you see when you multiply by 10^n ? Explain how your pattern works for any given exponent.

Vocabulary

Tier 2

- pattern
- base

Tier 3

- powers of 10
- exponent
- decimal point

Spanish Cognates

base

exponente

punto decimal

Standards Connections

5.NBT.2 → 5.NBT.5, 5.NBT.7

5.NBT.2 Examples:

Students might write,

$$36 \times 10 = 36 \times 10^1 = 360$$

$$36 \times 10 \times 10 = 36 \times 10^2 = 3600$$

$$36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36,000$$

$$36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360,000$$

Students might think and/or say, "I noticed that every time I multiplied by 10 I placed a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left. When I multiplied 36 by 10, the 30 became 300. The 6 became 60 (or the 36 became 360).

(Adapted from Arizona 2012)

Focus, Coherence, and Rigor:

Students can use their understanding of the structure of whole numbers to generalize this understanding to decimals (MP.7) and explain the relationship between the numerals (MP.6) (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

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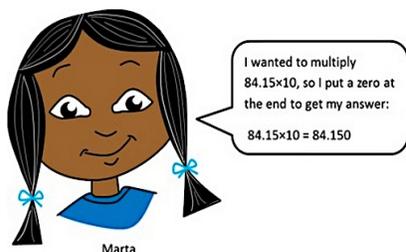
Standard Explanation

Powers of 10 is a fundamental aspect of the base-ten system. Students extend their understanding of place to explain patterns in the number of zeros of the product when multiplying a number by powers of 10, including the placement of the decimal point. New at grade five is the use of whole number exponents to denote powers of 10 (5.NBT.2▲). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

5.NBT.2 Illustrative Tasks:

- Marta's Multiplication Error, <https://www.illustrativemathematics.org/illustrations/1524>

Marta made an error while finding the product 84.15×10 .



In your own words, explain Marta's misunderstanding. Please explain what she should do to get the correct answer and include the correct answer in your response.

- 5.NBT.1 Multiplying Decimals by 10, <https://www.illustrativemathematics.org/illustrations/1620>
 - Explain why $0.4 \times 10 = 4$.
 - Explain why $3.4 \times 10 = 34$.

Draw pictures to illustrate your explanations.

The purpose of this task is to help students understand and explain why multiplying a decimal number by 10 shifts all the digits one place to the left. This understanding builds on work around place-value that students have done with whole numbers in 3rd and 4th grade (see especially 3.NBT.3 and 4.NBT.1). This is an instructional task that the teacher can give students before discussing this "digit shifting" principle for multiplying decimal numbers by 10.

The task begins with a decimal number with only one non-zero digit so that students can see how to it works in the simplest case. Note that the seeing that 10 groups of (4 groups of 0.1) is equal to 4 groups of (10 groups of 0.1) requires both the commutative and associative properties of multiplication.

5.NBT.A Understand the place value system

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

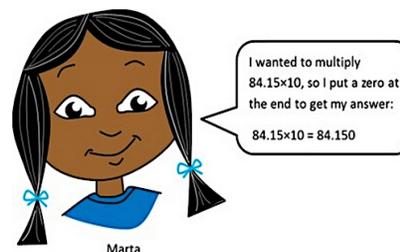
Standard Explanation

Powers of 10 is a fundamental aspect of the base-ten system. Students extend their understanding of place to explain patterns in the number of zeros of the product when multiplying a number by powers of 10, including the placement of the decimal point. New at grade five is the use of whole number exponents to denote powers of 10 (5.NBT.2▲). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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5.NBT.A Understand the place value system.

5.NBT.3 Read, write, and compare decimals to thousandths.

- a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
- b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Essential Skills and Concepts:

- Read decimals to the thousandths
- Write decimals in number, word, and expanded form
- Compare decimals using inequality symbols
- Explain decimal comparisons using place value understanding

Question Stems and Prompts:

- ✓ Write this number in word form and expanded form.
- ✓ How are the two types of expanded form related to one another? What strengths does each variation have?
 - $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$
 - $347.392 = 300 + 40 + 7 + 3/10 + 9/100 + 2/1000$
- ✓ How do you know _____ is greater/less than _____?

Vocabulary

Tier 2

- word form

Tier 3

- place value
- tenths
- hundredths
- thousandths
- inequality symbols
- expanded form
- standard form

Spanish Cognates

forma estandar

Standards Connections

5.NBT.3 → 5.NBT.4

5.NBT.3 Examples:

If represents 1, then represents $\frac{1}{10}$ and represents $\frac{1}{100}$.

"Explain why the following both represent the number 0.23"

--	--

"Well, I see that the 20 hundredths in the picture on the right can be grouped into 2 sets of 10 hundredths. That means these 2 groups represent 2 tenths, or $\frac{2}{10}$. There are 3 hundredths left, so altogether there are $\frac{2}{10} + \frac{3}{100}$ "

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5.NBT.A Understand the place value system.**5.NBT.3** Read, write, and compare decimals to thousandths.**Common Misconception.**

Some students relate comparing decimals with the idea “the longer the number the greater the number.” With whole numbers, a five-digit number is always greater than a one-, two-, three-, or four-digit number. However, when comparing decimals, a number with one decimal place may be greater than a number with two or three decimal places

Standard Explanation

Students build on understandings from fourth grade to read, write, and compare decimals to thousandths (5.NBT.3 ▲). They connect this work with prior understanding of decimal notations for fractions and addition of fractions with denominators of 10 and 100. Students use concrete models or drawings and number lines to extend this understanding to decimals to the thousandths. Models may include base-ten blocks, place value charts, grids, pictures, drawings, manipulatives and technology. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$).

Base-10 blocks can be a powerful tool for seeing these representations. For instance, if a “flat” is used to represent 1 (the whole or unit), then a “stick” represents $1/10$, and a small “cube” represents $1/100$. Students can be challenged to make sense of a number like 0.23 as being represented by both $2/10 + 3/100$ and $23/100$.

Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

For students who are not able to read, write, and represent multi-digit numbers, working with decimals will be challenging. Money is a good medium to provide meaning for decimals (e.g., dimes can represent tenths, pennies represent hundredths, and a penny circle with a $\frac{1}{10}$ sliver in it can represent thousandths), as well as base-10 blocks. Reading decimals can confuse some students because numbers to the left of the decimal are read based on the place value of the digit farthest to the left of the decimal (e.g., 462 is read as “four hundred sixty two”); however, decimals are read as whole numbers with the hundreds, tens, and ones said and then the place value of the digit farthest to the right of the decimal (e.g., .46 is read as 46 hundredths). Decimals are read as if they are fractions: You read the number as the numerator and then say the denominator. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

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5.NBT.A Understand the place value system

5.NBT.4 Use place value understanding to round decimals to any place.

Standard Explanation

Students use place value understanding to round decimals to any place (5.NBT.4▲). When rounding a decimal to a given place, students may identify two possible answers and use their understanding of place value to compare the given number to the possible answers. Students can use benchmark numbers (e.g., 0, 0.5, 1, and 1.5) to support their work.

Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Essential Skills and Concepts:

- Understand and explain the standard algorithm for multiplication
- Relate and compare various methods for multiplication to the standard algorithm
- Fluently multiply multi-digit numbers using the standard algorithm

Question Stems and Prompts:

- ✓ How do the various strategies for multiplication relate to the standard algorithm?
- ✓ Explain your thinking process as you use the algorithm.

Vocabulary

Tier 2

- fluent/fluency
- strategies

Tier 3

- multiply
- standard algorithm

Spanish Cognates

estrategias

multiplicar

Standards Connections

5.NBT.5 → 5.NBT.6, 5.NBT.7

5.NBT.5 Examples:

FLUENCY
<p>In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., “fluently” multiply multi-digit whole numbers using the standard algorithm (5.NBT.5▲). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.</p> <p>The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.</p>

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

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Essential Skills and Concepts:

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Vocabulary

Tier 2

- fluent/fluency
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Tier 3

- multiply
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5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Standard Explanation

In grades three and four, students used various strategies to multiply. In grade five students fluently multiply multi-digit whole numbers using the standard algorithm (5.NBT.5 ▲). Generally the standards distinguish strategies from algorithms. In particular, the “standard algorithm” refers here to multiplying numbers digit-by-digit and recording the products piece-by-piece. Note that the method of recording the algorithm is not the same as the algorithm itself, in the sense that the “partial products” method, which lists every single digit-by-digit product separately, is a completely valid recording method for the “standard algorithm.” Ultimately, the standards call for understanding the standard algorithm in terms of place value, and this should be the most important goal for instruction.

In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.

In previous grades, students built a conceptual understanding of multiplication with whole numbers as they applied multiple strategies to compute and solve problems. Students can continue to use different strategies and methods from previous years as long as they are efficient, but they must also understand and be able to use the standard algorithm. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

Example: Find the product 123×34

When students apply the standard algorithm, they decompose 34 into $30 + 4$. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products. The ways in which students are taught to record this method may vary, but all should emphasize the place-value nature of the algorithm. For example, one might write

$$\begin{array}{r} 123 \\ \times 34 \\ \hline 492 \leftarrow \text{this is the product of 4 and 123} \\ 3690 \leftarrow \text{this is the product of 30 and 123} \\ \hline 4182 \leftarrow \text{this is the sum of the two partial products} \end{array}$$

Note that a further decomposition of 123 into $100 + 20 + 3$ and recording of the partial products would also be acceptable.

(Adapted from Arizona 2012)

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

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5.NBT.B Understand the place value system.

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Skills and Concepts:

- Divide numbers using equations, rectangular arrays, and/or area models
- Explain division strategies and/or illustrations
- Relate multiplication to division and explain how this relationship helps you as you are dividing

Question Stems and Prompts:

- ✓ How are the various strategies for division related?
- ✓ Explain your thinking process as you completed your division.
- ✓ How can you use the relationship between multiplication and division to help you divide? To check your answer for reasonableness?

Vocabulary

Tier 3

- divide
- array
- area model

Spanish Cognates

- dividir
- modelo de area

Standards Connections

5.NBT.5 → 5.NBT.6, 5.NBT.7

5.NBT.6 Examples:

Example 1: Find the quotient $2682 \div 25$

- Using expanded notation: $2682 \div 25 = (2000 + 600 + 80 + 2) \div 25$
- Using an understanding of the relationship between 100 and 25, a student might think:
 - I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
 - 600 divided by 25 has to be 24.
 - Since 3×25 is 75, I know that 80 divided by 25 is 3 with a remainder of 5. (Note that a student might divide into 82 and not 80)
 - I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
 - $80 + 24 + 3 = 107$. So, the answer is 107 with a remainder of 7.
- Using an equation that relates division to multiplication, $25 \times n = 2682$, a student might estimate the answer to be slightly larger than 100 by recognizing that $25 \times 100 = 2500$.

Example 2: Find the quotient $9984 \div 64$

An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.

<p>Area model:</p> <div style="text-align: center;"> <table style="margin: auto;"> <tr><td></td><td style="text-align: center;">64</td></tr> <tr><td style="text-align: right;">100</td><td style="border: 1px solid black; padding: 2px;">6400</td></tr> <tr><td style="text-align: right;">50</td><td style="border: 1px solid black; padding: 2px;">3200</td></tr> <tr><td style="text-align: right;">5</td><td style="border: 1px solid black; padding: 2px;">320</td></tr> <tr><td style="text-align: right;">1</td><td style="border: 1px solid black; padding: 2px;">64</td></tr> </table> </div> <p>So the quotient is $100 + 50 + 5 + 1 = 156$.</p>		64	100	6400	50	3200	5	320	1	64	<p>Recording:</p> $ \begin{array}{r} 64 \overline{) 9984} \\ \underline{-6400} \quad (100 \times 64) \\ 3584 \\ \underline{-3200} \quad (50 \times 64) \\ 384 \\ \underline{-320} \quad (5 \times 64) \\ 64 \\ \underline{-64} \quad (1 \times 64) \\ 0 \end{array} $
	64										
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Question Stems and Prompts:

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Vocabulary

Tier 3

- divide
- array
- area model

Spanish Cognates

- dividir
- modelo de area

Standards Connections

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(Adapted from Arizona 2012)

5.NBT.B.6

Standard Explanation

In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

In grade five students extend division to include quotients of whole numbers with up to four-digit dividends and two-digit divisors using various strategies, and they illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (5.NBT.6▲). When the two-digit divisor is a "familiar" number, students might use various strategies based on place value understanding.

To help students understand the use of place value when dividing with two digit divisors, students can begin with simpler examples, such as dividing 150 by 30. Clearly the answer is 5 since this is 15 tens divided by 3 tens. However, when dividing 1500 by 30, students need to think of this as 150 tens divided by 3 tens, which is 50. This illustrates why when using the division algorithm the 5 would go in the tens place of the quotient.

The extension from one-digit divisors to two-digit divisors requires care (5.NBT.6▲). This is a major milestone along the way to reaching fluency with the standard algorithm in grade six. Division strategies in grade five extend the grade four methods to 2-digit divisors. Students continue to break the dividend into base-ten units and find the quotient place by place, starting from the highest place. They illustrate and explain their calculations using equations, rectangular arrays, and/or area models. Estimating the quotients is a difficult new aspect of dividing by a 2-digit number. Even if students round appropriately, the resulting estimate may need to be adjusted up or down. Students may write any needed new group from multiplying within the division or add it in mentally or write the multiplication out to the side, if necessary. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Focus, Coherence, and Rigor:

When students break divisors and dividends into sums of multiples of base-ten units (5.NBT.6▲), they also develop important mathematical practices such as how to see and make use of structure (MP.7) and attend to precision (MP.6). (PARCC 2012).

5.NBT.B.6

Standard Explanation

In fourth grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

In grade five students extend division to include quotients of whole numbers with up to four-digit dividends and two-digit divisors using various strategies, and they illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (5.NBT.6▲). When the two-digit divisor is a "familiar" number, students might use various strategies based on place value understanding.

To help students understand the use of place value when dividing with two digit divisors, students can begin with simpler examples, such as dividing 150 by 30. Clearly the answer is 5 since this is 15 tens divided by 3 tens. However, when dividing 1500 by 30, students need to think of this as 150 tens divided by 3 tens, which is 50. This illustrates why when using the division algorithm the 5 would go in the tens place of the quotient.

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When students break divisors and dividends into sums of multiples of base-ten units (5.NBT.6▲), they also develop important mathematical practices such as how to see and make use of structure (MP.7) and attend to precision (MP.6). (PARCC 2012).

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Essential Skills and Concepts:

- Add, subtract, multiply, and divide decimals using strategies and models
- Explain calculations using concrete models or drawings

Question Stems and Prompts:

- ✓ How are the various strategies used by your classmates related? What are the similarities/differences in the strategies?
- ✓ How do the models or drawings support your understanding of operations with decimals?
- ✓ Explain your strategy for adding, subtracting, multiplying or dividing decimals.

Vocabulary

Tier 2

- concrete models

Tier 3

- decimal
- base ten blocks
- properties of operations

Spanish Cognates

modelos concretos

decimal

propiedades de operaciones

Standards Connections

5.NBT.7 → 5.MD.1

5.NBT.7 Examples:

<p>Examples: Estimate</p> <p>3.6 + 1.7. A student can make good use of rounding to estimate that since 3.6 rounds up to 4 and 1.7 rounds up to 2, the answer should be close to 4 + 2 = 6.</p> <p>5.4 – 0.8. Students can again round and argue that since 5.4 rounds down to 5 and 0.8 rounds up to 1, the answer should be close to 5 – 1 = 4.</p> <p>6 × 2.4. A student might estimate an answer between 12 and 18 since 6 × 2 is 12 and 6 × 3 is 18.</p>
<p>Example 1: (Model for decimal subtraction) Find 4 – 0.3. Explain how you found your solution. "Since I'm subtracting 3 tenths from 4 wholes, it would help to divide one of the wholes into tenths. The other 3 wholes don't need to be divided up. I can see there are 3 wholes and 7 tenths leftover, or 3.7."</p>
<p>Example 2: Use an area model to demonstrate that $\frac{1}{10}$ of $\frac{1}{10}$ is $\frac{1}{100}$. "If I use my 10×10 grid and set the whole grid to equal 1 square unit, then I can see that when each length of the grid is divided into ten equal parts, each small square must be representing a $\frac{1}{10} \times \frac{1}{10}$ square. But there are 100 of these small squares in the whole, so each little square must have area $\frac{1}{100}$ square units."</p>
<p>Example 3: Use an area model to demonstrate that $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100}$. "Just like in the previous problem, I use my 10×10 grid to represent 1 whole, with dimensions 1 unit by 1 unit. If I break up each side length into ten equal parts, then I can create a smaller rectangle of dimensions 3 tenths of a unit by 4 tenths of a unit. It looks something like this:"</p>

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

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Standard Explanation

In grade five students build on work with comparing decimals in fourth grade and begin to add, subtract, multiply, and divide decimals to hundredths (5.NBT.7▲). Students focus on reasoning about operations with decimals using concrete models, drawings, various strategies, and explanations. They extend the models and written models they developed for whole numbers in grades one through four to decimal values. Students might estimate answers based on their understanding of operations and the value of the numbers. (MP.7, MP.8)

Students must understand and be able to explain that when adding decimals they add tenths to tenths and hundredths to hundredths. When students add in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. Students reinforce their understanding of adding decimals by connecting to prior understanding of adding fractions with denominators of 10 and 100 from fourth grade. Students understand that when adding and subtracting a whole number the decimal point is at the end of the whole number. Students use various models to support their understanding of decimal operations. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

5.NBT.7 Illustrative Task:

- What is $23 \div 5$?, 5.NBT.7 and 5.NF.3
<https://www.illustrativemathematics.org/illustrations/292>
- a. Jessa has 23 one-dollar bills that she wants to divide equally between her 5 children.
 - i. How much money will each receive? How much money will Jessa have left over?
 - ii. Jessa exchanged the remaining one-dollar bills for dimes. If she divides the money equally between her 5 children, how much money will each child get?
- b. A website has games available to purchase for \$5 each. If Lita has \$23, how many games can she purchase? Explain.
- c. A jug holds 5 gallons of water. How many jugs can Mark fill with 23 gallons of water? Explain.
- d. A class of 23 children will take a field trip. Each car can take 5 children. How many cars are needed to take all the children on the field trip? Explain.
- e. Write a division problem for $31 \div 4$ where the answer is a mixed number. Show how to solve your problem.

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

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5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)

Essential Skills and Concepts:

- Create equivalent fractions to find common denominators
- Add and subtract fractions including mixed numbers with like and unlike denominators

Question Stems and Prompts:

- ✓ What strategies can you use to find common denominators?
- ✓ What if two students find different common denominators before doing their calculations? How will this impact their solutions?
- ✓ Explain your thinking while adding/subtracting these fractions.

Vocabulary

Tier 3

- fraction
- mixed numbers
- equivalent fractions
- estimate
- numerator
- denominator
- benchmark fractions
- unlike denominators
- common denominators

Spanish Cognates

- fracción
- numero mixto
- fracciones equivalente
- estimar
- numerador
- denominador

Standards Connections

5.NF.1 → 5.NF.2, 5.NBT.7

5.NF.1 Examples:

<p>Examples:</p> $\frac{2}{5} + \frac{7}{8} = \frac{2}{5} \cdot \frac{8}{8} + \frac{7}{8} \cdot \frac{5}{5} = \frac{16}{40} + \frac{35}{40} = \frac{51}{40}$ $3\frac{1}{4} - \frac{1}{6} = 3\frac{6}{24} - \frac{4}{24} = 3\frac{2}{24} \text{ or } 3\frac{1}{12}$

(Adapted from Progressions 3-5 NF 2011)

Ella completed $\frac{3}{8}$ of a puzzle during recess on Tuesday. Joy completed $\frac{2}{6}$ of the same puzzle during recess on Wednesday.

How much of the puzzle did the two girls complete?

Use what you know about fractions to explain how you found your answer. Use numbers and/or words in your explanation.

Howard County Public School System,
<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.NF.1>

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5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.

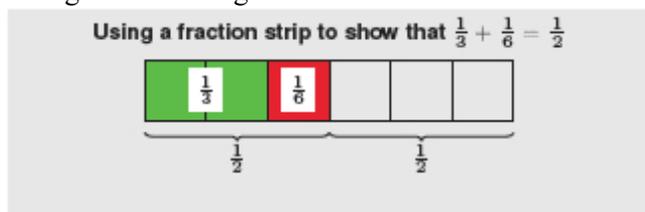
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

Standard Explanation

Student proficiency with fractions is essential to success in algebra at later grades. In grade five a critical area of instruction is developing fluency with addition and subtraction of fractions, including adding and subtracting fractions with unlike denominators.

Students find a common denominator by finding the product of both denominators. For $1/3 + 1/6$, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm. Student should first solve problems that require changing one of the fractions and progress to changing both fractions. Students understand that multiplying the denominators will always give a common denominator but may not result in the smallest denominator; however, it is not necessary to find a least common denominator to calculate sums and differences of fractions.

To add or subtract fractions with unlike denominators, students need an understanding of how to create equivalent fractions with the same denominators before adding or subtracting.



(NF Progression 3 – 5)

5.NF.1 Illustrative Task:

- Finding Common Denominators to Add, <https://www.illustrativemathematics.org/illustrations/848>
 - a. To add fractions, we usually first find a common denominator.
 - i. Find two different common denominators for $\frac{1}{3}$ and $\frac{1}{15}$.
 - ii. Use each common denominator to find the value of $\frac{1}{3} + \frac{1}{15}$. Draw a picture that shows your solution.
 - b. Find $\frac{3}{4} + \frac{1}{5}$. Draw a picture that shows your solution.
 - c. Find $\frac{14}{8} + \frac{15}{12}$.

5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.

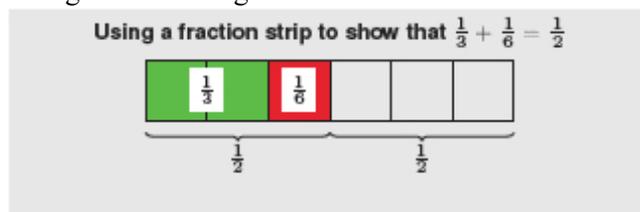
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5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Essential Skills and Concepts:

- Solve word problems by adding and subtracting fractions
- Use visual fraction models or equations to represent and solve the problems
- Estimate the solutions using benchmark fractions and number sense

Question Stems and Prompts:

- ✓ How does the model or equation you used represent the problem situation?
- ✓ Is your answer reasonable? How might you use benchmark fractions and number sense to decide?
- ✓ Compare and explain the various strategies and models used to solve the word problem.

Vocabulary

Tier 3

- fraction
- estimate
- benchmark fraction

Spanish Cognates

- fracción
- estimar

Standards Connections

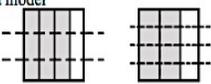
5.NF.3 → 5.NF.4a, 5.NF.5

5.NF.2 Examples:

Focus, Coherence, and Rigor:
When students meet standard (5.NF.2▲), they bring together the threads of fraction equivalence (learned in grades three through five) and addition and subtraction (learned in kindergarten through grade four) to fully extend addition and subtraction to fractions. (Adapted from PARCC 2012).

Jerry was making two different types of cookies. One recipe needed $3/4$ cup of sugar and the other needed $2/3$ cup of sugar. How much sugar did he need to make both recipes?

- Mental estimation:
A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to $1/2$ and state that both are larger than $1/2$ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.
- Area model



$\frac{3}{4}$ cup of sugar $\frac{2}{3}$ cup of sugar
 $\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$

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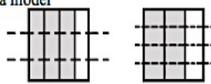
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5.NF.A.2

Standard Explanation

Students make sense of fractional quantities when solving word problems involving addition and subtraction of fractions referring to the same whole using a variety of strategies (5.NF.2 ▲).

This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as $\frac{7}{8}$ is greater than $\frac{3}{4}$ because $\frac{7}{8}$ is missing only $\frac{1}{8}$ and $\frac{3}{4}$ is missing $\frac{1}{4}$ so $\frac{7}{8}$ is closer to a whole. Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

5.NF.2 Illustrative Tasks:

- Do These Add Up?,

<https://www.illustrativemathematics.org/illustrations/481>

For each of the following word problems, determine whether or not $(\frac{2}{5} + \frac{3}{10})$ represents the problem. Explain your decision.

- A farmer planted $\frac{2}{5}$ of his forty acres in corn and another $\frac{3}{10}$ of his land in wheat. Taken together, what fraction of the 40 acres had been planted in corn or wheat?
- Jim drank $\frac{2}{5}$ of his water bottle and John drank $\frac{3}{10}$ of his water bottle. How much water did both boys drink?
- Allison has a batch of eggs in the incubator. On Monday $\frac{2}{5}$ of the eggs hatched, By Wednesday, $\frac{3}{10}$ more of the original batch hatched. How many eggs hatched in all?
- Two fifths of the cross-country team arrived at the weight room at 7 a.m. Ten minutes later, $\frac{3}{10}$ of the team showed up. The rest of the team stayed home. What fraction of the team made it to the weight room that day?
- Andy made 2 free throws out of 5 free throw attempts. Jose made 3 free throws out of 10 free throw attempts. What is the fraction of free throw attempts that the two boys made together?
- Two fifths of the students in the fifth grade want to be in the band. Three tenths of the students in the fifth grade want to play in the orchestra. What fraction of the students in the fifth grade want to be in one of the two musical groups?
- There are 150 students in the fifth grade in Washington Elementary School. Two fifths of the students like soccer best and $\frac{3}{10}$ of them like basketball best. What fraction like soccer or basketball best?
- The fifth grade at Lincoln School has two mixed-sex soccer teams, Team A and Team B. If $\frac{2}{5}$ of Team A are girls and $\frac{3}{10}$ of Team B are girls, what fraction of the players from the two teams are girls?
- Wesley ran $\frac{2}{5}$ of a mile on Monday and $\frac{3}{10}$ of a mile on Tuesday. How far did he run those two days?

- Salad Dressing,

<https://www.illustrativemathematics.org/illustrations/1172>

Aunt Barb's Salad Dressing Recipe

- $\frac{1}{3}$ cup olive oil
- $\frac{1}{6}$ cup balsamic vinegar
- a pinch of herbs
- a pinch of salt

Makes 6 servings

- How many cups of salad dressing will this recipe make? Write an equation to represent your thinking. Assume that the herbs and salt do not change the amount of dressing.
- If this recipe makes 6 servings, how much dressing would there be in one serving? Write a number sentence to represent your thinking.

5.NF.A.2

Standard Explanation

Students make sense of fractional quantities when solving word problems involving addition and subtraction of fractions referring to the same whole using a variety of strategies (5.NF.2 ▲).

This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as $\frac{7}{8}$ is greater than $\frac{3}{4}$ because $\frac{7}{8}$ is missing only $\frac{1}{8}$ and $\frac{3}{4}$ is missing $\frac{1}{4}$ so $\frac{7}{8}$ is closer to a whole. Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

5.NF.2 Illustrative Tasks:

- Do These Add Up?,

<https://www.illustrativemathematics.org/illustrations/481>

For each of the following word problems, determine whether or not $(\frac{2}{5} + \frac{3}{10})$ represents the problem. Explain your decision.

- A farmer planted $\frac{2}{5}$ of his forty acres in corn and another $\frac{3}{10}$ of his land in wheat. Taken together, what fraction of the 40 acres had been planted in corn or wheat?
- Jim drank $\frac{2}{5}$ of his water bottle and John drank $\frac{3}{10}$ of his water bottle. How much water did both boys drink?
- Allison has a batch of eggs in the incubator. On Monday $\frac{2}{5}$ of the eggs hatched, By Wednesday, $\frac{3}{10}$ more of the original batch hatched. How many eggs hatched in all?
- Two fifths of the cross-country team arrived at the weight room at 7 a.m. Ten minutes later, $\frac{3}{10}$ of the team showed up. The rest of the team stayed home. What fraction of the team made it to the weight room that day?
- Andy made 2 free throws out of 5 free throw attempts. Jose made 3 free throws out of 10 free throw attempts. What is the fraction of free throw attempts that the two boys made together?
- Two fifths of the students in the fifth grade want to be in the band. Three tenths of the students in the fifth grade want to play in the orchestra. What fraction of the students in the fifth grade want to be in one of the two musical groups?
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North Carolina Department of Instruction, 5th Grade Mathematics, Unpacked Content Updated August 2012
5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Essential Skills and Concepts:

- Explain that fractions represent division using visual models and equations
- Solve word problems with fraction and mixed number solutions
- Create contexts that illustrate fractions as division

Question Stems and Prompts:

- ✓ How can a fraction be interpreted as division?
- ✓ Create and explain a division situation that can be represented by a fraction.

Vocabulary

Tier 3

- fraction
- division

Spanish Cognates

- fracción
- división

Standards Connections

5.NF.3 → 5.NF.4a, 5.NF.5

5.NF.3 Examples:

For example: Sharing 5 objects equally among three shares, showing that $5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$

If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $1/3$ of itself to each share. Thus each share consists of 5 pieces, each of which is $1/3$ of an object, so each share is $5 \times 1/3 = 5/3$ of an object. (Progressions 3-5 NF 2012)

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5.NF.B.3

Standard Explanation

In grade five students connect fractions with division, understanding that $5 \div 3 = 5/3$ or, more generally, $a/b = a \div b$ for whole numbers a and b , with b not equal to zero (5.NF.3 ▲).

Students can explain this by working with their understanding of division as equal sharing. Students solve related word problems and demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as “three-fifths” and, after experiences with sharing problems, students generalize that dividing 3 into 5 equal parts ($3 \div 5$ also written as $\frac{3}{5}$) results in the fraction $\frac{3}{5}$ (3 of 5 equal parts). Students should also create story contexts to represent problems involving division of whole numbers. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

5.NF.3 Illustrative Tasks:

- How Much Pie?,
<https://www.illustrativemathematics.org/illustrations/858>

Tags: Lesson Plan Included

After a class potluck, Emily has three equally sized apple pies left and she wants to divide them into eight equal portions to give to eight students who want to take some pie home.

- Draw a picture showing how Emily might divide the pies into eight equal portions. Explain how your picture shows eight equal portions.
- What fraction of a pie will each of the eight students get?
- Explain how the answer to (b) is related the division problem $3 \div 8$.

- Converting Fractions of a Unit to a Smaller Unit, 5.NF.3 and 5.MD.1,

<https://www.illustrativemathematics.org/illustrations/293>

- Five brothers are going to take turns watching their family's new puppy. How much time will each brother spend watching the puppy in a single day if they all watch him for an equal length of time? Write your answer
 - Using only hours,
 - Using a whole number of hours and a whole number of minutes, and
 - Using only minutes.
- Mrs. Hinojosa had 75 feet of ribbon. If each of the 18 students in her class gets an equal length of ribbon, how long will each piece be? Write your answer
 - Using only feet,
 - Using a whole number of feet and a whole number of inches, and
 - Using only inches.
- Wesley walked 11 miles in 4 hours. If he walked the same distance every hour, how far did he walk in one hour? Write your answer
 - Using only miles,
 - Using a whole number of miles and a whole number of feet, and
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5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Essential Skills and Concepts:

- Multiply a fraction or whole number by a fraction
- Find the area of rectangles with fractional edge lengths
- Create and solve word problems involving the multiplication of fractions

Question Stems and Prompts:

- ✓ What strategies/models did you use to multiply a fraction or whole number by a fraction?
- ✓ How can you apply your knowledge of area to multiplying fractions?

Vocabulary

Tier 2

- area

Tier 3

- fraction
- multiply

Spanish Cognates

área

fracción

multiplicar

Standards Connections

5.NF.4 → 5.NF.5, 5.NF.7, 5.NBT.7

5.NF.4 Examples:

<p>Examples:</p> <p>When students multiply fractions such as in the problem $\frac{3}{5} \times 35$, they can think of the operation in more than one way:</p> <ul style="list-style-type: none"> • As $3 \times (35 \div 5)$, or $3 \times \frac{35}{5}$. (This is equivalent to $3 \times (\frac{1}{5} \times 35)$ and expresses the idea in standard 5.NF.4.b▲). • As $(3 \times 35) \div 5$, or $105 \div 5$. (This is equivalent to $\frac{105}{5}$.) <p>Students may be challenged to write a story problem for this operation.</p> <p>"Mark's mother said he could have $\frac{3}{5}$ of the peanuts she bought for him and his younger brother to share. If she bought a bag of 35 peanuts, how many peanuts does Mark receive?"</p>

5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

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Essential Skills and Concepts:

- Multiply a fraction or whole number by a fraction
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- Create and solve word problems involving the multiplication of fractions

Question Stems and Prompts:

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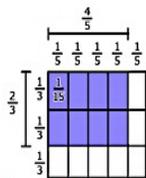
5.NF.B.4

Standard Explanation

Students apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction (5.NF.4 ▲). Students multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately and solve problems in both contextual and non-contextual situations. Students reason about how to multiply fractions using fraction strips and number line diagrams. Using an understanding of multiplication by a fraction, students develop an understanding of a general formula for the product of two fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

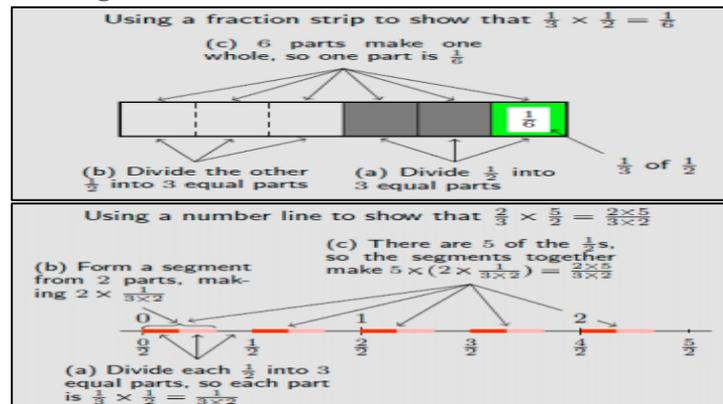
This standard extends student’s work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as $\frac{3}{5}$ actually could be represented as 3 pieces that are each one-fifth ($3 \times \frac{1}{5}$). This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

Building on previous understandings of multiplication, students find the area of a rectangle with fractional side lengths and represent fraction products as areas. (CA Mathematics Framework, adopted Nov. 6, 2013)

<p>Finally, when students move to examples like $\frac{2}{3} \times \frac{4}{5}$, they see that the division of the side lengths into fractional parts creates a division of the unit area into fractional parts as well. Students will discover that the fractional parts of the unit area are related to the denominators of the original fractions. Here, a 1×1 square is divided into thirds in one direction and fifths in another. This results in the unit square itself being divided into fifteenths. This reasoning shows why $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.</p>	<p>"I created a unit square, divided it into fifths in one direction and thirds in the other. This allows me to shade a rectangle of dimensions $\frac{2}{3}$ and $\frac{4}{5}$. I noticed that 15 of the new little rectangles make up the entire unit square, so they must be fifteenths ($\frac{1}{15}$). Altogether, I had 2×4 of those fifteenths. So my answer is $\frac{8}{15}$."</p> 
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(Adapted from Arizona 2012)

NF Progression Information:



Using a fraction strip to show that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

(c) 6 parts make one whole, so one part is $\frac{1}{6}$

(b) Divide the other $\frac{1}{2}$ into 3 equal parts

(a) Divide $\frac{1}{3}$ into 3 equal parts

$\frac{1}{3}$ of $\frac{1}{2}$

Using a number line to show that $\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$

(b) Form a segment from 2 parts, making $2 \times \frac{1}{3 \times 2}$

(c) There are 5 of the $\frac{1}{3}$, so the segments together make $5 \times (2 \times \frac{1}{3 \times 2}) = \frac{2 \times 5}{3 \times 2}$

(a) Divide each $\frac{1}{3}$ into 3 equal parts, so each part is $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

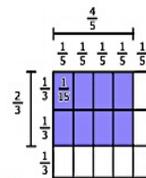
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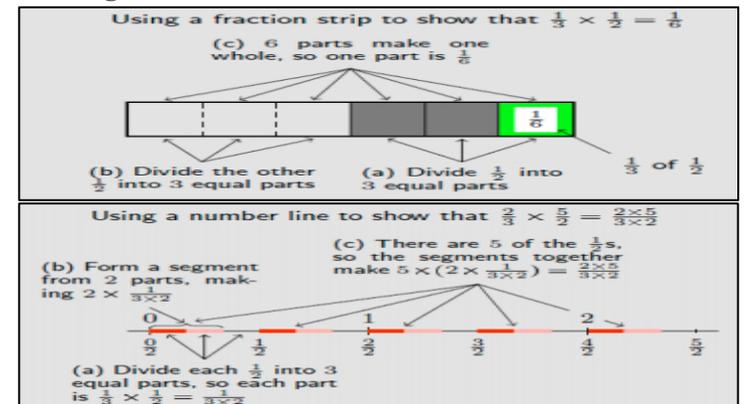
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Using a number line to show that $\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$

(b) Form a segment from 2 parts, making $2 \times \frac{1}{3 \times 2}$

(c) There are 5 of the $\frac{1}{3}$, so the segments together make $5 \times (2 \times \frac{1}{3 \times 2}) = \frac{2 \times 5}{3 \times 2}$

(a) Divide each $\frac{1}{3}$ into 3 equal parts, so each part is $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.5 Interpret multiplication as scaling (resizing), by:

- Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Essential Skills and Concepts:

- Understand and explain the relationship between products of numbers and fractions greater than 1
- Understand and explain the relationship between products of numbers and fractions less than 1
- Understand the principle of fraction equivalence when multiplying a fraction by 1 as n/n

Question Stems and Prompts:

- ✓ How do you know if the product will be greater than or less than 1?
- ✓ How can looking at the sizes of the factors help you to make reasonable estimates?
- ✓ What happens when you multiply a number by n/n ? Will this always work? Explain your thinking.

Vocabulary

Tier 3

- | | |
|------------------------|------------------------|
| • factors | factores |
| • scaling | escalamiento |
| • fraction equivalence | fracciones equivalente |

Spanish Cognates

Standards Connections

5.NF.4 → 5.NF.5, 5.NF.7, 5.NBT.7

5.NF.5 Examples:

<p>Examples:</p> <p>"I know $\frac{3}{4} \times 7$ is less than 7, because I make 4 equal shares from 7 but I only take 3 of them ($\frac{3}{4}$ is a fractional part less than one). If I'm taking a fractional part of 7 that is less than 1, the answer should be less than 7."</p>
<p>"I know that $2\frac{2}{3} \times 8$ should be more than 8, because 2 groups of 8 is 16 and $2\frac{2}{3} > 2$. Also, I know the answer should be less than $24 = 3 \times 8$, since $2\frac{2}{3} < 3$."</p>
<p>"I can show by equivalent fractions that $\frac{3}{4} = \frac{3 \times 5}{4 \times 5}$. But I also see that $\frac{5}{5} = 1$, so the result should still be equal to $\frac{3}{4}$."</p>

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5.NF.B.5**Standard Explanation**

This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.

In preparation for grade six work in ratios and proportional reasoning, students interpret multiplication as scaling (resizing) (5.NF.5 ▲) by examining how numbers change as they multiply by fractions. Students should have ample opportunities to examine the following cases: a) that when multiplying by a fraction greater than 1, the number increases, and b) that when multiplying by a fraction less the one, the number decreases. This is a new interpretation of multiplication and one that needs extensive discussion and explanation by students. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

NF Progression Information:

The understanding of multiplication as scaling is an important opportunity for students to reasoning abstractly (MP2). Previous work with multiplication by whole numbers enables students to see multiplication by numbers bigger than 1 as producing a larger quantity, as when a recipe is doubled, for example. Grade 5 work with multiplying by unit fractions, and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity, as when the budget of a large state university is multiplied by 1/2, for example.

The special case of multiplying by 1, which leaves a quantity unchanged, can be related to fraction equivalence by expressing 1 as n/n , as explained on page 7. (Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 14)

5.NF.5 Illustrative Tasks:

- Running a Mile,
<https://www.illustrativemathematics.org/illustrations/22>

Curt and Ian both ran a mile. Curt's time was $\frac{8}{9}$ Ian's time. Who ran faster? Explain and draw a picture.

- Fundraising,
<https://www.illustrativemathematics.org/illustrations/150>
Cai, Mark, and Jen were raising money for a school trip.

- Cai collected $2\frac{1}{2}$ times as much as Mark.
- Mark collected $\frac{2}{3}$ as much as Jen.

Who collected the most? Who collected the least? Explain.

5.NF.B.5**Standard Explanation**

This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.

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5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Essential Skills and Concepts:

- Solve word problems involving the multiplication of fractions and mixed numbers
- Explain strategies for solving using visual fraction models and/or equations

Question Stems and Prompts:

- ✓ Explain your thinking and the strategy used for solving the word problem. Discuss different strategies for solving.
- ✓ How can you represent this problem using a visual fraction model or equation? Where are the parts of the problem in your representation?

Vocabulary

Tier 3

- fraction
- mixed number
- equation
- visual fraction models

Spanish Cognates

- fracción
- numero mixto
- ecuación

Standards Connections

5.NF.6 → 5.NF.4, 5.NF. 7, 5.MD.2

5.NF.6 Examples:

There are $2\frac{1}{2}$ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. $\frac{2}{5}$ of the students on each bus are girls. How many busses would it take to carry *only* the girls?

Student 1
I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving $2\frac{1}{2}$ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls. When I added up the shaded pieces, $\frac{2}{5}$ of the 1st and 2nd bus were both shaded, and $\frac{1}{5}$ of the last bus was shaded.

$\frac{2}{5} + \frac{2}{5} + \frac{1}{5} = \frac{5}{5} = 1$ whole bus.

Student 2
 $2\frac{1}{2} \times \frac{2}{5} =$
I split the $2\frac{1}{2}$ into 2 and $\frac{1}{2}$
 $2 \times \frac{2}{5} = \frac{4}{5}$
 $\frac{1}{2} \times \frac{2}{5} = \frac{2}{10}$
I then added $\frac{4}{5}$ and $\frac{2}{10}$. That equals 1 whole bus load.

5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

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5.NF.B.6

Standard Explanation

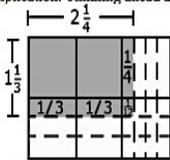
This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Students apply their understanding of multiplication of fractions and mixed numbers to solve real-world problems using visual models or equations (5.NF.6▲).
(CA Mathematics Framework, adopted Nov. 6, 2013)

5.NF.6 Example:

Example:

Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?
A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by $1\frac{1}{3}$ instead of $2\frac{1}{4}$.



The explanation may include the following:

- First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.
- When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.
- Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.
- $\frac{1}{3}$ times 2 is $\frac{2}{3}$.
- $\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.
- $\frac{2}{3}$ times $\frac{1}{3}$ is $\frac{2}{9}$.
- So the answer is $2\frac{1}{4} + \frac{2}{9} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

5.NF.6 Illustrative Task(s):

- Running to School,
<https://www.illustrativemathematics.org/illustrations/294>
The distance between Rosa’s house and her school is $\frac{3}{4}$ mile. She ran $\frac{1}{3}$ of the way to school.
How many miles did she run?

- To Multiply or Not to Multiply?,
<https://www.illustrativemathematics.org/illustrations/609>
Some of the problems below can be solved by multiplying $\frac{1}{8} \times \frac{2}{3}$, while others need a different operation. Select the ones that can be solved by multiplying these two numbers. For the remaining, tell what operation is appropriate. In all cases, solve the problem (if possible) and include appropriate units in the answer.

- Two-fifths of the students in Anya’s fifth grade class are girls. One-eighth of the girls wear glasses. What fraction of Anya’s class consists of girls who wear glasses?
- A farm is in the shape of a rectangle $\frac{1}{8}$ of a mile long and $\frac{2}{3}$ of a mile wide. What is the area of the farm?
- There is $\frac{2}{3}$ of a pizza left. If Jamie eats another $\frac{1}{8}$ of the original whole pizza, what fraction of the original pizza is left over?
- In Sam’s fifth grade class, $\frac{1}{8}$ of the students are boys. Of those boys, $\frac{2}{3}$ have red hair. What fraction of the class is red-haired boys?
- Only $\frac{1}{20}$ of the guests at the party wore both red and green. If $\frac{1}{8}$ of the guests wore red, what fraction of the guests who wore red also wore green?

5.NF.B.6

Standard Explanation

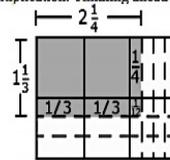
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- Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.
- $\frac{1}{3}$ times 2 is $\frac{2}{3}$.
- $\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.
- $\frac{2}{3}$ times $\frac{1}{3}$ is $\frac{2}{9}$.
- So the answer is $2\frac{1}{4} + \frac{2}{9} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

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5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.¹

- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.
- Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?

Essential Skills and Concepts:

- Solve problems involving division of a unit fraction by a whole number or a whole number by a unit fraction
- Understand and interpret fraction division
- Create and solve fraction division word problems using fraction models and/or equations

Question Stems and Prompts:

- ✓ Explain how you solved your division of fractions problem.
- ✓ How does your model/equation relate to the story problem?
- ✓ How can you use the relationship between multiplication and division to help you divide fractions?

Vocabulary

Tier 3

- fraction
- division
- equation
- visual fraction models

Spanish Cognates

- fracción
- división
- ecuación

¹ Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

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5.NF.B.7

Standards Connections

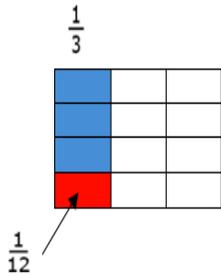
5.NF.6 → 5.NF.4, 5.NF. 7, 5.MD.2

Standard Explanation

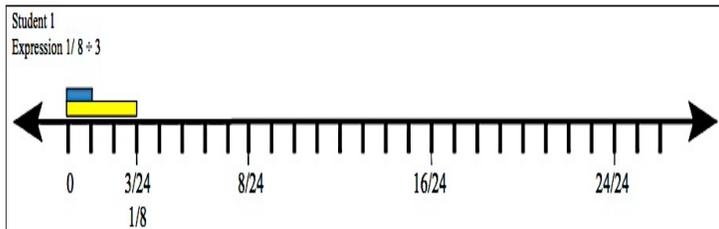
Students apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions (5.NF.7 ▲), a new concept at fifth grade. Students will extend their learning about division of fractions in simpler cases here in grade five to the general case in grade six (division of a fraction by a fractions is not a requirement at this grade). Students use visual fractions models to show the quotient and solve related real-world problems. (CA Mathematics Framework, adopted Nov. 6, 2013)

5.NF.7 Examples:

Knowing the number of groups/shares and finding how many/much in each group/share
 Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?
 The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.



You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?



5.NF.7 Illustrative Task:

- Dividing by One-Half, <https://www.illustrativemathematics.org/illustrations/12>

Solve the four problems below. Which of the following problems can be solved by finding $3 \div \frac{1}{2}$?

- Shauna buys a three-foot-long sandwich for a party. She then cuts the sandwich into pieces, with each piece being $\frac{1}{2}$ foot long. How many pieces does she get?
- Phil makes 3 quarts of soup for dinner. His family eats half of the soup for dinner. How many quarts of soup does Phil's family eat for dinner?
- A pirate finds three pounds of gold. In order to protect his riches, he hides the gold in two treasure chests, with an equal amount of gold in each chest. How many pounds of gold are in each chest?
- Leo used half of a bag of flour to make bread. If he used 3 cups of flour, how many cups were in the bag to start?

5.NF.B.7

Standards Connections

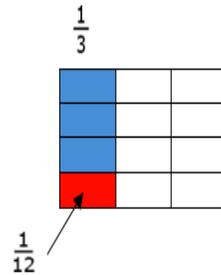
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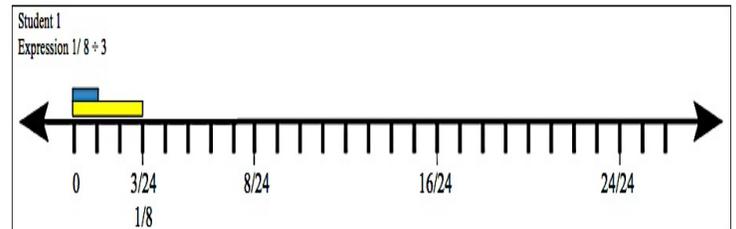
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5.MD.A Convert like measurement units within a given measurement system.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Essential Skills and Concepts:

- Know conversions within measurement systems
- Convert measurements within a given system
- Use measurement conversions to solve real-world and multi-step problems

Question Stems and Prompts:

- ✓ How many feet (m, mm, oz, etc) are equal to _____ inches (cm, m, lb, etc.)?
- ✓ How would you use conversions in this problem?

Vocabulary

Tier 3

- | | |
|------------------------------|------------------------|
| • volume, liquid volume | volumen |
| • hour, minute, second | hora, minuto, segundo |
| • mass | |
| • length | |
| • kilometer (km) | kilómetro |
| meter (m), centimeter (cm) | metro, centímetro |
| • kilogram (kg), gram (g) | kilogramo, gramo |
| • liter (L), milliliter (mL) | litro, milímetro |
| • ounce (oz), pound (lb) | onza |
| • cup (c), pint (pt) | pinta |
| • quart (qt), gallon (gal) | cuarto de galón, galón |

Standards Connections

5.MD.1 ← 5.NBT.7

Focus, Coherence, and Rigor:

Students' work with conversions within the metric system (**5.MD.1**) provides opportunities for practical applications of place value understanding and supports major work at the grade in the cluster "Understand the place value system" (**5.NBT.1▲**).

5.MD.1 Examples:

In the long jump, Karen can jump 51 inches, Debbie can jump 4 feet 4 inches, and Margaret can jump 1 yard, 1foot, 1 inch. Who can jump the farthest?

2 gallons, 1 quart, 3 pints _____ 7 quarts, 12 pints

Use what you know about customary measurement to explain how you found your answer.

5.MD.1 Illustrative Task:

- Minutes and Days,
<https://www.illustrativemathematics.org/illustrations/878>

What time was it 2011 minutes after the beginning of January 1, 2011?

5.MD.A Convert like measurement units within a given measurement system.

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Vocabulary

Tier 3

- | | |
|------------------------------|------------------------|
| • volume, liquid volume | volumen |
| • hour, minute, second | hora, minuto, segundo |
| • mass | |
| • length | |
| • kilometer (km) | kilómetro |
| meter (m), centimeter (cm) | metro, centímetro |
| • kilogram (kg), gram (g) | kilogramo, gramo |
| • liter (L), milliliter (mL) | litro, milímetro |
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Standards Connections

5.MD.1 ← 5.NBT.7

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5.MD.B Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Essential Skills and Concepts:

- Create a line plot using fraction intervals
- Solve problems using data from the line plots
- Interpret and explain the data displayed in the line plot

Question Stems and Prompts:

- ✓ What intervals did you use to create your line plot? Why?
- ✓ What information does the line plot give you?

Vocabulary

Tier 3

- data/data set
- line plot
- fraction intervals

Spanish Cognates

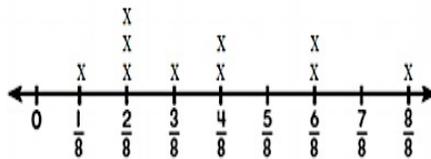
- datos
- intervalos de fracción

Standards Connections

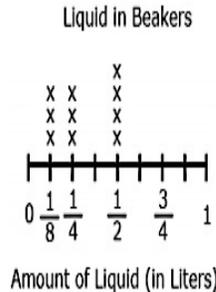
5.MD.2 ← 5.NF.2, 5.NF.6, 5.NF.7

5.MD.2 Examples:

Example:
Students measured objects in their desk to the nearest $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ of an inch then displayed data collected on a line plot. How many object measured $\frac{1}{4}$? $\frac{1}{2}$? If you put all the objects together end to end what would be the total length of all the objects?



Example:
Ten beakers, measured in liters, are filled with a liquid.



The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

5.MD.B Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Essential Skills and Concepts:

- Create a line plot using fraction intervals
- Solve problems using data from the line plots
- Interpret and explain the data displayed in the line plot

Question Stems and Prompts:

- ✓ What intervals did you use to create your line plot? Why?
- ✓ What information does the line plot give you?

Vocabulary

Tier 3

- data/data set
- line plot
- fraction intervals

Spanish Cognates

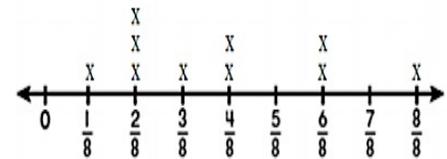
- datos
- intervalos de fracción

Standards Connections

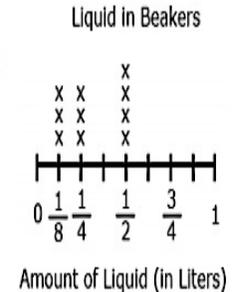
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5.MD.B Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Standard Explanation

This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

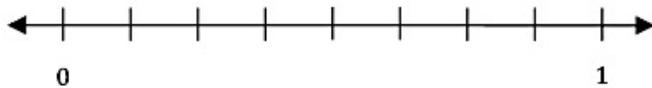
5.MD.2 Illustrative Task:

- Fractions on a Line Plot,
<https://www.illustrativemathematics.org/illustrations/1563>

You and your partner will need fraction cards made from this set:

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

- a. Label the line-plot below with $\frac{1}{8}$'s. Cut out and divide the cards evenly between the two players, laying them face-down. Each partner will choose one of their face-down cards and turn it over. The team will then add their fractions together. For each turn, each team will record their sum on the line plot.



Each team should have 12 data points marked on their line plot.

- b. Look at the line plot. Which values came up the most? Which values did not come up?
- c. The tick marks on the number line correspond to eighths. Which of the eighths will never come up as a sum of two of these cards? Why?
- d. You want to improve the game so that it is possible for two fractions to sum to $\frac{7}{8}$. Name one fraction card that you could add to the deck and explain why your new card would now make it possible to have $\frac{7}{8}$ as a sum of two cards.

5.MD.B Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

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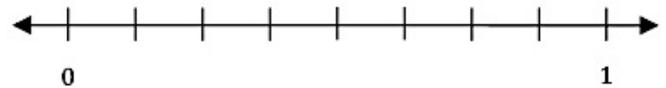
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$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
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5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
- A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

Essential Skills and Concepts:

- Understand that volume is an attribute of three-dimensional figures
- Recognize that a cube with a 1 unit side length is "one cubic unit" of volume
- Explain how to find the volume using unit cubes

Question Stems and Prompts:

- ✓ What is volume?
- ✓ How do you know if a shape has volume?
- ✓ How can you find volume? What units are used?

Vocabulary

Tier 2

- attribute

Tier 3

- volume
- unit cube
- one cubic unit
- solid figures
- three-dimensional

Spanish Cognates

atributo

volumen

tridimensional

Standards Connections

5.MD.1 ← 5.NBT.7

5.MD.C Illustrative Task:

- Box of Clay, <https://www.illustrativemathematics.org/illustrations/1031>

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box has twice the height, three times the width, and the same length as the first box. How many grams of clay can it hold?

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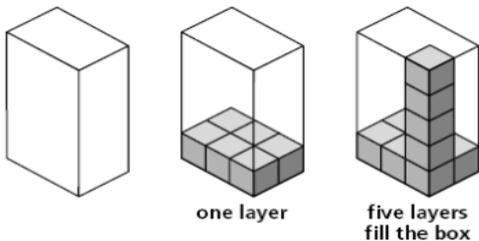
5.MD.C.3

Standard Explanation

Students develop an understanding of volume and relate volume to multiplication and addition. Volume introduces three-space dimension, a significant challenge to some students' spatial structuring and also a complexity in the nature of the materials measured. (5.MD.3 ▲) Solid units are “packed,” such as cubes in a three-dimensional array, whereas a liquid “fills” three-dimensional space, taking the shape of the container. “Packing” volume is more difficult than area concepts in early grades (e.g., iterating a unit to measure length and measuring area by tiling). Helping students think of volume as the number of cubes in n layers with a given area can be simpler than thinking of all three dimensions. (Adapted from PARCC 2012 and Progressions K-5 MD, measurement part 2012).

Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a “unit cube” (5.MD.3 ▲). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (5.MD.4 ▲). Students can also build up a rectangular prism with cubes to see the volume; it is easier to see the cubes in this method.

In grade three students measured and estimated liquid volume and worked with area measurement. At grade five, the concept of volume can be extended from area by relating earlier work covering an area to the bottom of cube with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer. (*CA Mathematics Framework*, adopted Nov. 6, 2013)



- (3×2) represented by first layer
- $(3 \times 2) \times 5$ represented by number of 3×2 layers
- $(3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) = 6 + 6 + 6 + 6 + 6 = 30$
(6 representing the size/area of one layer)

Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume or building up without the containers. Students may also use drawings or interactive computer software to simulate the same filling process. It is helpful for students use with concrete manipulatives before moving to pictorial representations.

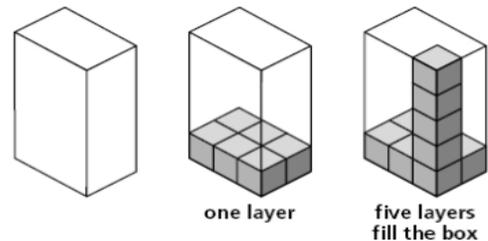
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5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Essential Skills and Concepts:

- Recognize that a cube with a 1 unit side length is "one cubic unit" of volume
- Explain how to find the volume using unit cubes
- Find the volume by counting unit cubes

Question Stems and Prompts:

- ✓ How can you find the volume of this shape?
- ✓ What units are used?
- ✓ Explain how you found the volume.

Vocabulary

- Tier 2
- base
- improvised (units)

Tier 3

- volume
- length
- width
- height
- rectangular prism
- cube
- cubic cm
- cubic in
- cubic ft

Spanish Cognates

base

volumen

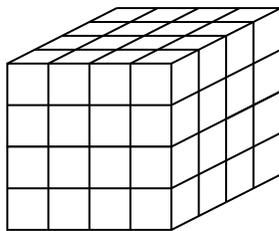
prisma rectángulo
cubo
centímetro cúbico

Standards Connections

5.MD.4 → 5.MD.5a

5.MD.4 Example:

Find the volume of each figure.



V = _____ units³

Explain how you found your solution.

Howard County Public School System,

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.MD.4>

5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

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Spanish Cognates

base

volumen

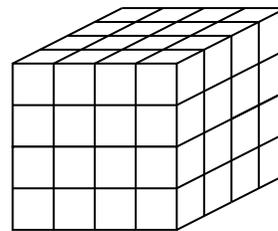
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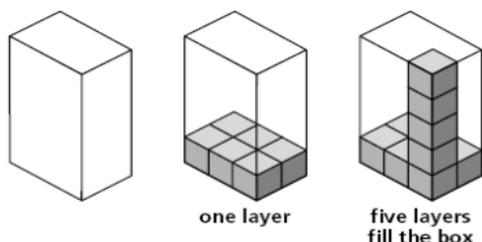
5.MD.C.4

Standard Explanation

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Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a “unit cube” (5.MD.3▲). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (5.MD.4▲). Students can also build up a rectangular prism with cubes to see the volume; it is easier to see the cubes in this method.

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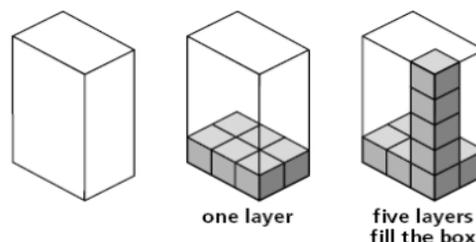
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5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Essential Skills and Concepts:

- Connect the ideas of volume to multiplication/addition
- Solve real world and mathematical problems involving volume
- Find the volume of a prism by packing it with cubes
- Use the volume formulas $V = l \times w \times h$ and $V = b \times h$
- Find the volume of two connected prisms by adding the volumes of the non-overlapping parts

Question Stems and Prompts:

- ✓ How are the volume formulas related?
- ✓ Would you get the same volume by packing the prism with unit cubes as you do with the formulas? Why or why not?
- ✓ How can you find the volume of a three-dimensional figures that contains several prisms?

Vocabulary

Tier 3

• volume	volumen
• length	
• width	
• height	
• base	base
• rectangular prism	prisma rectangular
• cube	cubo
• cubic units: cubic cm, cubic formulas	in, cubic ft formulas

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5.MD.C.5

Standards Connections

5.MD.5a → 5.MD.5b, 5.MD.5b → 5.MD.5c

Standard Explanation

Students measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units (5.MD.5 ▲). (CA Mathematics Framework, adopted Nov. 6, 2013)

5.MD.5 Illustrative Tasks:

- You Can Multiply Three Numbers in Any Order, <https://www.illustrativemathematics.org/illustrations/163>

Make sure you have plenty of snap cubes.

- Build a rectangular prism that is 2 cubes high, 3 cubes wide, and 5 cubes long.
- We will say that the volume of one cube is 1 cubic unit. What is the volume of the rectangular prism?
- The volume of the cube is $2 \times 3 \times 5$ cubic units. The expression

$$2 \times (3 \times 5)$$

can be interpreted as 2 groups with 3×5 cubes in each group. 3×5 can be interpreted as 3 groups with 5 cubes in each groups. How can you see the rectangular prism as being made of 2 groups with (3 groups of 5 cubes in each)?

- Explain how you can see each of these products by looking at the rectangular prism in different ways:

$$2 \times (5 \times 3)$$

$$3 \times (2 \times 5)$$

$$3 \times (5 \times 2)$$

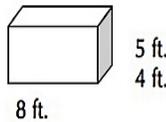
$$5 \times (2 \times 3)$$

$$5 \times (3 \times 2)$$

- Cari’s Aquarium, <https://www.illustrativemathematics.org/illustrations/1308>

Cari is the lead architect for the city’s new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.

- a. Cari’s first tank is 4 feet wide, 8 feet long and 5 feet high. How many cubic feet of water can her tank hold?



- b. Cari knows that a certain species of fish needs at least 240 cubic feet of water in their tank. Create 3 separate tanks that hold exactly 240 cubic feet of water. (Ex: She could design a tank that is 10 feet wide, 4 feet long and 6 feet in height.)
- c. In the back of the aquarium, Cari realizes that the ceiling is only 10 feet high. She needs to create a tank that can hold exactly 100 cubic feet of water. Name one way that she could build a tank that is not taller than 10 feet.

5.MD.C.5

Standards Connections

5.MD.5a → 5.MD.5b, 5.MD.5b → 5.MD.5c

Standard Explanation

Students measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units (5.MD.5 ▲). (CA Mathematics Framework, adopted Nov. 6, 2013)

5.MD.5 Illustrative Tasks:

- You Can Multiply Three Numbers in Any Order, <https://www.illustrativemathematics.org/illustrations/163>

Make sure you have plenty of snap cubes.

- Build a rectangular prism that is 2 cubes high, 3 cubes wide, and 5 cubes long.
- We will say that the volume of one cube is 1 cubic unit. What is the volume of the rectangular prism?
- The volume of the cube is $2 \times 3 \times 5$ cubic units. The expression

$$2 \times (3 \times 5)$$

can be interpreted as 2 groups with 3×5 cubes in each group. 3×5 can be interpreted as 3 groups with 5 cubes in each groups. How can you see the rectangular prism as being made of 2 groups with (3 groups of 5 cubes in each)?

- Explain how you can see each of these products by looking at the rectangular prism in different ways:

$$2 \times (5 \times 3)$$

$$3 \times (2 \times 5)$$

$$3 \times (5 \times 2)$$

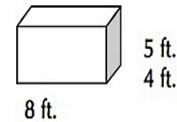
$$5 \times (2 \times 3)$$

$$5 \times (3 \times 2)$$

- Cari’s Aquarium, <https://www.illustrativemathematics.org/illustrations/1308>

Cari is the lead architect for the city’s new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.

- a. Cari’s first tank is 4 feet wide, 8 feet long and 5 feet high. How many cubic feet of water can her tank hold?



- b. Cari knows that a certain species of fish needs at least 240 cubic feet of water in their tank. Create 3 separate tanks that hold exactly 240 cubic feet of water. (Ex: She could design a tank that is 10 feet wide, 4 feet long and 6 feet in height.)
- c. In the back of the aquarium, Cari realizes that the ceiling is only 10 feet high. She needs to create a tank that can hold exactly 100 cubic feet of water. Name one way that she could build a tank that is not taller than 10 feet.



5.G.A Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Essential Skills and Concepts:

- Understand that the coordinate plane is create by two perpendicular lines (axes) that cross at the origin
- Plot points using coordinates and moving on the coordinate plane

Question Stems and Prompts:

- ✓ What are the parts of the coordinate plane?
- ✓ What direction does the x-axis run? Y-axis?
- ✓ Describe how to move to plot a given coordinate (x, y).

Vocabulary

Tier 2

- origin
- horizontal
- vertical
- points
- lines

Spanish Cognates

- origen
- horizontal
- vertical
- puntos
- líneas

Tier 3

- coordinate system
- coordinate plane
- first quadrant
- axis/axes
- x-axis
- y-axis
- intersection of lines
- ordered pairs
- coordinates
- x-coordinate
- y-coordinate

- sistema de coordenadas
- plano de coordenadas
- primero cuadrante
- eje/ejes
- eje x
- eje y
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Standards Connections

5.G.1 → 5.G.2

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Standard Explanation

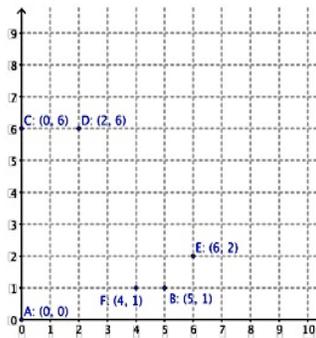
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Students need opportunities to create a coordinate grid, connect ordered pairs of coordinates to points on the grid, and describe how to get to the location (e.g., initially, an ordered pair (2, 3) could be described as a distance “2 from the origin along the x-axis and then 3 units up from the y-axis” or “right 2 and up 3”). For example:

Students might use a classroom size coordinate system to physically locate the coordinate points. For example, to locate the ordered pair (5, 3) students start at the origin point (0,0), then walk 5 units along the x-axis to find the first number in the pair (5), and then walk up 3 units for the second number in the pair (3). They continue this process to locate all the points in the following chart. Students recognize that ordered pairs name points in the plane.

Students graph and label the points below in a coordinate system.

- A (0, 0)
- B (5, 1)
- C (0, 6)
- D (2, 6)
- E (6, 2)
- F (4, 1)



California *Mathematics Framework*, adopted by the California State Board of Education November 6, 2013, <http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp>

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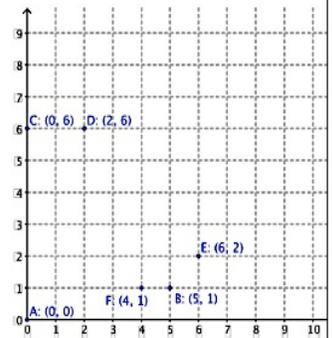
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Standards Connections

5.G.1 → 5.G.2

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Standards Connections

5.G.1 → 5.G.2

5. G.A Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Standard Explanation

In grade five students build on their previous work with number lines to use two perpendicular number lines to define a coordinate system (5.G.1). Students gain an understanding of the structure of the coordinate system. They learn the two axes make it possible to locate points on a coordinate plane and the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). This is the first time students work with coordinate planes, and at grade five this work is limited to the first quadrant.

Students need opportunities to create a coordinate grid, connect ordered pairs of coordinates to points on the grid, and describe how to get to the location (e.g., initially, an ordered pair (2, 3) could be described as a distance “2 from the origin along the x-axis and then 3 units up from the y-axis” or “right 2 and up 3”).

Students represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane (5.G.2). (*CA Mathematics Framework*, adopted Nov. 6, 2013)

5.G.2 Illustrative Task:

- Meerkat Coordinate Plane Task, <https://www.illustrativemathematics.org/illustrations/1516>

Greetings from the Kalahari Desert in South Africa! In this activity, you will learn a lot about the Kalahari’s most playful residents: meerkats.

- a. The following ordered pairs show the height of a typical meerkat at different times during the first 20 months of life. Graph the corresponding points and see what you can discover about meerkats. Once you have graphed them all, connect the points in the order they are given to form a line graph.



See if you can graph these ordered pairs:

- (0 months, 3 inches)
- (2 months, 5 inches)
- (4 months, 6 inches)
- (6 months, 7 inches)
- (8 months, 8 inches)
- (10 months, 9 inches)
- (12 months, 10 inches)
- (14 months, 12 inches)
- (16 months, 12 inches)
- (18 months, 12 inches)
- (20 months, 12 inches)

5. G.A Graph points on the coordinate plane to solve real-world and mathematical problems.

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5.G.B Classify two-dimensional figures into categories based on their properties.

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Essential Skills and Concepts:

- Understand that attributes of a category belong to all subcategories of that category
- Understand the relationships between shapes within the same category
- Explain the relationships between shapes within the same category
- Compare shapes by discussing their shared and unique attributes

Question Stems and Prompts:

- ✓ How are a parallelogram and a square related?
- ✓ What common attributes are shared with all quadrilaterals?
- ✓ What common attributes are shared with all triangles?
- ✓ Which attributes are unique to shapes within the hierarchy?
- ✓ How are the shapes in this category alike and different?

Vocabulary

Tier 2

- category
- subcategory
- hierarchy

Tier 3

- quadrilateral
- trapezoid
- parallelogram
- rectangle
- rhombus
- square
- triangles
- equilateral
- isosceles
- scalene
- right triangle
- obtuse triangle
- acute triangle

Spanish Cognates

- categoría
- subcategoría

- cuadrilátero
- trapecio
- paralelogramo
- rectángulo
- rombo

- triángulo
- equilátero
- isósceles
- escaleno
- triángulo obtuso
- triángulo agudo

Standards Connections

5.G.3 → 5.G.4

5.G.B Classify two-dimensional figures into categories based on their properties.

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Standards Connections

5.G.3 → 5.G.4

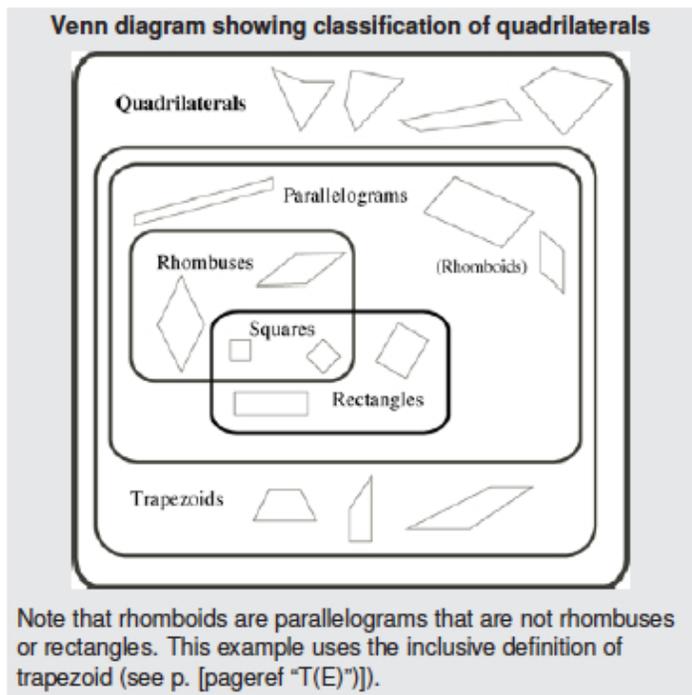
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Standard Explanation

In prior years, students described and compared properties of two-dimensional shapes and built, drew, and analyzed these shapes. In grade five students broaden their understanding to reason about the attributes (properties) of two-dimensional shapes and to classify these shapes in a hierarchy based on properties (5.G.4). Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). For example, students conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides. In this way, they relate certain categories of shapes as subclasses of other categories (5.G.3).

For example:



(Progressions K-6 G 2012 and KATM 5th 525 FlipBook 2012)

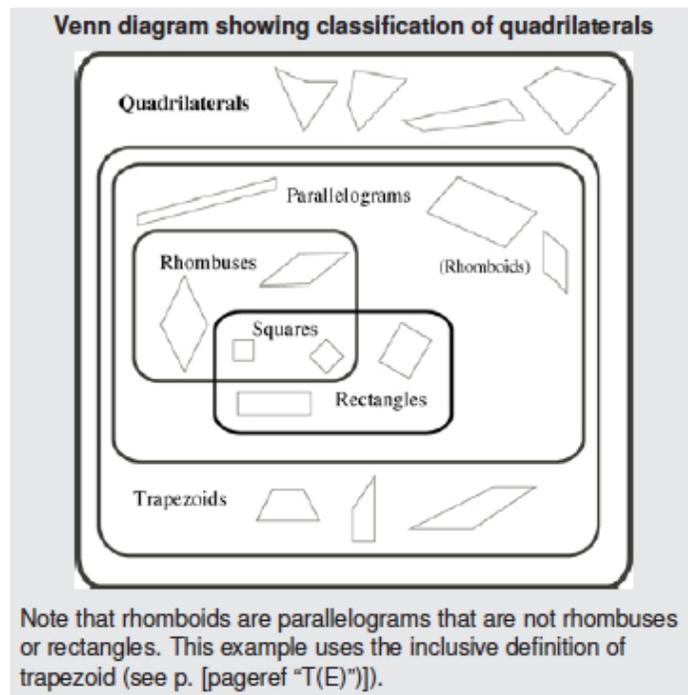
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5.G.B Classify two-dimensional figures into categories based on their properties.

5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

Essential Skills and Concepts:

- Understand and explain the hierarchy of shapes
- Understand the relationships between shapes within the same category
- Compare shapes using the hierarchy and describe common and unique attributes between them

Question Stems and Prompts:

- ✓ How are a parallelogram and a square related?
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Standards Connections

5.G.4 ← 5.G.3

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5.G.4 Illustrative Task:

- What is a Trapezoid? (Part 2)

<https://www.illustrativemathematics.org/illustrations/1505>

Niko and Carlos are studying parallelograms and trapezoids. They agree that a parallelogram is a quadrilateral with 2 pairs of parallel sides. Niko says,

A trapezoid has one pair of parallel sides and a parallelogram has two pairs of parallel sides. So a trapezoid is also a parallelogram.

Carlos says,

*No - a trapezoid can have **only one** pair of parallel sides.*

Niko says,

*That's not true. A trapezoid has **at least one** pair of parallel sides, but it can also have another.*

- With a partner, discuss the difference between Niko's definition and Carlos' definition for a trapezoid.
- Some people use Niko's definition for a trapezoid, and some people use Carlos' definition. Which statements below go with Niko's definition? Which statements go with Carlos' definition?
 - All parallelograms are trapezoids.
 - Some parallelograms are trapezoids.
 - No parallelograms are trapezoids.
 - All trapezoids are parallelograms.
 - Some trapezoids are parallelograms.
 - No trapezoids are parallelograms.

5.G.B Classify two-dimensional figures into categories based on their properties.

5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

Standard Explanation

In prior years, students described and compared properties of two-dimensional shapes and built, drew, and analyzed these shapes. In grade five students broaden their understanding to reason about the attributes (properties) of two-dimensional shapes and to classify these shapes in a hierarchy based on properties (5.G.4). Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). For example, students conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides. In this way, they relate certain categories of shapes as subclasses of other categories (5.G.3). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

5.G.4 Illustrative Task:

- What is a Trapezoid? (Part 2)

<https://www.illustrativemathematics.org/illustrations/1505>

Niko and Carlos are studying parallelograms and trapezoids. They agree that a parallelogram is a quadrilateral with 2 pairs of parallel sides. Niko says,

A trapezoid has one pair of parallel sides and a parallelogram has two pairs of parallel sides. So a trapezoid is also a parallelogram.

Carlos says,

*No - a trapezoid can have **only one** pair of parallel sides.*

Niko says,

*That's not true. A trapezoid has **at least one** pair of parallel sides, but it can also have another.*

- With a partner, discuss the difference between Niko's definition and Carlos' definition for a trapezoid.
- Some people use Niko's definition for a trapezoid, and some people use Carlos' definition. Which statements below go with Niko's definition? Which statements go with Carlos' definition?
 - All parallelograms are trapezoids.
 - Some parallelograms are trapezoids.
 - No parallelograms are trapezoids.
 - All trapezoids are parallelograms.
 - Some trapezoids are parallelograms.
 - No trapezoids are parallelograms.

Resources for the CCSS 5th Grade Bookmarks

California *Mathematics Framework*, adopted by the California State Board of Education November 6, 2013, <http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp>

Student Achievement Partners, Achieve the Core <http://achievethecore.org/>, Focus by Grade Level, <http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level>

Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics Tucson, AZ: Institute for Mathematics and Education, University of Arizona (Drafts)

- K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking (2011, May 29)
- K – 5, Number and Operations in Base Ten (2012, April 21)
- K – 3, Categorical Data; Grades 2 – 5, Measurement Data* (2011, June 20)
- K – 5, Geometric Measurement (2012, June 23)
- K – 6, Geometry (2012, June 23)
- Number and Operations – Fractions, 3 – 5 (2013, September 19)

Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, <http://www.illustrativemathematics.org/>

Student Achievement Partners, Achieve the Core <http://achievethecore.org/>, Focus by Grade Level, <http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level>

North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, <http://www.ncpublicschools.org/acre/standards/common-core-tools/-unmath>

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) <http://www.katm.org/baker/pages/common-core-resources.php>

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Arizona’s College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona’s College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication,
<http://www.azed.gov/azccrs/mathstandards/>

Howard County Public School System, Elementary Mathematics Office, Standards for Mathematical Practice for Parents, Draft 2011,
[https://grade3commoncoremath.wikispaces.hcpss.org/file/view/SFMP for Parents.docx/286906254/SFMP for Parents.docx](https://grade3commoncoremath.wikispaces.hcpss.org/file/view/SFMP%20for%20Parents.docx/286906254/SFMP%20for%20Parents.docx)

Howard County Public School System, Elementary and Secondary Mathematics Offices, Wiki Content and Resources, Elementary by grade level
<https://grade5commoncoremath.wikispaces.hcpss.org/home>, and Secondary
<https://secondarymathcommoncore.wikispaces.hcpss.org>

Long Beach Unified School District, Math Cognates, retrieved on 7/14/14,
http://www.lbschools.net/Main_Offices/Curriculum/Areas/Mathematics/XCD/ListOfMathCognates.pdf

A Graph of the Content Standards, Jason Zimba, June 7, 2012, <http://tinyurl.com/ccssmgraph>

Arizona’s College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona’s College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication,
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