Supporting California's Standards

Mathematics Bookmarks

Standards Reference to Support Planning and Instruction
http://commoncore.tcoe.org

8th Grade

Tulare County Office of Education

Tim A. Hire, County Superintendent of Schools

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Grade-Level Introduction

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

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(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various

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California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Eighth grade students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

Students:	Teachers:
• Analyze and explain the meaning of the problem • Actively engage in problem solving (Develop, carry out, and refine a plan) • Show patience and positive attitudes • Ask if their answers make sense • Check their answers with a different method	• Pose rich problems and/or ask open ended questions • Provide wait-time for processing/finding solutions • Circulate to pose probing questions and monitor student progress • Provide opportunities and time for cooperative problem solving and reciprocal teaching

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2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

Students:	Teachers:
• Represent a problem with symbols • Explain their thinking • Use numbers flexibly by applying properties of operations and place value • Examine the reasonableness of their answers/calculations	• Ask students to explain their thinking regardless of accuracy • Highlight flexible use of numbers • Facilitate discussion through guided questions and representations • Accept varied solutions/representations

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3. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

In eighth grade, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying and probing questions	• Provide opportunities for students to listen to or read the conclusions and arguments of others • Establish and facilitate a safe environment for discussion • Ask clarifying and probing questions • Avoid giving too much assistance (e.g., providing answers or procedures)

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4. **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying questions	• Allow time for the process to take place (model, make graphs, etc.) • Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) • Make appropriate tools available • Create an emotionally safe environment where risk taking is valued • Provide meaningful, real world, authentic, performance-based tasks (non traditional work problems)

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5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

Students:	Teachers:
• Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	• Make appropriate tools available for learning (calculators, concrete models, digital resources, pencil/paper, compass, protractor, etc.) • Use tools with their instruction

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6. **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Eighth grade students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

Students:	Teachers:
• Calculate accurately and efficiently • Explain their thinking using mathematics vocabulary • Use appropriate symbols and specify units of measure	• Recognize and model efficient strategies for computation • Use (and challenging students to use) mathematics vocabulary precisely and consistently

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7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

Students:	Teachers:
• Look for, develop, and generalize relationships and patterns • Apply reasonable thoughts about patterns and properties to new situations	• Provide time for applying and discussing properties • Ask questions about the application of patterns • Highlight different approaches for solving problems

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8. **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Students:	Teachers:
• Look for methods and shortcuts in patterns and repeated calculations • Evaluate the reasonableness of results and solutions	• Provide tasks and problems with patterns • Ask about possible answers before, and reasonableness after computations

8. **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Students:	Teachers:
• Look for methods and shortcuts in patterns and repeated calculations • Evaluate the reasonableness of results and solutions	• Provide tasks and problems with patterns • Ask about possible answers before, and reasonableness after computations

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Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connection between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major3 [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. The ▲ symbol will indicate standards in a Major Cluster in the narrative.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

8th Grade – CCSS for Mathematics

Grade 8 Overview

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8th Grade – CCSS for Mathematics

CCSS Where to Focus Grade 8 Mathematics

Not all of the content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have a greater emphasis is not to say that anything in the standards can be safely neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 8

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters □ Supporting Clusters ○ Additional Clusters

8.NS.A	□	Know that there are numbers that are not rational, and approximate them by rational numbers.
8.EE.A	■	Work with radicals and integer exponents.
8.EE.B	■	Understand the connections between proportional relationships, lines, and linear equations.
8.EE.C	■	Analyze and solve linear equations and pairs of simultaneous linear equations.
8.F.A	■	Define, evaluate, and compare functions.
8.F.B	■	Use functions to model relationships between quantities.
8.G.A	■	Understand congruence and similarity using physical models, transparencies, or geometry software.
8.G.B	■	Understand and apply the Pythagorean Theorem.
8.G.C	○	Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
8.SP.A	□	Investigate patterns of association in bivariate data.

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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8th Grade – CCSS for Mathematics

8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Essential Skills and Concepts:
- ☐ Understand that numbers are classified as rational or irrational
- ☐ Know that all numbers can be written in decimal form
- ☐ Show that all rational numbers can be written as a repeating or terminating decimal
- ☐ Convert repeating decimals into fraction form

Question Stems and Prompts:
- ✓ Is this number rational or irrational? How do you know?
- ✓ Write the number as a decimal.
- ✓ Convert the repeating decimal into a fraction or mixed number.
- ✓ Provide several examples of rational numbers and explain why they are rational.
- ✓ Provide several examples of irrational numbers and explain why they are irrational.

Vocabulary Spanish Cognates

Tier 2
- convert convertir
- rational racional
- irrational irracional

Tier 3
- decimal expansion expansión decimal
- repeating decimal
- terminating decimal

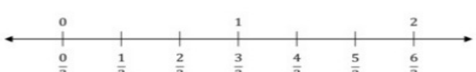
Standards Connections

7.NS.2d → *8.NS.1*, 8.NS.1 – *8.NS.2* – 8.EE.2

8.NS.1 Illustrative Tasks:
- Repeating or Terminating, https://www.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/1541

Tiffany said,

I know that 3 thirds equals 1 so $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$.



I also know that $\frac{1}{3}=0.333\ldots$ *where the 3's go on forever. But if I add them up as decimals, I get* $0.999\ldots$.

$$\begin{array}{r} 0.333\ldots \\ 0.333\ldots \\ +0.333\ldots \\ \hline 0.999\ldots \end{array}$$

I just added up the tenths, then the hundredths, then the thousands, and so on. What went wrong?

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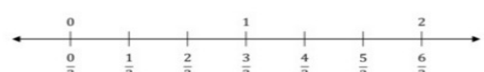

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8.NS.A.1

Standard Explanation

In seventh grade, adding, subtracting, multiplying, and dividing rational numbers was the culmination of numerical work with the four basic operations. The number system continues to develop in grade eight, expanding to the real numbers by introducing irrational numbers, and develops further in high school, expanding to become the complex numbers with the introduction of imaginary numbers (Adapted from PARCC 2012).

In grade eight, students learn that not all numbers can be expressed in the form a/b where a and b are positive or negative whole numbers with $b \neq 0$. Such numbers are called irrational, and students explore cases of both rational and irrational numbers and their decimal expansions to begin to understand the distinction. That rational numbers have eventually repeating decimal expansions is a direct result of how one uses long division to produce a decimal expansion. The full reasoning for why the converse is true, that eventually repeating decimals represent numbers that are rational, is beyond the scope of this grade. But students can use algebraic reasoning to show that eventually repeating decimals represent rational numbers in certain simple cases.

Since every decimal is of one of the two forms eventually repeating or non-repeating, this leaves irrational numbers as precisely those numbers whose decimal expansions do not have a repeating pattern. Students understand this informally in grade eight, and they experience approximating irrational numbers by rational numbers in simple cases. Ultimately, students should come to an informal understanding that the set of real numbers is comprised of rational numbers and irrational numbers. They will continue to work with irrational numbers and their rational approximations when solving equations as $x^2 = 18$ and in problems involving the Pythagorean Theorem. In the Expressions and Equations domain that follows, students learn to use radicals to represent such numbers. (Adapted from California Department of Education A Look at Grades Seven and Eight in California Public Schools: Transitioning to the Common Core State Standards in English Language Arts and Mathematics [CDE Transition Document] 2012, Arizona 2012, and N. Carolina 2013) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Why Rational Numbers Have Terminating or Repeating Decimal Expansions	8.NS.1
In each step of the standard algorithm to divide a by b, a partial quotient and a remainder are determined; the requirement is that each remainder is smaller than the divisor (b). In simpler examples, students will notice (or be led to notice) that once a remainder is repeated, the decimal repeats from that point onward, as in $\frac{1}{6} = 0.16666... = 0.1\overline{6}$ or $\frac{3}{11} = 0.272727... = 0.\overline{27}$. If a student imagines using long division to convert the fraction $\frac{3}{13}$ to a decimal without going through the tedium of actually producing the decimal, it can be reasoned that the possible remainders are 1 through 12. Consequently, a remainder that has already occurred will present itself by the thirteenth remainder, and therefore a repeating decimal results.	

8th Grade – CCSS for Mathematics

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8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi2$). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Essential Skills and Concepts:

- ☐ Approximate irrational numbers using rational numbers
- ☐ Compare irrational numbers using rational approximations
- ☐ Estimate the value of irrational numbers
- ☐ Plot the approximate locations of irrational numbers on a number line

Question Stems and Prompts:

- ✓ What is the approximate value of this irrational number? How do you know?
- ✓ Compare these two irrational numbers. Justify your comparison with numbers and words.
- ✓ Where would this irrational number be located on the number line?

Vocabulary Spanish Cognates

Tier 2

- approximate aproximado
- estimate estimación
- rational racional
- irrational irracional

Tier 3

- decimal expansion expansión decimal
- truncate the decimal

Standards Connections

8.NS.2 – *8.NS.1* – 8.EE.2

8.NS.2 Illustrative Tasks:

- Comparing Rational and Irrational Numbers, https://www.illustrativemathematics.org/illustrations/336

For each pair of numbers, decide which is larger without using a calculator. Explain your choices.

a. π^2 or 9

b. $\sqrt{50}$ or $\sqrt{51}$

c. $\sqrt{50}$ or 8

d. -2π or -6

- Placing A Square Root on the Number Line, https://www.illustrativemathematics.org/illustrations/336

Place $\sqrt{28}$ on a number line, accurate to one decimal point.

8th Grade – CCSS for Mathematics

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Ultimately, students should come to an informal understanding that the set of real numbers is comprised of rational numbers and irrational numbers. They will continue to work with irrational numbers and their rational approximations when solving equations as $x^2 = 18$ and in problems involving the Pythagorean Theorem. In the Expressions and Equations domain that follows, students learn to use radicals to represent such numbers. (Adapted from California Department of Education A Look at Grades Seven and Eight in California Public Schools: Transitioning to the Common Core State Standards in English Language Arts and Mathematics [CDE Transition Document] 2012, Arizona 2012, and N. Carolina 2013) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.NS.2 Examples:

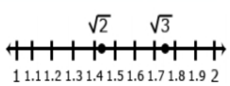
Example: Finding Better and Better Approximations of $\sqrt{2}$ 8.NS.2

The following reasoning may be used to approximate simple irrational square roots.

- Since $1^2 < 2 < 2^2$, then $\sqrt{1^2} < \sqrt{2} < \sqrt{2^2}$, which leads to $1 < \sqrt{2} < 2$. This means that $\sqrt{2}$ must be between 1 and 2.
- Since $1.4^2 = 1.96$ and $1.5^2 = 2.25$, students know by guessing and checking that $\sqrt{2}$ is between 1.4 and 1.5.
- Through additional guessing and checking, and by using a calculator, students see that since $1.41^2 = 1.9881$ and $1.42^2 = 2.0164$, $\sqrt{2}$ is between 1.41 and 1.42.

Continuing in this manner yields better and better approximations of $\sqrt{2}$. When students investigate this process with calculators, they gain some familiarity with the idea that the decimal expansion of $\sqrt{2}$ never repeats. Students should graph successive approximations on number lines to reinforce their understanding of the number line as a tool for representing real numbers.

Example 1:
Compare $\sqrt{2}$ and $\sqrt{3}$



Solution: Statements for the comparison could include:
$\sqrt{2}$ and $\sqrt{3}$ are between the whole numbers 1 and 2
$\sqrt{3}$ is between 1.7 and 1.8
$\sqrt{2}$ is less than $\sqrt{3}$

Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational.

Example 2:
Find an approximation of $\sqrt{28}$

- Determine the perfect squares $\sqrt{28}$ is between, which would be 25 and 36.
- The square roots of 25 and 36 are 5 and 6 respectively, so we know that $\sqrt{28}$ is between 5 and 6.
- Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27.
- The estimate of $\sqrt{28}$ would be 5.27 (the actual is 5.29).

8th Grade – CCSS for Mathematics

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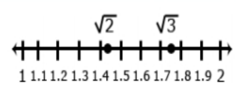

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8.EE.A Work with radicals and integer exponents.

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Essential Skills and Concepts:
- ☐ Solve problems involving integer exponents
- ☐ Create equivalent expressions using the properties of exponents
- ☐ Apply and explain the properties of exponents

Question Stems and Prompts:
- ✓ What is the equivalent expression for the given power?
- ✓ Explain the meaning of the power(s) and use this information to help you simplify the numerical expression.
- ✓ Simplify using the properties of exponents.

Vocabulary Spanish Cognates

Tier 2
- power
- base base

Tier 3
- equivalent equivalente
- numerical expressionexpresiones númerica
- exponent exponente
- integer

Standards Connections

6.EE.1 → *8.EE.1*
8.EE.1 → 8.EE.3, 8.EE.4

8.EE.1 Illustrative Task:
- Raising to the Zero and Negative Powers, https://www.illustrativemathematics.org/content-standards/8/EE/A/1/tasks/1438

In this problem c represents a positive number.

The quotient rule for exponents says that if m and n are positive integers with $m > n$, then

$$\frac{c^m}{c^n} = c^{m-n}.$$

After explaining to yourself why this is true, complete the following exploration of the quotient rule when $m \le n$:

a. What expression does the quotient rule provide for $\frac{c^m}{c^n}$ when $m = n$?

b. If $m = n$, simplify $\frac{c^m}{c^n}$ without using the quotient rule.

c. What do parts (a) and (b) above suggest is a good definition for c^0?

d. What expression does the quotient rule provide for $\frac{c^0}{c^n}$?

e. What expression do we get for $\frac{c^0}{c^n}$ if we use the value for c^0 found in part (c)?

f. Using parts (d) and (e), propose a definition for the expression c^{-n}.

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8th Grade – CCSS for Mathematics

8.EE.A Work with radicals and integer exponents.

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Essential Skills and Concepts:
- ☐ Solve problems involving integer exponents
- ☐ Create equivalent expressions using the properties of exponents
- ☐ Apply and explain the properties of exponents

Question Stems and Prompts:
- ✓ What is the equivalent expression for the given power?
- ✓ Explain the meaning of the power(s) and use this information to help you simplify the numerical expression.
- ✓ Simplify using the properties of exponents.

Vocabulary Spanish Cognates

Tier 2
- power
- base base

Tier 3
- equivalent equivalente
- numerical expressionexpresiones númerica
- exponent exponente
- integer

Standards Connections

6.EE.1 → *8.EE.1*
8.EE.1 → 8.EE.3, 8.EE.4

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d. What expression does the quotient rule provide for $\frac{c^0}{c^n}$?

e. What expression do we get for $\frac{c^0}{c^n}$ if we use the value for c^0 found in part (c)?

f. Using parts (d) and (e), propose a definition for the expression c^{-n}.

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8.EE.A Work with radicals and integer exponents.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Essential Skills and Concepts:

- ☐ Solve problems involving square and cube roots
- ☐ Evaluate the square roots of perfect squares
- ☐ Evaluate the cube roots of perfect cubes
- ☐ Know that $\sqrt{2}$ is irrational
- ☐ Know other common irrational numbers

Question Stems and Prompts:

- ✓ Find the square root/cube root.
- ✓ How do you know if a root is rational or irrational?
- ✓ Give an example of an irrational number and explain how you know that it is irrational.

Vocabulary Spanish Cognates

Tier 2

- rational racional
- irrational irracional

Tier 3

- square root
- cube root
- perfect square
- perfect cube cubo perfecto

Standards Connections

7.NS.3, 6.EE.5 → *8.EE.2*
8.NS.1 – *8.EE.2* – 8.NS.2
8.EE.2 – 8.G.6
8.EE.2 → 8.G.9

8th Grade – CCSS for Mathematics

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- perfect square
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Standards Connections

7.NS.3, 6.EE.5 → *8.EE.2*
8.NS.1 – *8.EE.2* – 8.NS.2
8.EE.2 – 8.G.6
8.EE.2 → 8.G.9

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8.EE.A.2

Standard Explanation

Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root $\sqrt{\ }$ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{x}$ are inverse operations. (Adapted from N. Carolina 2013)

Students do not learn the properties of rational exponents until high school. However, in grade eight they start to work systematically with the square root and cube root symbols, writing, for example, $\sqrt{64} = 8$ and $\sqrt[3]{5^3} = 5$. Since $\sqrt{p}$ is defined to only mean the positive solution to the equation x^2 = p (when it exists), it is not correct to say that $\sqrt{64} = \pm 8$. However, a correct solution to $x^2 = 64$ would be $x = \pm\sqrt{64} = \pm 8$. Students in grade eight are not in a position to prove that these are the only solutions, but rather use informal methods such as guess and check to verify them (Progressions 6-8 EE 2011).

Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students should generalize from many experiences that: (MP.2, MP.5, MP.6, and MP.7)

- Squaring a square root of a number returns the number back (e.g., $(\sqrt{5})^2 = 5$)
- Taking the square root of the square of a number sometimes returns the number back (e.g., $\sqrt{7^2} = \sqrt{49} = 7$, while $\sqrt{(-3)^2} = \sqrt{9} = 3 \neq -3$)
- Cubing a number and taking the cube root can be considered inverse operations.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

SBAC Sample Items:

Use the numbers shown to make the equations true. Each number can be used only once. To use a number, drag it to the appropriate box in an equation.

4 8 10 64 100 1,000

$\sqrt{\square} = \square$ $\sqrt[3]{\square} = \square$

Classify the numbers in the box as perfect squares and perfect cubes. To classify a number, drag it to the appropriate column in the chart. Numbers that are neither perfect squares nor perfect cubes should **not** be placed in the chart.

1 64 96 125 200 256 333 361

Perfect Squares but Not Perfect Cubes	Both Perfect Squares and Perfect Cubes	Perfect Cubes but Not Perfect Squares

8th Grade – CCSS for Mathematics

8.EE.A.2

Standard Explanation

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Perfect Squares but Not Perfect Cubes	Both Perfect Squares and Perfect Cubes	Perfect Cubes but Not Perfect Squares

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8.EE.A Work with radicals and integer exponents.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

Essential Skills and Concepts:
- ☐ Express very large or very small numbers using scientific notation
- ☐ Read and write numbers in scientific notation
- ☐ Write and explain multiplicative comparisons of numbers in scientific notation

Question Stems and Prompts:
- ✓ How would you rewrite this number using scientific notation?
- ✓ How many times larger/smaller is this amount than the other?
- ✓ Write the following numbers in scientific notation.
- ✓ Provide examples of real-life numbers that are written in scientific notation and explain why they need to be written this way.

Vocabulary Spanish Cognates

Tier 3
- scientific notation notación cientifica
- powers of ten

Standards Connections

5.NBT.2, 4.OA.2 → *8.EE.3*
8.EE.1 → *8.EE.3*
8.EE.3 – 8.EE.4

8th Grade – CCSS for Mathematics

8.EE.A Work with radicals and integer exponents.

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Vocabulary Spanish Cognates

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Standards Connections

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8.EE.1 → *8.EE.3*
8.EE.3 – 8.EE.4

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8.EE.A.3

Standard Explanation

Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents shown previously (8.EE.3–4▲) [adapted from CDE 2012d, ADE 2010, and NCDPI 2013b] (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Students recognize that if the exponent increases by one, the value increases 10 times. Likewise, if the exponent decreases by one, the value decreases 10 times. Students solve problems using addition, subtraction or multiplication, expressing the answer in scientific notation (Adapted from N. Carolina 2012).

Focus, Coherence, and Rigor

As students work with scientific notation, they learn to choose units of appropriate size for measurement of very large or very small quantities (**MP.2, MP.5, MP.6**).

8.EE.3 Illustrative Tasks:

- Orders of Magnitude, https://www.illustrativemathematics.org/content-standards/8/EE/A/3/tasks/1593

It is said that the average person blinks about 1000 times an hour. This is an *order-of-magnitude* estimate, that is, it is an estimate given as a power of ten. Consider:

- 100 blinks per hour, which is about two blinks per minute.
- 10,000 blinks per hour, which is about three blinks per second.

Neither of these are reasonable estimates for the number of blinks a person makes in an hour. Make order-of-magnitude estimates for each of the following:

a. Your age in hours.

b. The number of breaths you take in a year.

c. The number of heart beats in a lifetime.

d. The number of basketballs that would fill your classroom.

Can you think of others questions like these?

- Pennies to Heaven, https://www.illustrativemathematics.org/content-standards/8/EE/A/3/tasks/1291

A penny is about $\frac{1}{16}$ of an inch thick.

a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

8th Grade – CCSS for Mathematics

8.EE.A.3

Standard Explanation

Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents shown previously (8.EE.3–4▲) [adapted from CDE 2012d, ADE 2010, and NCDPI 2013b] (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Students recognize that if the exponent increases by one, the value increases 10 times. Likewise, if the exponent decreases by one, the value decreases 10 times. Students solve problems using addition, subtraction or multiplication, expressing the answer in scientific notation (Adapted from N. Carolina 2012).

Focus, Coherence, and Rigor

As students work with scientific notation, they learn to choose units of appropriate size for measurement of very large or very small quantities (MP.2, MP.5, MP.6).

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8th Grade – CCSS for Mathematics

8.EE.A Work with radicals and integer exponents.

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Essential Skills and Concepts:

- ☐ Add, subtract, multiply, and divide numbers in scientific notation (including those where both decimal and scientific notation are used)
- ☐ Select size appropriate measurements for real-life problems involving scientific notation
- ☐ Understand scientific notation values given while using a calculator or computer

Question Stems and Prompts:

- ✓ Interpret your solution after performing operations with scientific notation. What does your answer mean and how do you know?
- ✓ What units are most appropriate for this situation? How do you know?

Vocabulary	**Spanish Cognates**
Tier 3	
• scientific notation	notación cientifica
• decimal notation	notación decimal

Standards Connections

7.EE.3 → *8.EE.4*
8.EE.1 → *8.EE.4*
8.EE.3 – 8.EE.4

8.EE.4 Examples:

Example: Ants and Elephants 8.EE.4▲

An ant has a mass of approximately 4×10^{-3} grams, and an elephant has a mass of approximately 8 metric tons. How many ants does it take to have the same mass as an elephant?

(*Note:* 1 kg = 1000 grams, 1 metric ton = 1000 kg.)

Solution: To compare the masses of an ant and an elephant, we convert the mass of an elephant into grams:

$$8 \text{ metric tons} \times \frac{1000\text{kg}}{1 \text{ metric ton}} \times \frac{1000\text{g}}{1\text{kg}} = 8 \times 10^3 \times 10^3 \text{ grams} = 8\times10^6 \text{ grams}$$

If N represents the number of ants that have the same mass as an elephant, then $(4\times10^{-3})N$ is their total mass in grams. This should equal 8×10^6 grams, which yields a simple equation:

$(4\times10^{-3})N = 8\times10^6$, which means that

$$N = \frac{8\times10^6}{4\times10^{-3}} = 2\times10^{6-(-3)} = 2\times10^9$$

Therefore, 2×10^9 ants would have the same mass as an elephant.

Adapted from Illustrative Mathematics 2013f.

Focus, Coherence, and Rigor

The connection between the unit rate in a proportional relationship and the slope of its graph depends on a connection with the geometry of similar triangles (see standards **8.G.4–5▲**). The fact that a line has a well-defined slope—that the ratio between the rise and run for any two points on the line is always the same—depends on similar triangles.

Adapted from UA Progressions Documents 2011d.

8.EE.A Work with radicals and integer exponents.

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Essential Skills and Concepts:

- ☐ Add, subtract, multiply, and divide numbers in scientific notation (including those where both decimal and scientific notation are used)
- ☐ Select size appropriate measurements for real-life problems involving scientific notation
- ☐ Understand scientific notation values given while using a calculator or computer

Question Stems and Prompts:

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- ✓ What units are most appropriate for this situation? How do you know?

Vocabulary	**Spanish Cognates**
Tier 3	
• scientific notation	notación cientifica
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Standards Connections

7.EE.3 → *8.EE.4*
8.EE.1 → *8.EE.4*
8.EE.3 – 8.EE.4

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$(4\times10^{-3})N = 8\times10^6$, which means that

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Therefore, 2×10^9 ants would have the same mass as an elephant.

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8th Grade – CCSS for Mathematics

8.EE.A.4

Standard Explanation

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Students add and subtract with scientific notation. Students use laws of exponents to multiply or divide numbers written in scientific notation, writing the product or quotient in proper scientific notation. Students understand the magnitude of the number being expressed in scientific notation and choose an appropriate corresponding unit. Students understand scientific notation as generated on various calculators or other technology. Students enter scientific notation using E or EE (scientific notation), * (multiplication), and ^ (exponent) symbols.

8.EE.4 Illustrative Tasks:

- Choosing Appropriate Units, https://www.illustrativemathematics.org/content-standards/8/EE/A/4/tasks/1981

a. A computer has 128 gigabytes of memory. One gigabyte is 1×10^9 bytes. A floppy disk, used for storage by computers in the 1970's, holds about 80 kilobytes. There are 1000 bytes in a kilobyte. How many kilobytes of memory does a modern computer have? How many gigabytes of memory does a floppy disk have? Express your answers both as decimals and using scientific notation.

b. George told his teacher that he spent over 21,000 seconds working on his homework. Express this amount using scientific notation. What would be a more appropriate unit of time for George to use? Explain and convert to your new units.

- Giantburgers, https://www.illustrativemathematics.org/content-standards/8/EE/A/4/tasks/113

This headline appeared in a newspaper.

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about 8×10^3 Giantburger restaurants in America.
- Each restaurant serves on average 2.5×10^3 people every day.
- There are about 3×10^8 Americans.

Explain your reasons and show clearly how you figured it out.

8.EE.A.4

Standard Explanation

Students expand their exponent work as they perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation to express very large or very small numbers. Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents shown previously (8.EE.3–4▲) [adapted from CDE 2012d, ADE 2010, and NCDPI 2013b] (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Essential Skills and Concepts:
- ☐ Graph proportional relationships and unit rate, understanding that the unit rate is the slope of the graph
- ☐ Compare graphs, tables and equations of two different proportional relationships
- ☐ Sketch and interpret graphs of proportional relationships

Question Stems and Prompts:
- ✓ What is the unit rate of this graph? How do you know?
- ✓ Describe how to find the slope from different representations (i.e. table, graph, equation, etc.).
- ✓ Compare these two proportional relationships to determine which has a greater unit rate.

Vocabulary	**Spanish Cognates**
Tier 2	
• slope	
• origin	origen
Tier 3	
• proportional relationship.	relación proporcional
• unit rate	

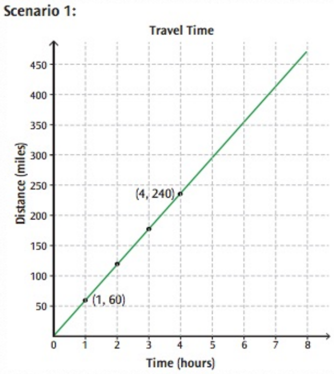
Standards Connections
7.RP.2 → *8.EE.5*
8.EE.5 – 8.EE.6
8.EE.5 → 8.F.2

8.EE.5 Example:

Example **8.EE.5▲**

Compare the scenarios below to determine which represents a greater speed. Include in your explanation a description of each scenario that discusses unit rates.

Scenario 1:



Scenario 2: The equation for the distance y in miles as a function of the time x in hours is:

$$y = 55x$$

Solution: "The unit rate in Scenario 1 can be read from the graph; it is 60 miles per hour. In Scenario 2, I can see that this looks like an equation $y = kx$, and in that type of equation the unit rate is the constant k. Therefore, the speed in Scenario 2 is 55 miles per hour. So the person traveling in Scenario 1 is moving at a faster rate."

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

8th Grade – CCSS for Mathematics

8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Essential Skills and Concepts:
- ☐ Graph proportional relationships and unit rate, understanding that the unit rate is the slope of the graph
- ☐ Compare graphs, tables and equations of two different proportional relationships
- ☐ Sketch and interpret graphs of proportional relationships

Question Stems and Prompts:
- ✓ What is the unit rate of this graph? How do you know?
- ✓ Describe how to find the slope from different representations (i.e. table, graph, equation, etc.).
- ✓ Compare these two proportional relationships to determine which has a greater unit rate.

Vocabulary	**Spanish Cognates**
Tier 2	
• slope	
• origin	origen
Tier 3	
• proportional relationship.	relación proporcional
• unit rate	

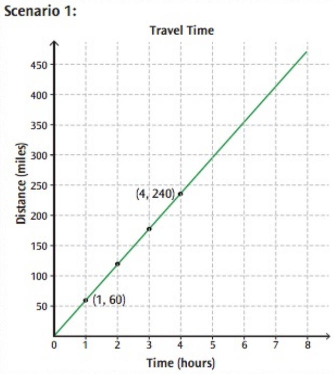
Standards Connections
7.RP.2 → *8.EE.5*
8.EE.5 – 8.EE.6
8.EE.5 → 8.F.2

8.EE.5 Example:

Example **8.EE.5▲**

Compare the scenarios below to determine which represents a greater speed. Include in your explanation a description of each scenario that discusses unit rates.

Scenario 1:



Scenario 2: The equation for the distance y in miles as a function of the time x in hours is:

$$y = 55x$$

Solution: "The unit rate in Scenario 1 can be read from the graph; it is 60 miles per hour. In Scenario 2, I can see that this looks like an equation $y = kx$, and in that type of equation the unit rate is the constant k. Therefore, the speed in Scenario 2 is 55 miles per hour. So the person traveling in Scenario 1 is moving at a faster rate."

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

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8.EE.B.5

Standard Explanation

Students build on their work with unit rates from sixth grade and proportional relationships in seventh grade to compare graphs, tables, and equations of proportional relationships (8.EE.5▲). Students identify the unit rate (or slope) to compare two proportional relationships represented in different ways (e.g., as graph of the line through the origin, a table exhibiting a constant rate of change, or an equation of the form $y = kx$). Students interpret the unit rate in a proportional relationship (e.g., r miles per hour) as the slope of the graph. They understand that the slope of a line represents a constant rate of change (CA *Mathematics Framework,* adopted Nov. 6, 2013).

8.EE.5 Illustrative Tasks:

- Coffee by the Pound, https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/129

Lena paid $18.96 for 3 pounds of coffee.

a. What is the cost per pound for this coffee?

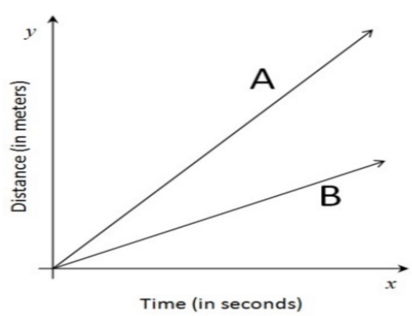
b. How many pounds of coffee could she buy for $1.00?

c. Draw a graph in the coordinate plane of the relationship between the number of pounds of coffee and the total cost.

d. In this situation, what is the meaning of the slope of the line you drew in part (c)?

- Comparing Speeds in Graphs and Equations, https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/57

The graphs below show the distance two cars have traveled along the freeway over a period of several seconds. Car A is traveling 30 meters per second.



Which equation from those shown below is the best choice for describing the distance traveled by car B after x seconds? Explain.

Which equation from those shown below is the best choice for describing the distance traveled by car B after x seconds? Explain.

a. $y = 85x$

b. $y = 60x$

c. $y = 30x$

d. $y = 15x$

8th Grade – CCSS for Mathematics

8.EE.B.5

Standard Explanation

Students build on their work with unit rates from sixth grade and proportional relationships in seventh grade to compare graphs, tables, and equations of proportional relationships (8.EE.5▲). Students identify the unit rate (or slope) to compare two proportional relationships represented in different ways (e.g., as graph of the line through the origin, a table exhibiting a constant rate of change, or an equation of the form $y = kx$). Students interpret the unit rate in a proportional relationship (e.g., r miles per hour) as the slope of the graph. They understand that the slope of a line represents a constant rate of change (CA *Mathematics Framework,* adopted Nov. 6, 2013).

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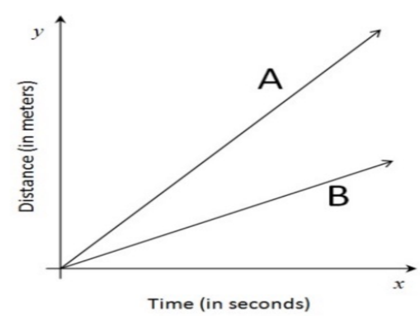
- Comparing Speeds in Graphs and Equations, https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/57

The graphs below show the distance two cars have traveled along the freeway over a period of several seconds. Car A is traveling 30 meters per second.



Which equation from those shown below is the best choice for describing the distance traveled by car B after x seconds? Explain.

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a. $y = 85x$

b. $y = 60x$

c. $y = 30x$

d. $y = 15x$

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8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

8.EE.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Essential Skills and Concepts:
- ☐ Create equations for lines that pass through or do not pass through the origin.
- ☐ Explain the created equations in terms of their slope and y-intercept.
- ☐ Understand similar triangles and use them to explain why the slope is same between two points.

Question Stems and Prompts:
- ✓ Looking at these two triangles, are they similar? How do you know? Explain.
- ✓ Which formula will use you use for this line? How do you know?
- ✓ Create an equation for each line on the graph. Explain your equation describing how you used the slope and y-intercept.

Vocabulary	**Spanish Cognates**
Tier 2	
• slope	
• origin	origen
• similar	similar
Tier 3	
• similar triangle	

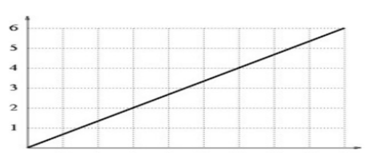
Standards Connections
7.G.1, 7.RP.2 → *8.EE.6*
8.EE.6 – 8.EE.5
8.EE.6 → 8.EE.8, 8.F.2, 8.F.3

8.EE.6 Illustrative Task:
- Slopes Between Points on a Line, https://www.illustrativemathematics.org/content-standards/8/EE/B/6/tasks/1537

The slope between two points is calculated by finding the change in y-values and dividing by the change in x-values. For example, the slope between the points (7, -15) and (-8, 22) can be computed as follows:

- The difference in the y-values is $-15 - 22 = -37$.
- The difference in the x-values is $7 - (-8) = 15$.
- Dividing these two differences, we find that the slope is $-\frac{37}{15}$.

Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.



Eva finds the slope between the points (0,0) and (3,2). Carl finds the slope between the points (3,2) and (6,4). Maria finds the slope between the points (3,2) and (9,6). They have each drawn a triangle to help with their calculations (shown below).

8th Grade – CCSS for Mathematics

8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

8.EE.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Essential Skills and Concepts:
- ☐ Create equations for lines that pass through or do not pass through the origin.
- ☐ Explain the created equations in terms of their slope and y-intercept.
- ☐ Understand similar triangles and use them to explain why the slope is same between two points.

Question Stems and Prompts:
- ✓ Looking at these two triangles, are they similar? How do you know? Explain.
- ✓ Which formula will use you use for this line? How do you know?
- ✓ Create an equation for each line on the graph. Explain your equation describing how you used the slope and y-intercept.

Vocabulary **Spanish Cognates**
Tier 2
• slope
• origin origen
• similar similar
Tier 3
• similar triangle

Standards Connections
7.G.1, 7.RP.2 → *8.EE.6*
8.EE.6 – 8.EE.5
8.EE.6 → 8.EE.8, 8.F.2, 8.F.3

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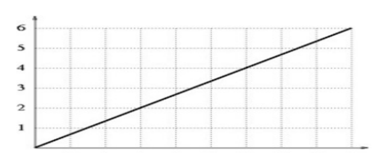

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8th Grade – CCSS for Mathematics

8.EE.B.6

Standard Explanation

Standard (8.EE.6▲) represents a convergence of several ideas in this and previous grade levels. Students have graphed proportional relationships and found the slope of the resulting line, interpreting it as the unit rate (8.EE.5▲). It is here that the language of "rise over run" comes into use. In the Functions domain, students will see that any linear equation $y = mx + b$ determines a function whose graph is a straight line (a linear function), and they verify that the slope of the line is equal to m (8.F.3). In standard (8.EE.6▲), students go further and explain why the slope m is the same through any two points on a line. They justify this fact using similar triangles, which are studied in standards (8.G.4-5▲).

In grade eight students build on previous work with proportional relationships, unit rates, and graphing to connect these ideas and understand that the points (x, y) on a non-vertical line are the solutions of the equation $y = mx + b$, where m is the slope of the line, as well as the unit rate of a proportional relationship in the case $b = 0$ (CA *Mathematics Framework,* adopted Nov. 6, 2013).

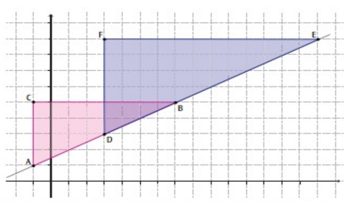
8.EE.6 Examples:

Example of Reasoning **8.EE.6▲**

Show that the slope is the same between any two points on a line.

In grade seven, students made scale drawings of figures and observed the proportional relationships between side lengths of such figures (**7.G.1▲**). In grade eight, students generalize this idea and study ***dilations*** of plane figures, and they define figures as being ***similar*** in terms of dilations (see standard **8.G.4▲**). Students discover that similar figures share a proportional relationship between side lengths, just as scale drawings did: there is a *scale factor* $k > 0$ such that corresponding side lengths of similar figures are related by the equation $s_1 = k \bullet s_2$. Furthermore, the ratio of two sides in one shape is equal to the ratio of the corresponding two sides in the other shape. Finally, standard **8.G.5▲** calls for students to informally argue that triangles with two corresponding angles of the same measure must be similar, and this is the final piece of the puzzle for using similar triangles to show that the slope is the same between any two points on the coordinate plane (**8.EE.6▲**).

Explain why the slopes between points A and B and points D and E are the same.



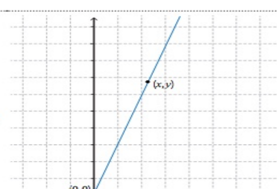
Solution: "$\angle A$ and $\angle D$ are equal because they are corresponding angles formed by the transversal crossing the vertical lines through points A and D. Since $\angle C$ and $\angle F$ are both right angles, the triangles are similar.

This means the ratios $\frac{AC}{BC}$ and $\frac{DF}{EF}$ are equal. But when you find the 'rise over the run,' these are the exact ratios you find, so the slope is the same between these two sets of points."

Additional Examples of Reasoning **8.EE.6▲**

Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Example 1: Explain how to derive the equation $y = 3x$ for the line of slope $m = 3$ shown at right.

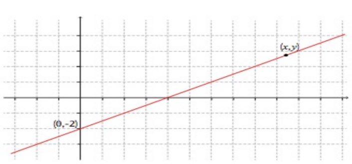


Solution: "I know that the slope is the same between any two points on a line. So I'll choose the origin $(0,0)$ and a generic point on the line, calling it (x, y). By choosing a generic point like this, I know that any point on the line will fit the equation I come up with. The slope between these two points is found by

$$3 = \frac{\text{rise}}{\text{run}} = \frac{y-0}{x-0} = \frac{y}{x}$$

This equation can be rearranged to $y = 3x$."

Example 2: Explain how to derive the equation $y = \frac{1}{2}x - 2$ for the line of slope $m = \frac{1}{2}$ with intercept $b = -2$.



Solution: "I know the slope is $\frac{1}{2}$, so I'll determine the equation of the line using the slope formula, with the point $(0,-2)$ and the generic point (x, y). The slope between these two points is found by

$$\frac{1}{2} = \frac{\text{rise}}{\text{run}} = \frac{y-(-2)}{x-0} = \frac{y+2}{x}$$

This can be rearranged to $y + 2 = \frac{1}{2}x$, which is the same as $y = \frac{1}{2}x - 2$."

8.EE.B.6

Standard Explanation

Standard (8.EE.6▲) represents a convergence of several ideas in this and previous grade levels. Students have graphed proportional relationships and found the slope of the resulting line, interpreting it as the unit rate (8.EE.5▲). It is here that the language of "rise over run" comes into use. In the Functions domain, students will see that any linear equation $y = mx + b$ determines a function whose graph is a straight line (a linear function), and they verify that the slope of the line is equal to m (8.F.3). In standard (8.EE.6▲), students go further and explain why the slope m is the same through any two points on a line. They justify this fact using similar triangles, which are studied in standards (8.G.4-5▲).

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8.EE.6 Examples:

Example of Reasoning **8.EE.6▲**

Show that the slope is the same between any two points on a line.

In grade seven, students made scale drawings of figures and observed the proportional relationships between side lengths of such figures (**7.G.1▲**). In grade eight, students generalize this idea and study ***dilations*** of plane figures, and they define figures as being ***similar*** in terms of dilations (see standard **8.G.4▲**). Students discover that similar figures share a proportional relationship between side lengths, just as scale drawings did: there is a *scale factor* $k > 0$ such that corresponding side lengths of similar figures are related by the equation $s_1 = k \bullet s_2$. Furthermore, the ratio of two sides in one shape is equal to the ratio of the corresponding two sides in the other shape. Finally, standard **8.G.5▲** calls for students to informally argue that triangles with two corresponding angles of the same measure must be similar, and this is the final piece of the puzzle for using similar triangles to show that the slope is the same between any two points on the coordinate plane (**8.EE.6▲**).

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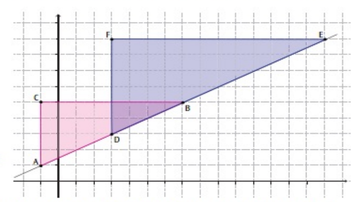

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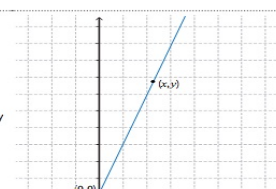
This means the ratios $\frac{AC}{BC}$ and $\frac{DF}{EF}$ are equal. But when you find the 'rise over the run,' these are the exact ratios you find, so the slope is the same between these two sets of points."

Additional Examples of Reasoning **8.EE.6▲**

Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Example 1: Explain how to derive the equation $y = 3x$ for the line of slope $m = 3$ shown at right.



Solution: "I know that the slope is the same between any two points on a line. So I'll choose the origin $(0,0)$ and a generic point on the line, calling it (x, y). By choosing a generic point like this, I know that any point on the line will fit the equation I come up with. The slope between these two points is found by

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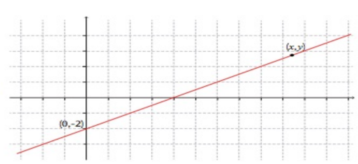

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8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Essential Skills and Concepts:

- ☐ Solve linear equations in one variable using equivalent equations, using the distributive property, and/or combining like terms
- ☐ Know that linear equations can have one solution, infinite solutions, or no solutions
- ☐ Give examples of linear equations with one solution, infinite solutions, or no solutions

Question Stems and Prompts:

- ✓ Solve this equation. Explain your thinking and provide justification for your steps.
- ✓ Does this equation have one solution, infinite solutions, or no solutions? How do you know?
- ✓ Provide an example of a linear equation with one solution, infinite solutions, and no solutions. Explain why each of the equations you provided has that type of solution.

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
• like terms	
Tier 3	
• linear equations	ecuación lineal
• coefficients	coeficientes
• distributive property	propiedad distributiva

Standards Connections

7.EE.1, 7.EE.4a → *8.EE.7b*

8.EE.7b – 8.SP.3

8.EE.7 Illustrative Task:

- Coupon vs. Discount, https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/583

You have a coupon worth $18 off the purchase of a scientific calculator. At the same time the calculator is offered with a discount of 15%, but no further discounts may be applied. For what tag price on the calculator do you pay the same amount for each discount?

8th Grade – CCSS for Mathematics

8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Essential Skills and Concepts:

- ☐ Solve linear equations in one variable using equivalent equations, using the distributive property, and/or combining like terms
- ☐ Know that linear equations can have one solution, infinite solutions, or no solutions
- ☐ Give examples of linear equations with one solution, infinite solutions, or no solutions

Question Stems and Prompts:

- ✓ Solve this equation. Explain your thinking and provide justification for your steps.
- ✓ Does this equation have one solution, infinite solutions, or no solutions? How do you know?
- ✓ Provide an example of a linear equation with one solution, infinite solutions, and no solutions. Explain why each of the equations you provided has that type of solution.

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
• like terms	
Tier 3	
• linear equations	ecuación lineal
• coefficients	coeficientes
• distributive property	propiedad distributiva

Standards Connections

7.EE.1, 7.EE.4a → *8.EE.7b*

8.EE.7b – 8.SP.3

8.EE.7 Illustrative Task:

- Coupon vs. Discount, https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/583

You have a coupon worth $18 off the purchase of a scientific calculator. At the same time the calculator is offered with a discount of 15%, but no further discounts may be applied. For what tag price on the calculator do you pay the same amount for each discount?

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8.EE.C.7

Standard Explanation

Students have worked informally with one-variable linear equations as early as kindergarten. This important line of development culminates in grade eight as much of students' work involves analyzing and solving linear equations and pairs of simultaneous linear equations.

Grade eight students solve linear equations in one variable, including cases with one solution, infinitely many solutions, and no solutions (8.EE.7▲). Students show examples of each of these cases by successively transforming an equation into simpler forms ($x = a$, $a = a$, and $a = b$, where a and b represent different numbers). Solving some linear equations will require students to expand expressions using the distributive property and to collect like terms.

To be prepared for the higher mathematics courses students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade eight. Prior to grade eight, some standards identify fluency expectations at the grade level. In eighth grade linear algebra is an instructional focus and although the grade eight standards do not specifically identify fluency expectations, eighth grade students who can fluently solve linear equations (8.EE.7▲) and pairs of simultaneous linear equations (8.EE.8▲) will be better prepared to complete courses in higher mathematics. These fluencies and the conceptual understandings that support them are foundational for work in higher mathematics. Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade eight with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations.

Of particular importance for students to attain in grade eight are skills and understandings to work with radical and integer exponents (8.EE.1-4▲); understand connections between proportional relationships, lines, and linear 564 equations (8.EE.5-6▲); analyze and solve linear equations and pairs of simultaneous linear equations (8.EE.7-8▲); and define, evaluate, and compare functions (8.F.1-3▲). In addition, the skills and understandings to use functions to model relationships between quantities (8.F.4-5) will better prepare students to use mathematics to model real-world problems in the higher grades (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Solutions to One-Variable Equations — **8.EE.7a▲**

- When an equation has only one solution, there is only one value of the variable that makes the equation true (e.g., $12-4y=16$).
- When an equation has an infinite number of solutions, the equation is true for all real numbers and is sometimes referred to as an *identity*—for example, $7x+14=7(x+2)$. Solving this equation by using familiar steps might yield $14=14$, a statement that is true regardless of the value of x. Students should be encouraged to think about why this means that any real number solves the equation and relate it back to the original equation (e.g., the equation is showing the distributive property).
- When an equation has no solutions, it is sometimes described as *inconsistent*—for example, $5x-2=5(x+1)$. Attempting to solve this equation might yield $-2=5$, which is a false statement regardless of the value of x. Students should be encouraged to reason why there are no solutions to the equation; for example, they observe that the original equation is equivalent to $5x-2=5x+5$ and reason that it is never the case that $N-2=N+5$, no matter what N is.

Adapted from ADE 2010.

8th Grade – CCSS for Mathematics

8.EE.C.7

Standard Explanation

Students have worked informally with one-variable linear equations as early as kindergarten. This important line of development culminates in grade eight as much of students' work involves analyzing and solving linear equations and pairs of simultaneous linear equations.

Grade eight students solve linear equations in one variable, including cases with one solution, infinitely many solutions, and no solutions (8.EE.7▲). Students show examples of each of these cases by successively transforming an equation into simpler forms ($x = a$, $a = a$, and $a = b$, where a and b represent different numbers). Solving some linear equations will require students to expand expressions using the distributive property and to collect like terms.

To be prepared for the higher mathematics courses students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade eight. Prior to grade eight, some standards identify fluency expectations at the grade level. In eighth grade linear algebra is an instructional focus and although the grade eight standards do not specifically identify fluency expectations, eighth grade students who can fluently solve linear equations (8.EE.7▲) and pairs of simultaneous linear equations (8.EE.8▲) will be better prepared to complete courses in higher mathematics. These fluencies and the conceptual understandings that support them are foundational for work in higher mathematics. Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade eight with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations.

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8th Grade – CCSS for Mathematics

8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Essential Skills and Concepts:

- ☐ Understand that the solution of a system of linear equations is the point where the two equations intersect
- ☐ Solve systems of linear equations algebraically, graphically, or by inspection
- ☐ Solve real-world and mathematical problems involving systems of linear equations

Question Stems and Prompts:

- ✓ What does the solution of a system of linear equations mean? Explain your thinking.
- ✓ Solve the system of linear equations algebraically, graphically, or by inspection. Explain the method you chose for solving the system.
- ✓ What does the solution of this system mean in the context of its real-world situation?

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
• simultaneous	simultáneo
• intersect	
Tier 3	
• system of equations	sistema de ecuaciones
• simultaneous linear equations	
• point of intersection	punto de intersección

Standards Connections

6.EE.5, 7.EE.4a, 8.EE.6 → *8.EE.8*

8.EE.8 Illustrative Task:

- How Many Solutions?, https://www.illustrativemathematics.org/content-standards/8/EE/C/8/tasks/554

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Tier 2	
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8.EE.C.8

Standard Explanation

Grade eight students also analyze and solve pairs of simultaneous linear equations. Solving pairs of simultaneous linear equations builds on the skills and understandings students used to solve linear equations with one variable (8.EE.8 a-c▲), and systems of linear equations can also have one solution, infinitely many solutions, or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically. Grade eight students learn that for a system of linear equations:

- If the graphs of the lines meet at one point (the lines intersect), then there is one solution, the ordered pair of the point of intersection representing the solution of the system.
- If the graphs of the lines do not meet (the lines are parallel), the system has no solutions, and the slopes of these lines are the same.
- If the graphs of the lines are coincident (the graphs are exactly the same line) then the system has infinitely many solutions, the solutions being the set of all ordered pairs on the line.

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions.

Students solve real-world and mathematical problems leading to two linear equations in two variables. Below is an example of how reasoning about real-world situations can be used to introduce and make sense out of solving systems of equations by elimination. The technique of elimination can be used in general cases to solve systems of equations.

To be prepared for the higher mathematics courses students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade eight. Prior to grade eight, some standards identify fluency expectations at the grade level. In eighth grade linear algebra is an instructional focus and although the grade eight standards do not specifically identify fluency expectations, eighth grade students who can fluently solve linear equations (8.EE.7▲) and pairs of simultaneous linear equations (8.EE.8▲) will be better prepared to complete courses in higher mathematics. These fluencies and the conceptual understandings that support them are foundational for work in higher mathematics. Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade eight with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8th Grade – CCSS for Mathematics

8.EE.C.8

Standard Explanation

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8th Grade – CCSS for Mathematics

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8.F.A Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

Essential Skills and Concepts:
- ☐ Understand and explain what functions are
- ☐ Know the association between the input/x-value and output/y-value within functions
- ☐ Understand what the graph of a function looks like

Question Stems and Prompts:
- ✓ What is a function?
- ✓ Determine if the equation is a function using a table, equation or graph. Explain your thinking.
- ✓ Create an example of a function and explain how you know that it is a function.
- ✓ Create an example that is not a function and explain why it is not a function.

Vocabulary	**Spanish Cognates**
Tier 2	
• Function	función
• input	
• output	
Tier 3	
• ordered pair	par ordenado

Standards Connections
7.RP.2 → *8.F.1*
8.F.1 → 8.F.2, 8.F.3, 8.F.5

8.F.1 Example:

Example: Introduction to Functions 8.F.1▲

To introduce the concept of a function, a teacher might have students contrast two workers' wages at two different jobs, one with an hourly wage and the other based on a combination of an hourly wage and tips. Students read through scenarios and make a table for each of the two workers, listing hours worked and money earned during 20 different shifts varying from 3 to 8 hours in length. Students answer questions about the data, including the level of predictability of the wage of each worker, based on the number of hours worked. Students graph the data and observe the patterns of the graph. Next, the teacher could introduce the concept of a function and relate the tables and graphs from the activity to the idea of a function, emphasizing that an input value completely determines an output value. Students could then be challenged to find other quantities that are functions and to create and discuss corresponding tables and/or graphs.

SBAC Sample Item:

Fill in each x-value and y-value in the table below to create a relation that is **not** a function.

x	y

[1] Function notation is not required in Grade 8.

8th Grade – CCSS for Mathematics

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Tier 2
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8.F.A.1

Standard Explanation

In grade seven, students learned to determine if two quantities represented a proportional relationship. Proportional reasoning is a transitional topic, coming between arithmetic and algebra. Underlying the progression from proportional reasoning through algebra and beyond is the idea of a function— a rule that assigns to each input exactly one output. In grade eight a critical area of instruction is the concept of a function. Students are introduced to functions, and they learn proportional relationships are part of a broader group of linear functions.

In grade eight, students understand two main points in regards to functions (8.F.1▲):

- A *function* is a rule that assigns to each input exactly one output, and
- The *graph* of a function is the set of ordered pairs consisting of an input and the corresponding output.

In general, students understand that functions describe situations in which one quantity determines another. The main work in grade eight concerns linear functions, though students are exposed to non-linear functions to contrast them with linear functions. Thus, students may view a linear equation like $y = -.75x + 12$ as a rule that defines a quantity y whenever the quantity x is given. In this case, the function may describe the amount of money remaining after x turns when a student who starts with $12 plays a game that costs $.75 per turn. Or, students may view the formula for the area of a circle, $A = \pi r^2$ as a (non-linear) function in the sense that the area of a circle is dependent on its radius. Student work with functions at grade eight remains informal, but sets the stage for more formal work in the higher mathematics courses (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.F.1 Illustrative Task:

- Foxes and Rabbits, https://www.illustrativemathematics.org/content-standards/8/F/A/1/tasks/713

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of t corresponds to the beginning of the month.

t, month	1	2	3	4	5	6	7	8	9	10	11	12
R, number of rabbits	1000	750	567	500	567	750	1000	1250	1433	1500	1433	1250
F, number of foxes	150	143	125	100	75	57	50	57	75	100	125	143

a. According to the data in the table, is F a function of R? Is R a function of F? Explain.

b. Are either R or F functions of t? Explain.

8th Grade – CCSS for Mathematics

8.F.A.1

Standard Explanation

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8.F.A Define, evaluate, and compare functions.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Essential Skills and Concepts:
- ☐ Identify and describe functions from equations, graphs, and tables/ordered pairs
- ☐ Compare functions that are represented in different ways

Question Stems and Prompts:
- ✓ Determine if the equation is a function using a table, equation or graph. Explain your thinking.
- ✓ Which of these functions has a greater rate of change? How do you know?
- ✓ Compare these functions giving specific evidence about how they are similar and different.

Vocabulary	**Spanish Cognates**
Tier 2	
• properties	propiedades
• function	función
• verbal description	descripción verbal
Tier 3	
• algebraically	algebraicamente
• graphically	gráficamente
• function tables	tabla de funciones

Standards Connections
7.RP.2, 8.EE.5, 8.EE.6, 8.F.1 → *8.F.2*
8.F.2 → 8.F.3, 8.F.5

8.F.2 Examples:

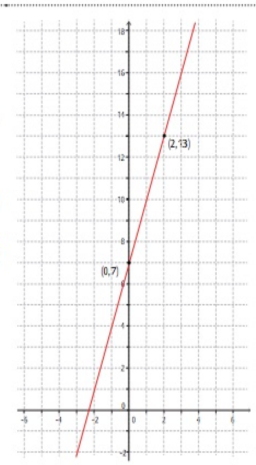
Example: Functions Represented Differently 8.F.2▲

Which function has a greater rate of change? (MP.1, MP.2, MP.4, MP.8)

Function 1: The function represented by the graph shown.

Function 2: The function whose input x and output y are related by the equation $y = 4x + 7$.

Solution: The graph of the function shows that when $x = 0$, the value of the function is $y = 7$, and when $x = 2$ the value of the function is $y = 13$. This means that function 1 increases by 6 units when x increases by 2 units. Function 2 also has an output of $y = 7$ when $x = 0$, but when $x = 2$, the value of function 2 is $y = 15$. This means that function 2 increases by 8 units when x increases by 2 units. Therefore, function 2 has a greater rate of change.



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8th Grade – CCSS for Mathematics

8.F.A Define, evaluate, and compare functions.

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Essential Skills and Concepts:
☐ Identify and describe functions from equations, graphs, and tables/ordered pairs
☐ Compare functions that are represented in different ways

Question Stems and Prompts:
✓ Determine if the equation is a function using a table, equation or graph. Explain your thinking.
✓ Which of these functions has a greater rate of change? How do you know?
✓ Compare these functions giving specific evidence about how they are similar and different.

Vocabulary / Spanish Cognates
Tier 2
• properties — propiedades
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• algebraically — algebraicamente
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• function tables — tabla de funciones

Standards Connections
7.RP.2, 8.EE.5, 8.EE.6, 8.F.1 → 8.F.2
8.F.2 → 8.F.3, 8.F.5

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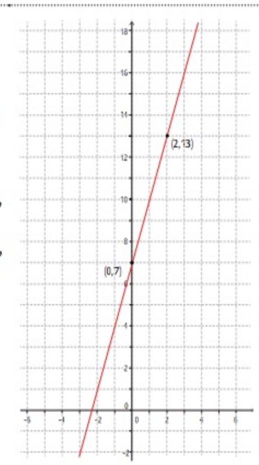

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Standard Explanation

Students are able to connect foundational understandings about functions to their work with proportional relationships. The same kinds of tables and graphs students used in seventh grade to recognize and represent proportional relationships between quantities are used in eighth grade when students compare the properties of two functions that are represented in different ways (e.g., numerically in tables, visually in graphs). Students also compare the properties of two functions that are represented algebraically or verbally (8.F.2▲) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.F.2 Illustrative Task:

- Battery Charging, https://www.illustrativemathematics.org/content-standards/8/F/A/2/tasks/641

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?

b. How much time would Sam need to charge the battery 100% on both devices?

8th Grade – CCSS for Mathematics

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8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Standard Explanation

Students are able to connect foundational understandings about functions to their work with proportional relationships. The same kinds of tables and graphs students used in seventh grade to recognize and represent proportional relationships between quantities are used in eighth grade when students compare the properties of two functions that are represented in different ways (e.g., numerically in tables, visually in graphs). Students also compare the properties of two functions that are represented algebraically or verbally (8.F.2▲) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.F.2 Illustrative Task:

- Battery Charging, https://www.illustrativemathematics.org/content-standards/8/F/A/2/tasks/641

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?

b. How much time would Sam need to charge the battery 100% on both devices?

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8th Grade – CCSS for Mathematics

8.F.A Define, evaluate, and compare functions.

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Essential Skills and Concepts:
- ☐ Know that the equation $y = mx + b$ is a linear function Understand the difference between a linear and nonlinear function
- ☐ Create and explain examples of linear and nonlinear functions

Question Stems and Prompts:
- ✓ What is the difference between functions that are linear and those that are not linear?
- ✓ Describe how you know that $y = mx + b$ is a linear function.
- ✓ Determine which of the functions listed are linear and which are not linear and explain your reasoning.
- ✓ Create examples of linear and nonlinear functions. Explain why you created your examples the way you did.

Vocabulary	**Spanish Cognates**
Tier 2	
• function	función
Tier 3	
• slope intercept form	
• linear function	función lineal
• nonlinear functions	función no lineal

Standards Connections
8.EE.6, 8.F.1, 8.F.2 → *8.F.3*
8.F.3 → 8.F.4, 8.F.5

8.F.A Define, evaluate, and compare functions.

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Essential Skills and Concepts:
- ☐ Know that the equation $y = mx + b$ is a linear function Understand the difference between a linear and nonlinear function
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Vocabulary	**Spanish Cognates**
Tier 2	
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Standards Connections
8.EE.6, 8.F.1, 8.F.2 → *8.F.3*
8.F.3 → 8.F.4, 8.F.5

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8^{th} Grade – CCSS for Mathematics

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Standard Explanation

Students' understanding of the equation $y = mx + b$ deepens as they learn that the equation defines a linear function whose graph is a straight line (8.F.3▲), a concept closely related to standard (8.EE.6▲). To avoid the mistaken impression that all functional relationships are linear, students also work with nonlinear functions and provide examples of nonlinear functions, recognizing that the graph of a nonlinear function is not a straight line. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs& tables to categorize functions as linear or non-linear. (Adapted from N. Carolina 2013)

8.F.3 Illustrative Task:

- Introduction to Linear Functions, https://www.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813

a. Decide which of the following points are on the graph of the function $y = 2x + 1$:

- $(0, 1), (2, 5), (\frac{1}{2}, 2), (2, -1), (-1, -1), (0.5, 1)$.
- Find 3 more points on the graph of the function.

b. Find several points that are on the graph of the function $y = 2x^2 + 1$.

- Plot the points in the coordinate plane. Is this a linear function?
- Support your conclusion.

c. Graph both functions and list as many differences between the two functions as you can.

SBAC Sample Item:

Samir was assigned to write an example of a linear functional relationship. He wrote this example for the assignment.

The relationship between the year and the population of a county when the population increases by 10% each year

Part A

Complete the table below to create an example of the population of a certain county that is increasing by 10% each year.

Year	Population of a Certain County
0	
1	
2	
3	
4	

8.F.A Define, evaluate, and compare functions.

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Standard Explanation

Students' understanding of the equation $y = mx + b$ deepens as they learn that the equation defines a linear function whose graph is a straight line (8.F.3▲), a concept closely related to standard (8.EE.6▲). To avoid the mistaken impression that all functional relationships are linear, students also work with nonlinear functions and provide examples of nonlinear functions, recognizing that the graph of a nonlinear function is not a straight line. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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8.F.B Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Essential Skills and Concepts:
- ☐ Create a function to model a linear relationship
- ☐ Determine the rate of change and initial value from a description, two (x, y) values, a table, or a graph
- ☐ Interpret the rate of change and initial value in terms of the context of the situation

Question Stems and Prompts:
- ✓ Write a function for the given relationship.
- ✓ Using a description, two (x, y) values, a table, or a graph, find the rate of change and initial value for this function.
- ✓ What do the rate of change and initial value mean in this real-world situation? Explain your thinking.

Vocabulary	**Spanish Cognates**
Tier 2	
• function	función
• slope	
• initial value	valor inicial
Tier 3	
• rate of change	
• graph gráfico	
• table of values	tabla de valores
• linear function	función lineal

Standards Connections
7.RP.2 → *8.F.4,* 8.SP.2, 8.SP.3 – *8.F.4* – 8.F.5

8.F.4 Example:

Example: Modeling With a Linear Function 8.F.4

A car rental company charges \$45 per day to rent a car as well as a one-time \$25 fee for the car's GPS navigation system. Write an equation for the cost in dollars, c, as a function of the number of days the car is rented, d. What is the initial value for this function?

Solution: There are several aids that may help students determine an equation for the cost:

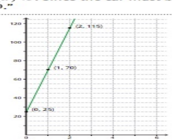
- A verbal description: "Each day adds \$45 to the cost, but there is a one-time \$25 GPS fee. This means that the cost should be \$25 plus \$45 times the number of days you rent the car, or $c = 25 + 45d$. Since a customer must rent the car for a minimum of 1 day, the initial value is $25 + 45 = 70$, which means it costs \$70 to rent the car for 1 day."

- A table: "I made a table to give me a feel for how much the car rental might cost after d days.

d (days)	c (dollars)
1	$70 = 25 + (1)45$
2	$115 = 25 + (2)45$
3	$160 = 25 + (3)45$
4	$205 = 25 + (4)45$
d	$c = 25 + (d)45$

The table helped me see that the cost in dollars is represented by $c = 25 + (d)45$. Since the car must be rented for 1 day or more, the initial value is when $d = 1$, which is $c = 70$, or \$70."

- A graph: "I made a rough graph and saw that the relationship between the cost and the days rented appeared to be linear. I found the slope of the line to be 45 (which is the cost per day) and the y intercept to be 25. This means the equation is $c = 45d + 25$. It is important to see that even though the y intercept of the graph is 25, that is not the initial value—because the initial value is when someone rents the car for 1 day. The point on the graph is $(1, 70)$ so the initial value is \$70."



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8th Grade – CCSS for Mathematics

8.F.B Use functions to model relationships between quantities.

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Essential Skills and Concepts:
☐ Create a function to model a linear relationship
☐ Determine the rate of change and initial value from a description, two (x, y) values, a table, or a graph
☐ Interpret the rate of change and initial value in terms of the context of the situation

Question Stems and Prompts:
✓ Write a function for the given relationship.
✓ Using a description, two (x, y) values, a table, or a graph, find the rate of change and initial value for this function.
✓ What do the rate of change and initial value mean in this real-world situation? Explain your thinking.

Vocabulary / Spanish Cognates
Tier 2
• function función
• slope
• initial value valor inicial
Tier 3
• rate of change
• graph gráfico
• table of values tabla de valores
• linear function función lineal

Standards Connections
7.RP.2 → 8.F.4, 8.SP.2, 8.SP.3 – 8.F.4 – 8.F.5

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• A table: "I made a table to give me a feel for how much the car rental might cost after d days.
d (days) | c (dollars)
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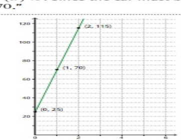

8th Grade – CCSS for Mathematics

8.F.B.4

Standard Explanation

In grade eight students learn to use functions to represent relationships between quantities. This work is also closely tied to MP.4 (Model with Mathematics). There are many real-world problems that can be modeled with linear functions, including instances of constant payment plans (e.g., phone plans), costs associated with running a business, and relationships between associated bivariate data (see standard 8.SP.3). Students also recognize that linear functions in which $b = 0$ are proportional relationships, something they have studied since grade six.

Standard 8.F.4 refers to students finding the initial value of a linear function. What is intended by this standard is that if f represents a linear function with a domain $[a, b]$, that is, the input values for f are between the values a and b, then the initial value for f would be $f(a)$. The term "initial value" takes its name from an interpretation of the independent variable as representing time, although the term can apply to any function. Note that formal introduction of the term domain does not occur until the higher mathematics courses, but teachers may desire to include this language if it clarifies these ideas for students. The example below will illustrate this definition (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.F.4 Illustrative Tasks:

- Video Streaming, https://www.illustrativemathematics.org/content-standards/8/F/B/4/tasks/247

You work for a video streaming company that has two monthly plans to choose from:

- Plan 1: A flat rate of $7 per month plus $2.50 per video viewed
- Plan 2: $4 per video viewed

a. What type of functions model this situation? Explain how you know.

b. Define variables that make sense in the context, and then write an equation with cost as a function of videos viewed, representing each monthly plan.

c. How much would 3 videos in a month cost for each plan? 5 videos?

d. Compare the two plans and explain what advice you would give to a customer trying to decide which plan is best for them, based on their viewing habits.

- Baseball Cards, https://www.illustrativemathematics.org/content-standards/8/F/B/4/tasks/552

A student has had a collection of baseball cards for several years. Suppose that B, the number of cards in the collection, can be described as a function of t, which is time in years since the collection was started. Explain what each of the following equations would tell us about the number of cards in the collection over time.

a. $B = 200 + 100t$

b. $B = 100 + 200t$

c. $B = 2000 - 100t$

d. $B = 100 - 200t$

8.F.B.4

Standard Explanation

In grade eight students learn to use functions to represent relationships between quantities. This work is also closely tied to MP.4 (Model with Mathematics). There are many real-world problems that can be modeled with linear functions, including instances of constant payment plans (e.g., phone plans), costs associated with running a business, and relationships between associated bivariate data (see standard 8.SP.3). Students also recognize that linear functions in which $b = 0$ are proportional relationships, something they have studied since grade six.

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8th Grade – CCSS for Mathematics

8.F.B Use functions to model relationships between quantities.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Essential Skills and Concepts:

- ☐ Analyze and describe the graph of a functional relationship
- ☐ Sketch a graph based upon a description of the features of the function

Question Stems and Prompts:

- ✓ Match the graph to the correct description of the function.
- ✓ Describe the graph of the function. Explain your thinking in terms of the context of the situation.
- ✓ Write a description of a function and then sketch a graph that matches our description.

Vocabulary	**Spanish Cognates**
Tier 2	
• increasing	
• decreasing	decreciente
• verbal description	descripción verbal
• sketch	
Tier 3	
• graph	gráfico
• linear function	función lineal
• nonlinear function	función no lineal

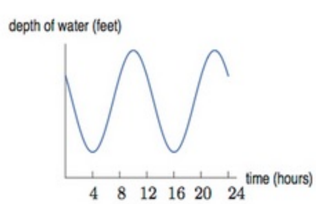
Standards Connections

8.F.1, 8.F.2, 8.F.3 → *8.F.5*
8.F.4 – *8.F.5*

8.F.5 Illustrative Task:

- Tides, https://www.illustrativemathematics.org/content-standards/8/F/B/5/tasks/628

The figure below gives the depth of the water at Montauk Point, New York, for a day in November.



a. How many high tides took place on this day?

b. How many low tides took place on this day?

c. How much time elapsed in between high tides?

8.F.B Use functions to model relationships between quantities.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

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Standards Connections

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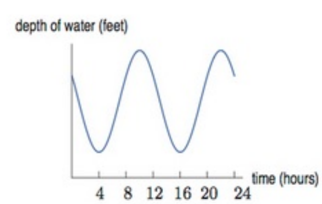

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8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Standard Explanation

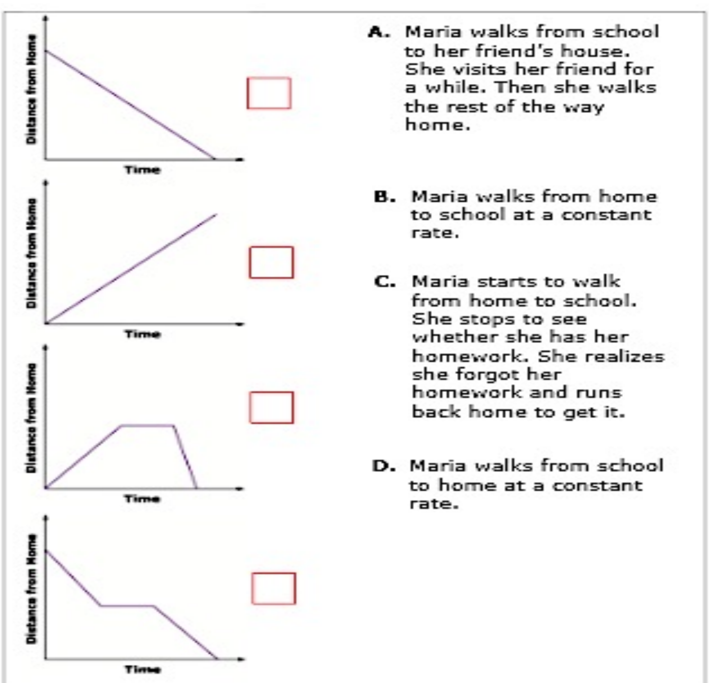
Students analyze graphs and then describe qualitatively the functional relationship between two quantities (e.g., where the function is increasing or decreasing, linear or nonlinear). They are able to sketch graphs that illustrate the qualitative features of functions that are described verbally (8.F.5) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

> **Focus, Coherence, and Rigor**
>
> Work in the cluster "Use functions to model relationships between quantities" involves functions for modeling linear relationships and computing a rate of change or initial value, which supports major work at grade eight with proportional relationships and setting up linear equations (**8.EE.5–8▲**).

SBAC Sample Item:

Each day, Maria walks from home to school and then from school to home. The graphs that follow show the distance that Maria is from home at different times during the walk.

Match the graphs to the descriptions of Maria's walk shown to the right of the graphs. Next to each graph, enter the letter (A, B, C, D) of the description that best matches the graph.



8th Grade – CCSS for Mathematics

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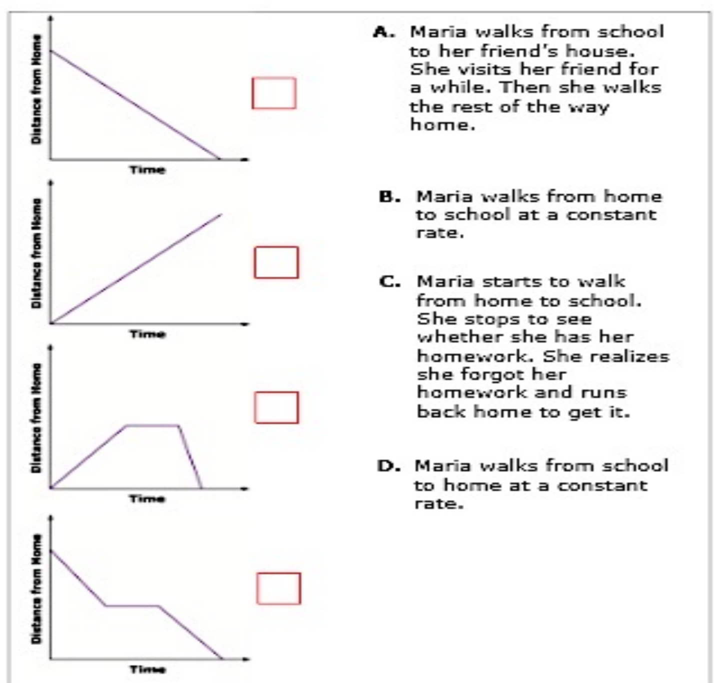

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8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Essential Skills and Concepts:

- ☐ Understand and explain rotations, reflections, and translations
- ☐ Describe the effects of rotations, reflections, and translations on lines, angles, and parallel lines

Question Stems and Prompts:

- ✓ After rotating the image, are the lines at the same angle? How do you know?
- ✓ Explore the reflected figures. Are the lengths of the line segments the same? Explain.
- ✓ How do you know that this image is a translation of the first image? Explain.

Vocabulary	**Spanish Cognates**
Tier 2	
• rotation	rotación
• reflection	reflexión
• translation	
Tier 3	
• line	línea
• angle	ángulo
• parallel lines	líneas paralelas
• line segment	segmento de línea
• rigid transformation	transformación rígida

Standards Connections

7.G.2, 7.G.5 → *8.G.1*
8.G.1 → 8.G.2, 8.G.3

8.G.1 Examples:

Characteristics of Rotations, Reflections, and Translations 8.G.1a–c▲

Students come to understand that the following transformations result in shapes that are *congruent* to one another.

- Students understand a *rotation* as the spinning of a figure around a fixed point known as the *center of rotation*. Unless specified otherwise, rotations are usually performed counterclockwise according to a particular angle of rotation.
- Students understand a *reflection* as the flipping of an object over a line known as the *line of reflection*.
- Students understand a *translation* as the shifting of an object in one direction for a fixed distance, so that any point lying on the shape moves the same distance in the same direction.

An example of an interactive online tool that shows transformation is Shodor Education's "Interactivate Transmographer" (http://www.shodor.org/interactivate/activities/Transmographer/ [Shodor Education Foundation, Inc. 2015]), which allows students to work with rotation, reflection, and translation.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

8th Grade – CCSS for Mathematics

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Essential Skills and Concepts:

- ☐ Understand and explain rotations, reflections, and translations
- ☐ Describe the effects of rotations, reflections, and translations on lines, angles, and parallel lines

Question Stems and Prompts:

- ✓ After rotating the image, are the lines at the same angle? How do you know?
- ✓ Explore the reflected figures. Are the lengths of the line segments the same? Explain.
- ✓ How do you know that this image is a translation of the first image? Explain.

Vocabulary / **Spanish Cognates**
Tier 2
• rotation — rotación
• reflection — reflexión
• translation
Tier 3
• line — línea
• angle — ángulo
• parallel lines — líneas paralelas
• line segment — segmento de línea
• rigid transformation — transformación rígida

Standards Connections

7.G.2, 7.G.5 → *8.G.1*
8.G.1 → 8.G.2, 8.G.3

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8.G.A.1

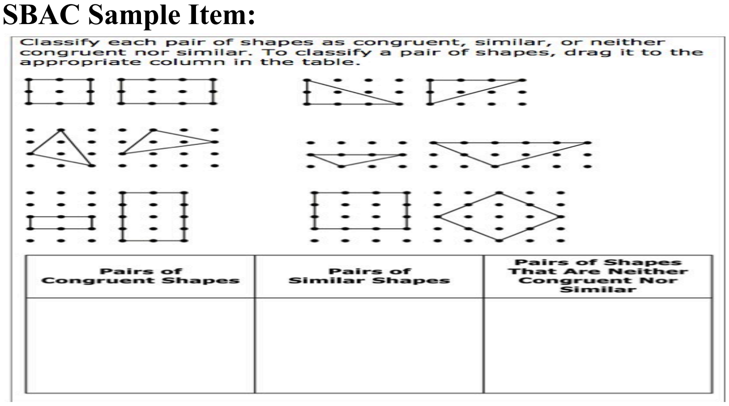
Standard Explanation

In grade seven, students solved problems involving scale drawings and informal geometric constructions, and they worked with two- and three-dimensional shapes to solve problems involving area, surface area, and volume. Students in grade eight complete their work on volume by solving problems involving cones, cylinders, and spheres. They also analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence and by understanding and applying the Pythagorean Theorem, which is a critical area of instruction at this grade level.

In this grade-eight Geometry domain, a major shift in the traditional curriculum occurs with the introduction of basic transformational geometry. In particular, the notion of congruence is defined differently than it has been in the past. Previously, two shapes were understood to be congruent if they had the “same size and same shape.” This imprecise notion is exchanged for a more precise one: that a two-dimensional figure is congruent to another if the second figure can be obtained from the first by a sequence of rotations, reflections, and translations. Students need ample opportunities to explore these three geometric transformations and their properties. The work in the Geometry domain is designed to provide a seamless transition to the Geometry conceptual category in higher mathematics courses, which begins by approaching transformational geometry from a more advanced perspective.

With the aid of physical models, transparencies, and geometry software, students in grade eight gain an understanding of transformations and their relationship to congruence of shapes. Through experimentation, students verify the properties of rotations, reflections, and translations, including discovering that these transformations change the position of a geometric figure but not its shape or size (8.G.1a–c▲). Finally, students come to understand that congruent shapes are precisely those that can be “mapped” one onto the other by using rotations, reflections, or translations (8.G.2▲) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

SBAC Sample Item:



8th Grade – CCSS for Mathematics

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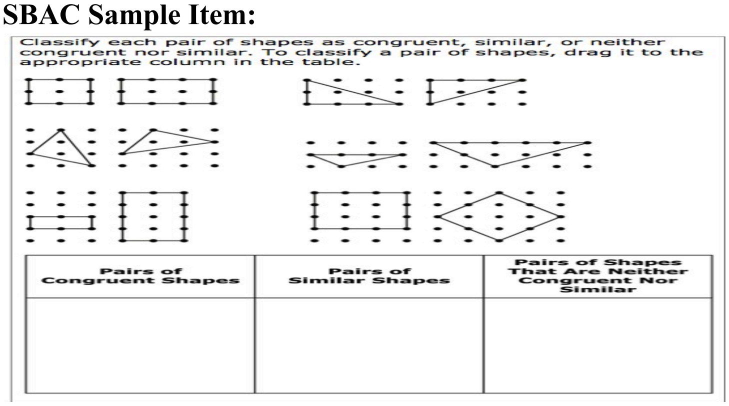

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8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Essential Skills and Concepts:

- ☐ Use rotations, reflections, and/or translations to create a congruent second figure from a given figure
- ☐ Understand congruence of two-dimensional figures
- ☐ Justify the congruence of two figures by describing the sequence of rotations, reflections, and/or translations needed to go from one figure to the other

Question Stems and Prompts:

- ✓ How do you know if two-dimensional figures are congruent?
- ✓ Describe how the second figure is congruent to the first.
- ✓ Is Figure A congruent to Figure A'? Explain how you know.
- ✓ Describe the sequence of transformations that created a congruent Figure A'.

Vocabulary Spanish Cognates

Tier 2

- rotation rotación
- reflection reflexión
- translation

Tier 3

- congruent congruente
- rigid transformation transformación rígida

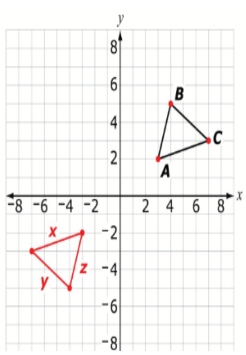
Standards Connections

8.G.1 → *8.G.2*
8.G.2 → 8.G.4, 8.G.5

SBAC Sample Item:

Triangle *ABC* on this coordinate grid was created by joining points *A* (3, 2), *B* (4, 5), and *C* (7, 3) with line segments.

Triangle *ABC* was reflected over the *x*-axis and then reflected over the *y*-axis to form the red triangle, where *x*, *y*, and *z* represent the lengths of the sides of the red triangle.



Click the appropriate boxes in the table to show which sides of the triangles have equal lengths.

	x	y	z
AB			
AC			
BC			

8th Grade – CCSS for Mathematics

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

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Vocabulary Spanish Cognates

Tier 2

- rotation rotación
- reflection reflexión
- translation

Tier 3

- congruent congruente
- rigid transformation transformación rígida

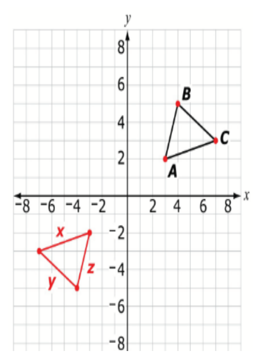
Standards Connections

8.G.1 → 8.G.2
8.G.2 → 8.G.4, 8.G.5

SBAC Sample Item:

Triangle ABC on this coordinate grid was created by joining points A (3, 2), B (4, 5), and C (7, 3) with line segments.

Triangle ABC was reflected over the x-axis and then reflected over the y-axis to form the red triangle, where x, y, and z represent the lengths of the sides of the red triangle.



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8th Grade – CCSS for Mathematics

8.G.A.2

Standard Explanation

With the aid of physical models, transparencies, and geometry software, students in grade eight gain an understanding of transformations and their relationship to congruence of shapes. Through experimentation, students verify the properties of rotations, reflections, and translations, including discovering that these transformations change the position of a geometric figure but not its shape or size (8.G.1a–c▲). Finally, students come to understand that congruent shapes are precisely those that can be "mapped" one onto the other by using rotations, reflections, or translations (8.G.2▲) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

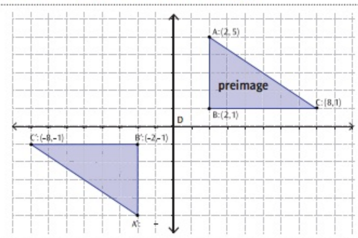
8.G.2 Examples:

Examples of Four Geometric Transformations 8.G.2▲

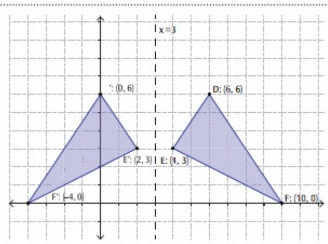
(Note that the original figure is called the *preimage*, and the new figure is called the *image*.)

Rotation: A figure can be rotated up to 360° about the center of rotation.

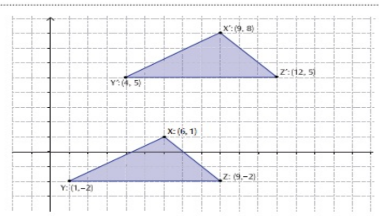
Consider when ΔABC is rotated 180° clockwise about the origin. The coordinates of ΔABC are $A(2,5)$, $B(2,1)$, and $C(8,1)$. When rotated 180°, the image triangle $\Delta A'B'C'$ has coordinates $A'(-2,-5)$, $B'(-2,-1)$, $C'(-8,-1)$. Each coordinate of the image is the opposite of its preimage point's coordinate.



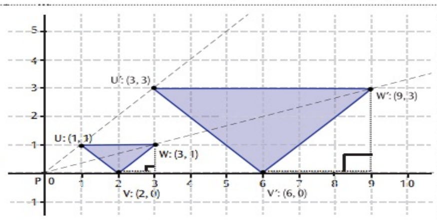
Reflection: In the picture shown, ΔDEF has been reflected across the line $x = 3$. Notice the change in the orientation of the points, in the sense that the counterclockwise order of the preimage D–E–F is reversed in the image to D'–F'–E'. Notice also that each point on the image is at the same distance from the line of reflection as its corresponding point on the preimage.



Translation: Here, ΔXYZ has been translated 3 units to the right and 7 units up. Orientation is preserved. It is not too difficult to see that under this transformation, a preimage point (x, y) yields the image point $(x+3, y+7)$.



Dilation: In the picture, ΔUVW has been dilated from the origin $P:(0,0)$ by a factor of $k=3$. The picture shows that the segments PU, PV, and PW have all been multiplied by the factor $k=3$, which results in a new triangle, $\Delta U'V'W'$. By definition, ΔUVW and $\Delta U'V'W'$ are similar triangles. Students should experiment and find that the ratios of corresponding side lengths satisfy $\frac{U'V'}{UV} = \frac{V'W'}{VW} = \frac{U'W'}{UW} = 3$, which corresponds to k. Students can apply the Pythagorean Theorem (**8.G.7–8▲**) to find the side lengths and justify this result. For example, they may find the lengths of VW and $V'W'$:



$$\overline{VW} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{and } \overline{V'W'} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

Students can check informally that $\sqrt{18} = 3\sqrt{2}$, as formal work with radicals has not yet begun in grade eight.

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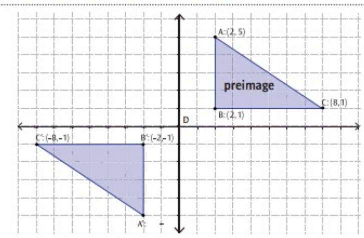

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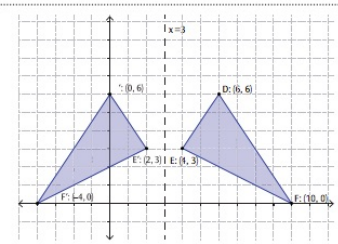

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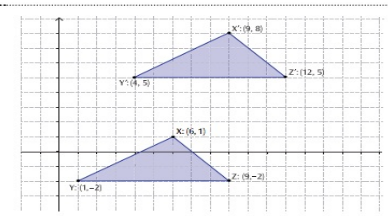

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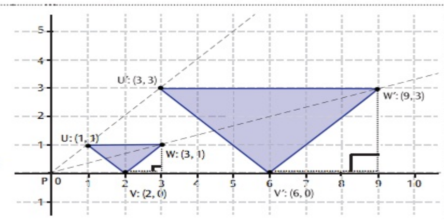

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8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Essential Skills and Concepts:
- ☐ Understand and explain dilations, rotations, reflections, and translations
- ☐ Use coordinates to describe a two-dimensional figure before and after dilations, translations, rotations, and reflections

Question Stems and Prompts:
- ✓ Which ordered pair represents point A after the dilation?
- ✓ Describe the figure using coordinates after a series of transformations or a single dilation, translation, rotation, and reflection.
- ✓ Create a copy of the given image using dilations, translations, rotations, and reflections. Describe your use of transformation to create your new image.

Vocabulary Spanish Cognates

Tier 2
- dilation dilatación
- rotation rotación
- reflection reflexión
- translation

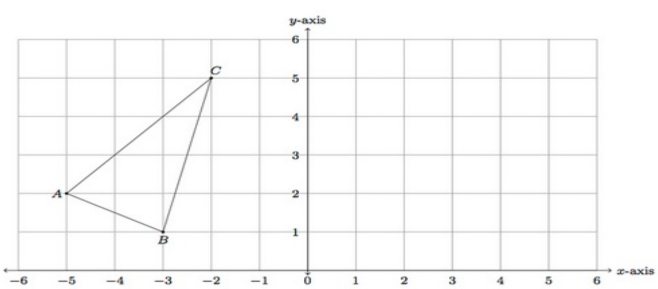
Tier 3
- rigid transformation transformación rígida

Standards Connections

8.G.1 → *8.G.3*
8.G.3 → 8.G.4

8.G.3 Illustrative Task:
- Reflecting Reflections, https://www.illustrativemathematics.org/content-standards/8/G/A/3/tasks/1243

Below is a picture of a triangle on a coordinate grid:



a. Draw the reflection of $\triangle ABC$ over the line $x = -2$. Label the image of A as A', the image of B as B' and the image of C as C'.

b. Draw the reflection of $\triangle A'B'C'$ over the line $x = 2$. Label the image of A' as A'', the image of B' as B'' and the image of C' as C''.

c. What single rigid transformation of the plane will map $\triangle ABC$ to $\triangle A''B''C''$? Explain.

8th Grade – CCSS for Mathematics

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8.G.3 → 8.G.4

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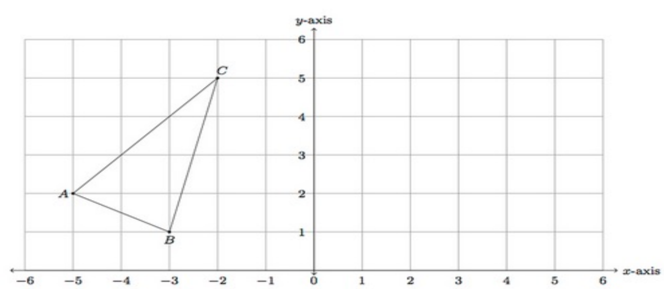

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8th Grade – CCSS for Mathematics

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Standard Explanation

Students come to understand that the following transformations result in shapes that are congruent to one another. Students understand a rotation as the spinning of a figure around a fixed point known as the center of rotation. Rotations are usually performed counterclockwise according to a certain angle of rotation, unless otherwise specified. Students understand a reflection as the flipping of an object over a line, known as the line of reflection. Students understand a translation as the shifting of an object in one direction a fixed distance, so that any point lying on the shape moves the same distance in the same direction. Students identify resulting coordinates from translations, reflections, and rotations (90º, 180º and 270º both clockwise and counterclockwise), recognizing the relationship between the coordinates and the transformation (North Carolina Math Unpacking Standards 2012). Note that the original figure is called the pre-image, while the new figure is called the image.

Students also study dilations in standard (8.G.3▲). A dilation with scale factor $k > 0$ can be thought of as a stretching (if $k > 1$) or shrinking (if $k < 1$) of an object. In a dilation, a point is specified from which the distance to the points of a figure are multiplied to obtain new points, and hence a new figure (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

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8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Essential Skills and Concepts:
- ☐ Use rotations, reflections, translations, and/or dilations to create a similar second figure from a given figure
- ☐ Understand similarity of two-dimensional figures
- ☐ Justify the similarity of two figures by describing the sequence of rotations, reflections, translations and/or dilations needed to go from one figure to the other

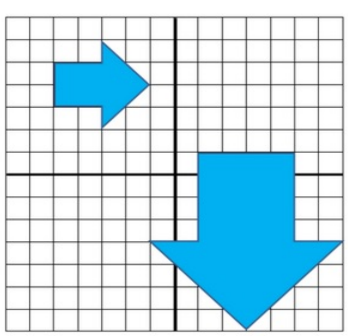
Question Stems and Prompts:
- ✓ How do you know if two-dimensional figures are similar?
- ✓ Describe how the second figure is similar to the first.
- ✓ Is Figure A similar to Figure A'? Explain how you know.
- ✓ Describe the sequence of transformations that resulted in a similar Figure A'.

Vocabulary	**Spanish Cognates**
Tier 2	
• dilation	dilatación
• rotation	rotación
• reflection	reflexión
• translation	
Tier 3	
• rigid transformation	transformación rígida

Standards Connections
8.G.2, 8.G.3 → *8.G.4*
8.G.4 → 8.G.5

8.G.4 Illustrative Task:
- Are they Similar?, https://www.illustrativemathematics.org/content-standards/8/G/A/4/tasks/1946

Determine, using rotations, translations, reflections, and/or dilations, whether the two polygons below are similar.



The intersection of the dark lines on the coordinate plane represents the origin (0,0) in the coordinate plane.

8th Grade – CCSS for Mathematics

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Essential Skills and Concepts:
- ☐ Use rotations, reflections, translations, and/or dilations to create a similar second figure from a given figure
- ☐ Understand similarity of two-dimensional figures
- ☐ Justify the similarity of two figures by describing the sequence of rotations, reflections, translations and/or dilations needed to go from one figure to the other

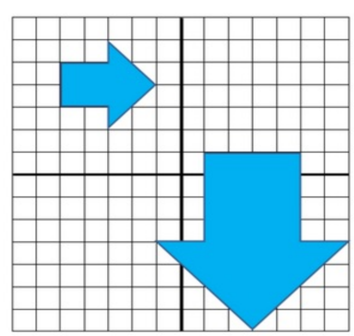
Question Stems and Prompts:
- ✓ How do you know if two-dimensional figures are similar?
- ✓ Describe how the second figure is similar to the first.
- ✓ Is Figure A similar to Figure A'? Explain how you know.
- ✓ Describe the sequence of transformations that resulted in a similar Figure A'.

Vocabulary	**Spanish Cognates**
Tier 2	
• dilation	dilatación
• rotation	rotación
• reflection	reflexión
• translation	
Tier 3	
• rigid transformation.	transformación rígida

Standards Connections
8.G.2, 8.G.3 → *8.G.4*
8.G.4 → 8.G.5

8.G.4 Illustrative Task:
- Are they Similar?, https://www.illustrativemathematics.org/content-standards/8/G/A/4/tasks/1946

Determine, using rotations, translations, reflections, and/or dilations, whether the two polygons below are similar.



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8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Standard Explanation

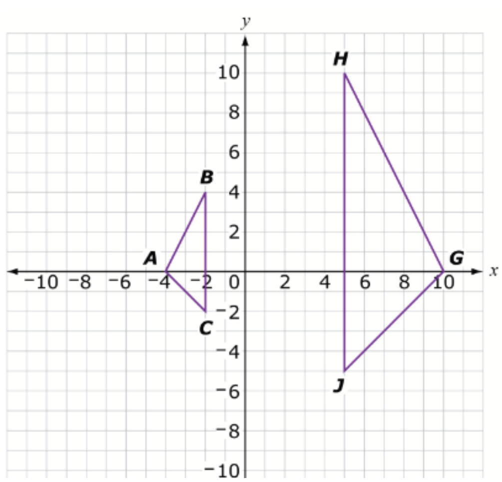
The definition of similar shapes is analogous to the new definition of congruence, but it has been refined to be more precise. Previously, shapes were said to be similar if they had the "same shape but not necessarily the same size." Now, two shapes are said to be similar if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations (8.G.4▲). By investigating dilations and using reasoning such as in the previous example, students learn that the following statements are true:

1. When two shapes are similar, the length of a segment AB in the first shape is multiplied by the scale factor k to give the length of the corresponding segment $A'B'$ in the second shape: $A'B' = k \bullet AB$.
2. Because the previous fact is true for all sides of a dilated shape, the ratio of the lengths of any two corresponding sides of the first and second shape is equal to k.
3. It is also true that the ratio of any two side lengths from the first shape is the same as the ratio of the corresponding side lengths from the second shape—for example, $\frac{AB}{BC} = \frac{A'B'}{B'C'}$. (Students can justify this algebraically, because fact 2 yields that $\frac{AB}{A'B'} = \frac{BC}{B'C'}$.)

(CA *Mathematics Framework*, adopted Nov. 6, 2013).

SBAC Sample Item:

A transformation is applied to $\triangle ABC$ to form $\triangle DEF$ (not shown).
Then, a transformation is applied to $\triangle DEF$ to form $\triangle GHJ$.



8th Grade – CCSS for Mathematics

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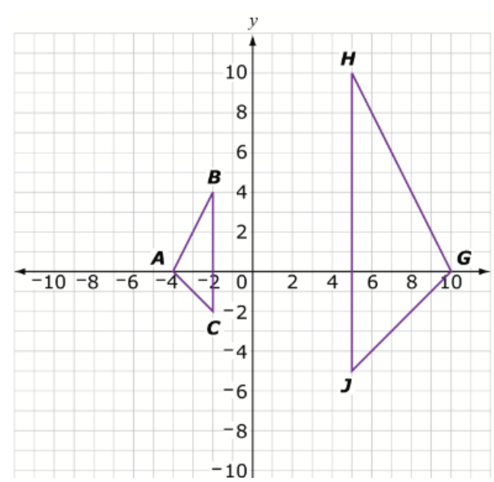

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8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Essential Skills and Concepts:
- ☐ Justify and explain your thinking about the angle sum of triangles and the exterior angle of triangles
- ☐ Justify and explain your thinking about the angles created by parallel lines cut by a transversal
- ☐ Understand the angle-angle criterion for similarity of triangles

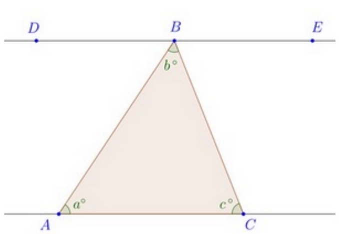
Question Stems and Prompts:
- ✓ Justify the relationships between the given triangles using the angle sum of triangles or the exterior angle of triangles.
- ✓ What are the relationships between the angles created by parallel lines cut by a transversal?
- ✓ Describe the angle-angle criterion for similarity. How can you use this to justify similar triangles?

Vocabulary	**Spanish Cognates**
Tier 3	
• angle sum	suma de los ángulos
• exterior angle	ángulo exterior
• parallel lines	líneas paralelas
• transversal	
• angle-angle criterion	criterio ángulo de ángulo

Standards Connections
8.G.2, 8.G.4 → *8.G.5*
8.G.5 → 8.EE.6

8.G.5 Illustrative Task:
- Triangle's Interior Angles, https://www.illustrativemathematics.org/content-standards/8/G/A/5/tasks/1501

Given that $\overleftrightarrow{DE} \parallel \overleftrightarrow{AC}$ in the diagram below, prove that $a + b + c = 180$.



Explain why this result holds for any triangle, not just the one displayed above.

8^{th} Grade – CCSS for Mathematics

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Essential Skills and Concepts:
- ☐ Justify and explain your thinking about the angle sum of triangles and the exterior angle of triangles
- ☐ Justify and explain your thinking about the angles created by parallel lines cut by a transversal
- ☐ Understand the angle-angle criterion for similarity of triangles

Question Stems and Prompts:
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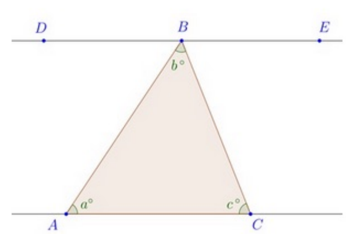

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8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

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Standard Explanation

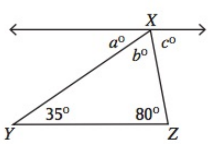
Students use informal arguments to establish facts about the angle sum and exterior angles of triangles (e.g., consecutive exterior angles are supplementary), the angles created when parallel lines are cut by a transversal (e.g., corresponding angles are congruent), and the angle–angle criterion for similarity of triangles (if two angles of a triangle are congruent to two angles of another triangle, the two triangles are similar) [8.G.5▲]. When coupled with the previous three properties of similar shapes, the angle–angle criterion for triangle similarity allows students to justify the fact that the slope of a line is the same between any two points on the line (CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.G.5 Example:

Example: The sum of the measures of the angles of a triangle is 180°.	8.G.5▲

In the figure shown, the line through point X is parallel to segment YZ.

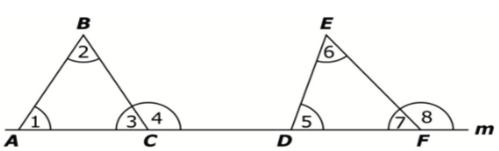
We know that $a = 35$ because it is the measure of an angle that is alternating with $\angle Y$. For a similar reason, $c = 80$. Because all lines have an angle measure of 180°, we know that $a + b + c = 180$, which leads to $b = 180 - (35 + 80) = 65$. So the sum of the measures of the angles in this triangle is 180°.



Adapted from ADE 2010.

SBAC Sample Item:

Triangle ABC and triangle DEF lie on line m.



not drawn to scale

The measure of $\angle 4$ is less than the measure of $\angle 8$.

For each comparison in this table, use the drop-down menu to select the symbol that correctly indicates the relationship between the first quantity and the second quantity.

First Quantity	Comparison	Second Quantity
$m\angle 3$		$m\angle 7$
$m\angle 3 + m\angle 2$		$m\angle 5 + m\angle 6$
$m\angle 1 + m\angle 2 + m\angle 3$		$m\angle 5 + m\angle 6 + m\angle 7$

8th Grade – CCSS for Mathematics

8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Standard Explanation

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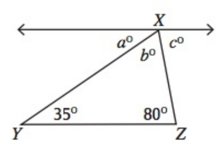

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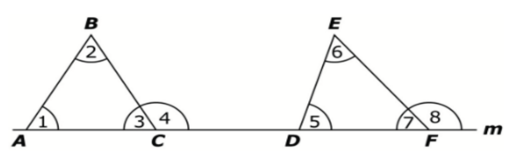

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8th Grade – CCSS for Mathematics

8.G.B Understand and apply the Pythagorean Theorem.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

Essential Skills and Concepts:

- ☐ Understand and explain the Pythagorean Theorem and its converse.
- ☐ Explain a proof of the Pythagorean Theorem and its converse.

Question Stems and Prompts:

- ✓ What is the Pythagorean Theorem?
- ✓ Explain the converse of the Pythagorean Theorem.
- ✓ Explain a proof of the Pythagorean Theorem (or its converse). Support your proof with words, examples, and/or models.

Vocabulary	**Spanish Cognates**
Tier 2	
• proof	
• converse	converso
Tier 3	
• pythagorean theorem	teorema de pitágoras

Standards Connections

7.G.6 → *8.G.6*

8.EE.2 – 8.G.6 – 8.G.7

8.G.6 Illustrative Task:

- Converse of the Pythagorean Theorem, https://www.illustrativemathematics.org/content-standards/8/G/B/6/tasks/724

A Pythagorean triple (a, b, c) is a set of three positive whole numbers which satisfy the equation

$$a^2 + b^2 = c^2.$$

Many ancient cultures used simple Pythagorean triples such as (3,4,5) in order to accurately construct right angles: if a triangle has sides of lengths 3, 4, and 5 units, respectively, then the angle opposite the side of length 5 units is a right angle.

a. State the Pythagorean Theorem and its converse.

b. Explain why this practice of constructing a triangle with side-lengths 3, 4, and 5 to produce a right angle uses the converse of the Pythagorean Theorem.

c. Explain, in this particular case, why the converse of the Pythagorean Theorem is true.

8.G.B Understand and apply the Pythagorean Theorem.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

Essential Skills and Concepts:

- ☐ Understand and explain the Pythagorean Theorem and its converse.
- ☐ Explain a proof of the Pythagorean Theorem and its converse.

Question Stems and Prompts:

- ✓ What is the Pythagorean Theorem?
- ✓ Explain the converse of the Pythagorean Theorem.
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Vocabulary	**Spanish Cognates**
Tier 2	
• proof	
• converse	converso
Tier 3	
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Standards Connections

7.G.6 → *8.G.6*

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8.G.B Understand and apply the Pythagorean Theorem.

8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

Standard Explanation

The Pythagorean Theorem is useful in practical problems, relates to grade-level work in irrational numbers, and plays an important role mathematically in coordinate geometry in higher mathematics. In grade eight, students explain a proof of the Pythagorean Theorem (8.G.6▲). There are many varied and interesting proofs of the Pythagorean Theorem. In grade eight students apply the theorem to determine unknown side lengths in right triangles (8.G.7▲) and to find the distance between two points in a coordinate system (8.G.8▲). Work with the Pythagorean Theorem will support students' work in high school-level coordinate geometry (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Focus, Coherence, and Rigor

Understanding, modeling, and applying (**MP.4**) the Pythagorean Theorem and its converse require that students look for and make use of structure (**MP.7**) and express repeated reasoning (**MP.8**). Students also construct and critique arguments as they explain a proof of the Pythagorean Theorem and its converse to others (**MP.3**).

Adapted from Charles A. Dana Center 2012.

8.G.6 Illustrative Task(s):

Converse of the Pythagorean Theorem. A Pythagorean triple (a,b,c) is a set of three positive whole numbers which satisfy the equation

$$a2+b2=c2.$$

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8th Grade – CCSS for Mathematics

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8.G.B Understand and apply the Pythagorean Theorem.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Essential Skills and Concepts:
- ☐ Understand and explain the Pythagorean Theorem
- ☐ Apply the Pythagorean Theorem to find unknown lengths
- ☐ Solve real-world and mathematical problems using the Pythagorean Theorem in two and three dimensions

Question Stems and Prompts:
- ✓ What is the Pythagorean Theorem?
- ✓ What are Pythagorean triples? How can they help you to find missing side lengths?
- ✓ Give examples of real world examples that could use the Pythagorean Theorem.
- ✓ The bases on a baseball diamond form a perfect square. Home plate is 90 feet away from first base and each of the bases is 90 feet apart from each other. If a player fields the ball on third base, how far must they throw the ball to get the out at first?

Vocabulary	**Spanish Cognates**
Tier 2	
• sides	
Tier 3	
• pythagorean theorem	teorema de pitágoras
• right triangle	
• right angle	
• hypotenuese	hipotenusa

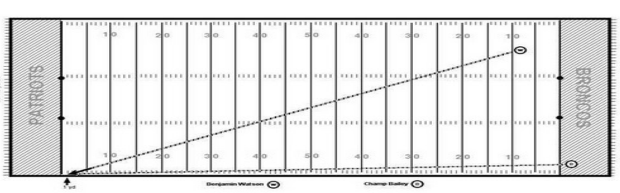
Standards Connections

8.G.6 – *8.G.7*
8.G.7 → 8.G.8

8.G.7 Illustrative Task:
- Running on the Football Field, https://www.illustrativemathematics.org/content-standards/8/G/B/7/tasks/655

During the 2005 Divisional Playoff game between The Denver Broncos and The New England Patriots, Bronco player Champ Bailey intercepted Tom Brady around the goal line (see the circled B). He ran the ball nearly all the way to other goal line. Ben Watson of the New England Patriots (see the circled W) chased after Champ and tracked him down just before the other goal line.

In the image below, each hash mark is equal to one yard: note too the the field is $53\frac{1}{3}$ yards wide.



a. How can you use the diagram and the Pythagorean Theorem to find approximately how many yards Ben Watson ran to track down Champ Bailey?

b. Use the Pythagorean Theorem to find approximately how many yards Watson ran in this play.

c. Which player ran further during this play? By approximately how many more yards?

8th Grade – CCSS for Mathematics

8.G.B Understand and apply the Pythagorean Theorem.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Essential Skills and Concepts:
☐ Understand and explain the Pythagorean Theorem
☐ Apply the Pythagorean Theorem to find unknown lengths
☐ Solve real-world and mathematical problems using the Pythagorean Theorem in two and three dimensions

Question Stems and Prompts:
✓ What is the Pythagorean Theorem?
✓ What are Pythagorean triples? How can they help you to find missing side lengths?
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✓ The bases on a baseball diamond form a perfect square. Home plate is 90 feet away from first base and each of the bases is 90 feet apart from each other. If a player fields the ball on third base, how far must they throw the ball to get the out at first?

Vocabulary / Spanish Cognates
Tier 2
• sides
Tier 3
• pythagorean theorem — teorema de pitágoras
• right triangle
• right angle
• hypotenuese — hipotenusa

Standards Connections
8.G.6 – 8.G.7
8.G.7 → 8.G.8

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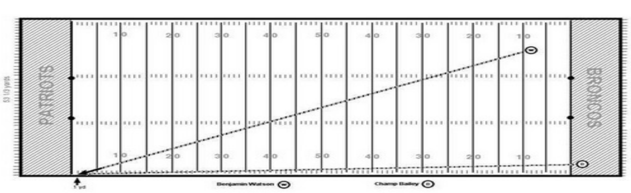
During the 2005 Divisional Playoff game between The Denver Broncos and The New England Patriots, Bronco player Champ Bailey intercepted Tom Brady around the goal line (see the circled B). He ran the ball nearly all the way to other goal line. Ben Watson of the New England Patriots (see the circled W) chased after Champ and tracked him down just before the other goal line.

In the image below, each hash mark is equal to one yard: note too the the field is $53\frac{1}{3}$ yards wide.



a. How can you use the diagram and the Pythagorean Theorem to find approximately how many yards Ben Watson ran to track down Champ Bailey?

b. Use the Pythagorean Theorem to find approximately how many yards Watson ran in this play.

c. Which player ran further during this play? By approximately how many more yards?

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8.G.B Understand and apply the Pythagorean Theorem.

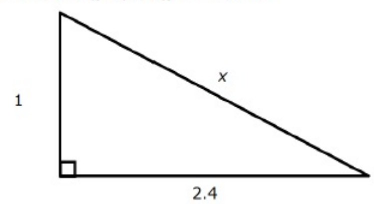
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Standard Explanation

The Pythagorean Theorem is useful in practical problems, relates to grade-level work in irrational numbers, and plays an important role mathematically in coordinate geometry in higher mathematics. In grade eight, students explain a proof of the Pythagorean Theorem (8.G.6▲). There are many varied and interesting proofs of the Pythagorean Theorem. In grade eight students apply the theorem to determine unknown side lengths in right triangles (8.G.7▲) and to find the distance between two points in a coordinate system (8.G.8▲). Work with the Pythagorean Theorem will support students' work in high school-level coordinate geometry (CA *Mathematics Framework*, adopted Nov. 6, 2013).

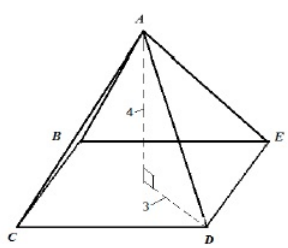
SBAC Sample Items:

Example Stem 1: A right triangle is shown.



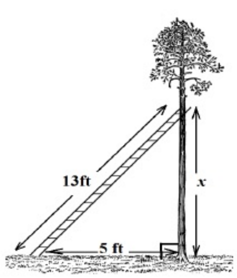
Enter the value of *x*.

Example Stem 2: A right square pyramid is shown. The height of the pyramid is 4 units. The distance from the center of the base of the pyramid to vertex D is 3 units, as shown.



Enter the length of segment *AD,* in units.

Stem 3: A 13-foot ladder is leaning on a tree. The bottom of the ladder is on the ground at a distance of 5 feet from the base of the tree. The base of the tree and the ground form a right angle as shown.



Enter the distance between the ground and the top of the ladder, *x*, in feet.

8th Grade – CCSS for Mathematics

8.G.B Understand and apply the Pythagorean Theorem.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

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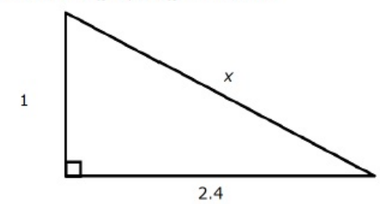

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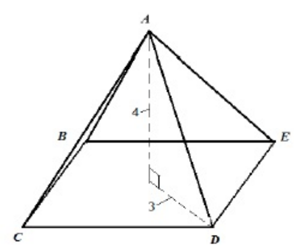

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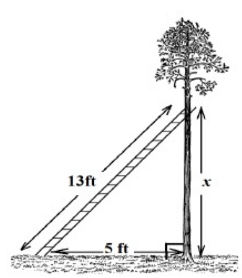

Enter the distance between the ground and the top of the ladder, *x*, in feet.

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8th Grade – CCSS for Mathematics

8.G.B Understand and apply the Pythagorean Theorem.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Essential Skills and Concepts:
- ☐ Understand and explain the Pythagorean Theorem
- ☐ Apply the Pythagorean Theorem to find the distance between two points on the coordinate plane

Question Stems and Prompts:
- ✓ Determine the distance between the two points (*x*,*y*) and (*x*,*y*) on the coordinate plane.
- ✓ Explain how to determine the distance between two points on the coordinate plane using the Pythagorean Theorem.

Vocabulary Spanish Cognates

Tier 3
- pythagorean theorem teorema de pitágoras
- coordinate system Sistema de coordenadas
- coordinate plane plano de coordenadas

Standards Connections

6.G.3, 8.G.7 → *8.G.8*

8.G.8 Illustrative Tasks:
- Finding the Distance Between Points, https://www.illustrativemathematics.org/content-standards/8/G/B/8/tasks/1919

  a. Plot the points (5,3), (-1,1), and (2,-3) in the coordinate plane and find the lengths of the three segments connecting the points.

  b. Find the distance between (5,9) and (-4,2) without plotting the points.

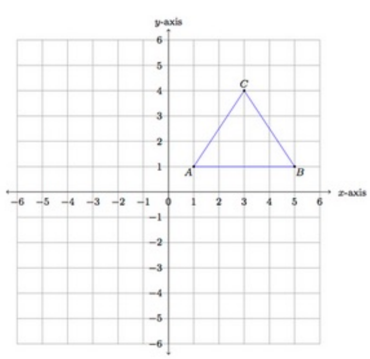
  c. If (u, v) and (s, t) are two distinct points in the plane, what is the distance between them? Explain how you know.

  d. Does your answer to (c) agree with your calculations in parts (a) and (b)? Explain.

- Finding Isosceles Triangles, https://www.illustrativemathematics.org/content-standards/8/G/B/8/tasks/1556

Mrs. Lu has asked students in her class to find isosceles triangles whose vertices lie on a coordinate grid. For each student example below, explain why the triangle is isosceles.

a. Jessica draws the following triangle:



8.G.B Understand and apply the Pythagorean Theorem.

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Essential Skills and Concepts:
- ☐ Understand and explain the Pythagorean Theorem
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Vocabulary Spanish Cognates

Tier 3
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6.G.3, 8.G.7 → *8.G.8*

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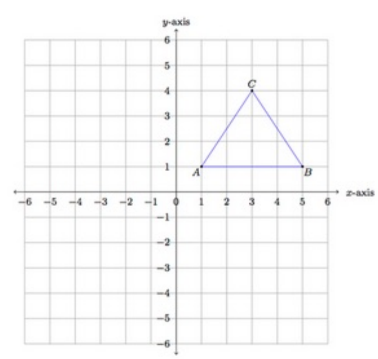

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8.G.B Understand and apply the Pythagorean Theorem.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Standard Explanation

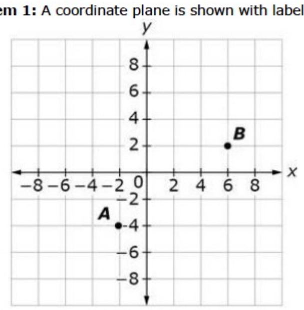
The Pythagorean Theorem is useful in practical problems, relates to grade-level work in irrational numbers, and plays an important role mathematically in coordinate geometry in higher mathematics. In grade eight, students explain a proof of the Pythagorean Theorem (8.G.6▲). There are many varied and interesting proofs of the Pythagorean Theorem. In grade eight students apply the theorem to determine unknown side lengths in right triangles (8.G.7▲) and to find the distance between two points in a coordinate system (8.G.8▲). Work with the Pythagorean Theorem will support students' work in high school-level coordinate geometry (CA *Mathematics Framework,* adopted Nov. 6, 2013).

SBAC Sample Items:

What is the distance between (0, 0) and (8, 15) on the *xy*-coordinate plane?

(A) 7 units
(B) 8 units
(C) 17 units
(D) 23 units

Example Stem 1: A coordinate plane is shown with labeled points.

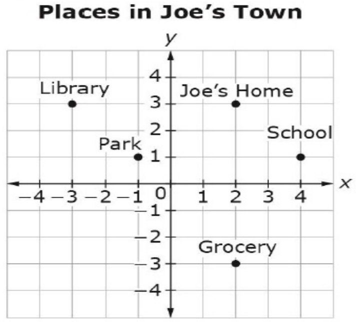


What is the distance between point *A* and point *B* on the coordinate plane?

A. 5
B. 6
C. 10
D. 14

Example Stem: The points show different locations in Joe's town. Each unit represents 1 mile.

Places in Joe's Town



What is the distance, in miles, between Joe's Home and the Park? Round your answer to the nearest tenth.

8th Grade – CCSS for Mathematics

8.G.B Understand and apply the Pythagorean Theorem.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Standard Explanation

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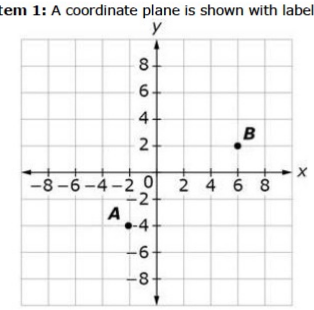

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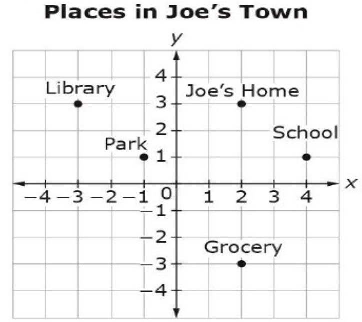

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8th Grade – CCSS for Mathematics

8.G.C Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Essential Skills and Concepts:
- ☐ Know and explain the formulas for the volumes of cones, cylinders, and spheres
- ☐ Apply the formulas for the volume of cones, cylinders and spheres
- ☐ Solve real-world and mathematical examples by using volume formulas

Question Stems and Prompts:
- ✓ What is the formula for the volume of a cone (cylinder, sphere)?
- ✓ How are the volume formulas related?
- ✓ Give examples of real-world problems where you could use the volume formulas.

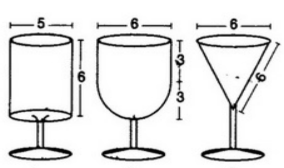
Vocabulary	**Spanish Cognates**
Tier 3	
• formula	fórmula
• volume	volumen
• cone	cono
• cylinder	cilindro
• sphere	esfera

Standards Connections
8.EE.2 → *8.G.9*

8.G.9 Illustrative Task:
- Glasses,

https://www.illustrativemathematics.org/content-standards/8/G/C/9/tasks/112

The diagram shows three glasses (not drawn to scale). The measurements are all in centimeters.



The bowl of glass 1 is cylindrical. The inside diameter is 5 cm and the inside height is 6 cm.

The bowl of glass 2 is composed of a hemisphere attached to cylinder. The inside diameter of both the hemisphere and the cylinder is 6 cm. The height of the cylinder is 3 cm.

The bowl of glass 3 is an inverted cone. The inside diameter is 6 cm and the inside slant height is 6 cm.

a. Find the vertical height of the bowl of glass 3.

b. Calculate the volume of the bowl of each of these glasses.

c. Glass 2 is filled with water and then half the water is poured out. Find the height of the water.

8.G.C Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

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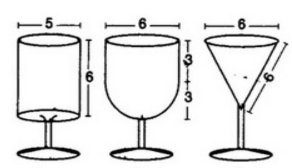

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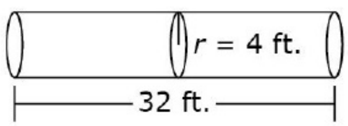
8.G.C.9

Standard Explanation

In grade seven students learned about the area of a circle. In eighth grade students learn the formulas for calculating the volumes of cones, cylinders, and spheres and use the formulas to solve real-world and mathematical problems (8.G.9). When students learn to solve problems involving volumes of cones, cylinders and spheres — together with their previous grade seven work in angle measure, area, surface area and volume — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning and multistep numerical problem solving, can be combined and used in flexible ways as part of modeling during high school and in college and careers (Adapted from PARCC 2012) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

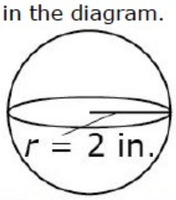
SBAC Sample Items:

Example Stem 1: This figure shows the dimensions of a tanker truck. The tank forms a cylinder with a length of 32 feet and radius of 4 feet.



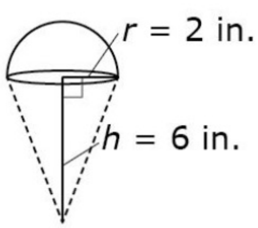
What is the volume, in cubic feet, of the tank? Round your answer to the nearest hundredth.

Example Stem 2: A baseball has a radius of 2 inches, as shown in the diagram.



What is the volume, in cubic inches, of the baseball? Round your answer to the nearest hundredth.

Example Stem 3: An ice cream cone has a height of 6 inches and a radius of 2 inches as shown. The ice cream completely fills the cone, as well as the half-sphere above the cone.



Which is closest to the total volume, in cubic inches, of the ice cream?

A. $\frac{16}{3}\pi$
B. 8π
C. $\frac{40}{3}\pi$
D. 20π

8th Grade – CCSS for Mathematics

8.G.C.9

Standard Explanation

In grade seven students learned about the area of a circle. In eighth grade students learn the formulas for calculating the volumes of cones, cylinders, and spheres and use the formulas to solve real-world and mathematical problems (8.G.9). When students learn to solve problems involving volumes of cones, cylinders and spheres — together with their previous grade seven work in angle measure, area, surface area and volume — they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning and multistep numerical problem solving, can be combined and used in flexible ways as part of modeling during high school and in college and careers (Adapted from PARCC 2012) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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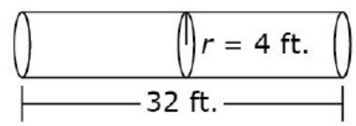

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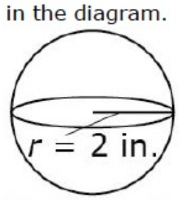

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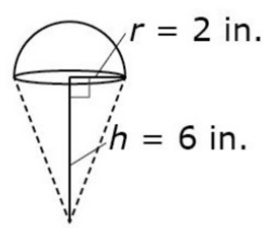

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8.SP.A Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Essential Skills and Concepts:
- ☐ Understand, identify and describe clustering, outliers, positive and negative association, linear associations and nonlinear associations.
- ☐ Interpret and explain bivariate data
- ☐ Create scatter plots and describe patterns of association

Question Stems and Prompts:
- ✓ What type of association is visible on the scatter plot? How do you know?
- ✓ Create a scatter plot for the given data.
- ✓ Describe patterns that you notice within the scatter plot.

Vocabulary	**Spanish Cognates**
Tier 2	
• clustering	
• positive	positivo
• negative	negativo
Tier 3	
• data	datos
• outliers	
• scatter plot	
• bivariate data	datos bivariado
• linear	lineal
• nonlinear	no lineal

Standards Connections
6.NS.8 → *8.SP.1*
8.SP.1 → 8.SP.2

8.SP.1 Example:

Example: Creating Scatter Plots 8.SP.1

Customer satisfaction is vital to the success of fast-food restaurants, and speed of service is a key component of that satisfaction. In order to determine the best staffing level, the owners of a local fast-food restaurant have collected the data below showing the number of staff members and the average time for filling an order. Describe the association between the number of staff and the average time for filling an order, and make a recommendation as to how many staff should be hired.

Number of staff members	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

8th Grade – CCSS for Mathematics

8.SP.A Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Essential Skills and Concepts:
- ☐ Understand, identify and describe clustering, outliers, positive and negative association, linear associations and nonlinear associations.
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• negative	negativo
Tier 3	
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8.SP.A.1

Standard Explanation

Building on work in earlier grades with univariate measurement data and analyzing data on line plots and histograms, grade-eight students begin to work with bivariate measurement data and use scatter plots to represent and analyze the data.

Bivariate measurement data represent two separate (but usually related) measurements. Scatter plots can show the relationship between the two measured variables. Collecting and analyzing bivariate measurement data help students to answer questions such as "How does more time spent on homework affect test grades?" and "What is the relationship between annual income and the number of years of formal education a person has?"

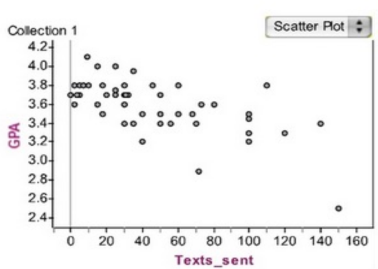
Students in grade eight construct and interpret scatter plots to investigate patterns of association between two quantities (8.SP.1). They also build on their previous knowledge of scatter plots to examine relationships between variables. Grade-eight students analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Additionally, they examine outliers to determine if data points are valid or represent a recording or measurement error.

Students can use tools such as those offered by the National Center for Education Statistics (http://nces.ed.gov/nceskids/createagraph/default.aspx [National Center for Education Statistics 2013]) to create a graph or generate data sets CA *Mathematics Framework*, adopted Nov. 6, 2013).

8.SP.1 Illustrative Task:

- Texting and Grades I, https://www.illustrativemathematics.org/content-standards/8/SP/A/1/tasks/975

Medhavi suspects that there is a relationship between the number of text messages high school students send and their academic achievement. To explore this, she asks each student in a random sample of 52 students from her school how many text messages he or she sent yesterday and what his or her grade point average (GPA) was during the most recent marking period. The data are summarized in the scatter plot of number of text messages sent versus GPA shown below.



Describe the relationship between number of text messages sent and GPA. Discuss both the overall pattern and any deviations from the pattern.

8th Grade – CCSS for Mathematics

8.SP.A.1

Standard Explanation

Building on work in earlier grades with univariate measurement data and analyzing data on line plots and histograms, grade-eight students begin to work with bivariate measurement data and use scatter plots to represent and analyze the data.

Bivariate measurement data represent two separate (but usually related) measurements. Scatter plots can show the relationship between the two measured variables. Collecting and analyzing bivariate measurement data help students to answer questions such as "How does more time spent on homework affect test grades?" and "What is the relationship between annual income and the number of years of formal education a person has?"

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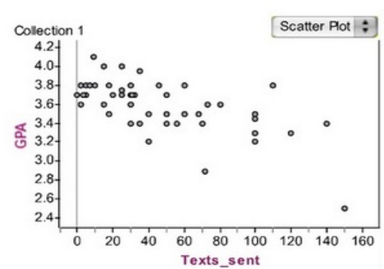

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8.SP.A Investigate patterns of association in bivariate data.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Essential Skills and Concepts:

- ☐ Model the relationship between two quantitative variables using a straight line
- ☐ Informally fit a straight line
- ☐ Assess the fit of the model by comparing it to the closeness of the data points to the line

Question Stems and Prompts:

- ✓ Describe the association between _______ and ______.
- ✓ What type of association does the scatter plot demonstrate? How do you know?
- ✓ Fit a line to the data within the scatter plot. What would the equation of this line be?
- ✓ How well does your line fit the data? How do you know?

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
Tier 3	
• data	datos
• scatter plot	
• linear association	asociación lineal
• line of best fit	

Standards Connections

8.SP.1 → *8.SP.2*
8.SP.2 – 8.F.4
8.SP.2 → 8.SP.3

8.SP.2 Example:

Example: Informally Determining a Line of Best Fit **8.SP.2**

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and the amount of gasoline used (in gallons). Describe the relationship between the variables. If the data are linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles traveled	0	75	120	160	250	300
Amount of gasoline used (gallons)	0	2.3	4.5	5.7	9.7	10.7

8th Grade – CCSS for Mathematics

8.SP.A Investigate patterns of association in bivariate data.

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- ✓ How well does your line fit the data? How do you know?

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
Tier 3	
• data	datos
• scatter plot	
• linear association	asociación lineal
• line of best fit	

Standards Connections

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Standard Explanation

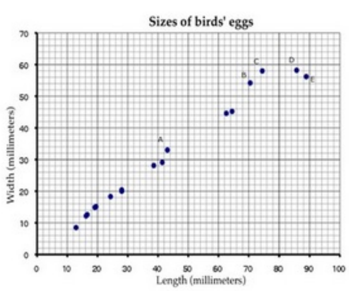
Grade-eight students know that straight lines are widely used to model relationships between two quantitative variables (8.SP.2). For scatter plots that appear to show a linear association, students informally fit a line (e.g., by drawing a line on the coordinate plane between data points) and informally assess the fit by judging the closeness of the data points to the straight line (CA *Mathematics Framework*, adopted Nov. 6, 2013).

For a data showing a linear pattern, students sketch a line through the "center" of the cloud of points that captures the essential nature of the trend, at first by use of an informal fitting procedure, perhaps as informal as laying a stick of spaghetti on the plot. How well the line "fits" the cloud of points is judged by how closely the points are packed around the line, considering that one or more outliers might have tremendous influence on the positioning of the line (Common Core Standards Writing Team, Draft Progressions 6-8 Statistics and Probability).

8.SP.2 Illustrative Task:

- Birds' Eggs, https://www.illustrativemathematics.org/content-standards/8/SP/A/2/tasks/41

This scatter diagram shows the lengths and widths of the eggs of some American birds.



a. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.

b. What does the graph show about the relationship between the lengths of birds' eggs and their widths?

8th Grade – CCSS for Mathematics

8.SP.A Investigate patterns of association in bivariate data.

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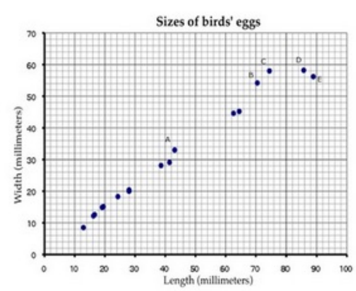

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8.SP.A Investigate patterns of association in bivariate data.

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

Essential Skills and Concepts:

- ☐ Understand how to use bivariate data to create a graph for the purpose of solving a problem
- ☐ Interpret the slope and intercept of a line.

Question Stems and Prompts:

- ✓ Is there an association between _______ and _____? How do you know?
- ✓ Given the data from _______ and ______. Make a scattered plot.

Vocabulary	**Spanish Cognates**
Tier 2	
• slope	
• intercept	
Tier 3	
• linear model	modelo lineal
• line of best fit	
• bivariate data	datos bivariado

Standards Connections

7.SP.2 → *8.SP.3*

8.EE.7b – *8.SP.3* – 8.F.4

8.SP.3 Example:

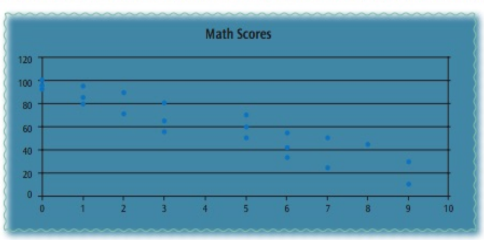
Example: Finding a Linear Model for a Data Set **8.SP.3**

Make a scatter plot by using data from students' math scores and absences. Informally fit a line to the graph and determine an approximate linear function that models the data. What would you expect to be the score of a student with 4 absences?

Solution:

Absences	3	5	1	1	3	6	5	3	0	7	8
Math scores	65	50	95	85	80	34	70	56	100	24	45
Absences	2	9	0	6	6	2	0	5	7	9	1
Math scores	71	30	95	55	42	90	92	60	50	10	80

Students would most likely use simple data software to make a scatter plot, finding a graph that looks like the following:



Students can use graphing software to find a line of best fit. Such a line might be $y=-8x+95$. They interpret this equation as defining a function that gives the approximate score of a student based on the number of his or her absences. Thus, a student with 4 absences should have a score of approximately $y=-8(4)+95=63$.

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

8th Grade – CCSS for Mathematics

8.SP.A Investigate patterns of association in bivariate data.

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Essential Skills and Concepts:
☐ Understand how to use bivariate data to create a graph for the purpose of solving a problem
☐ Interpret the slope and intercept of a line.

Question Stems and Prompts:
✓ Is there an association between _______ and _____? How do you know?
✓ Given the data from _______ and ______. Make a scattered plot.

Vocabulary / Spanish Cognates
Tier 2
• slope
• intercept
Tier 3
• linear model — modelo lineal
• line of best fit
• bivariate data — datos bivariado

Standards Connections
7.SP.2 → 8.SP.3
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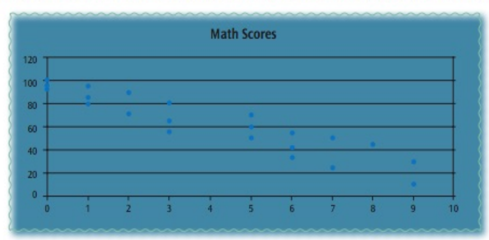

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8.SP.A.3

Standard Explanation

Students in grade eight solve problems in the context of bivariate measurement data by using the equation of a linear model (8.SP.3). They interpret the slope and the -intercept in the context of the problem. For example, in a linear model for a biology experiment, students interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 centimeters in the height of the plant (CA *Mathematics Framework,* adopted Nov. 6, 2013).

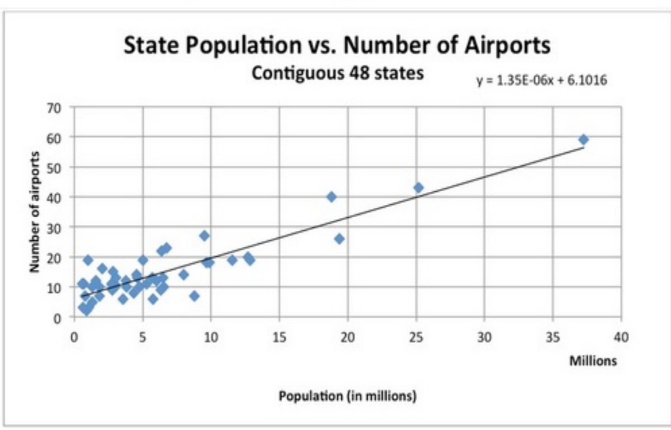
Focus, Coherence, and Rigor

Work in the Statistics and Probability cluster "Investigate patterns of association in bivariate data" involves looking for patterns in scatter plots and using linear models to describe data. This is directly connected to major work in the Expressions and Equations clusters (**8.EE.1–8▲**) and provides opportunities for students to model with mathematics (**MP.4**).

8.SP.3 Illustrative Task:

- Airports, Assessment Variation, https://www.illustrativemathematics.org/content-standards/8/SP/A/3/tasks/1370

The scatter plot below shows the relationship between the number of airports in a state and the population of that state according to the 2010 Census. Each dot represents a single state. The number of airports in each state comes from data on http://www.nationalatlas.gov/atlasftp.html?openChapters=chptrans#chptrans. The data for population comes from the 2010 census: http://www.census.gov/2010census/data/



a. How would you characterize the relationship between the number of airports in a state and the state's population? (Select one):

i. The variables are positively associated; states with higher populations tend to have fewer airports.

ii. The variables are negatively associated; states with higher populations tend to have fewer airports.

iii. The variables are positively associated; states with higher populations tend to have more airports.

8th Grade – CCSS for Mathematics

8.SP.A.3

Standard Explanation

Students in grade eight solve problems in the context of bivariate measurement data by using the equation of a linear model (8.SP.3). They interpret the slope and the -intercept in the context of the problem. For example, in a linear model for a biology experiment, students interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 centimeters in the height of the plant (CA *Mathematics Framework,* adopted Nov. 6, 2013).

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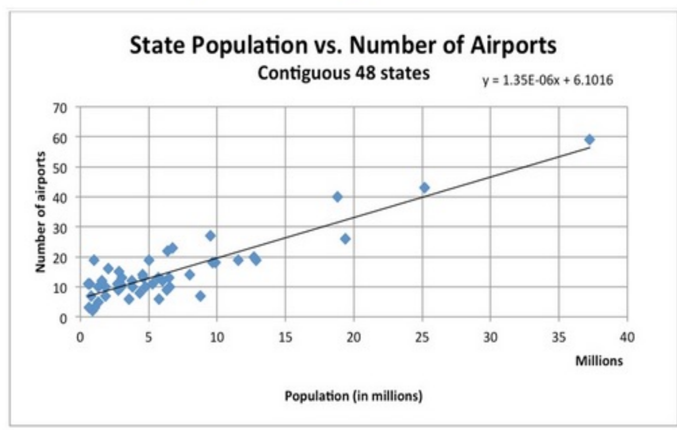

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8.SP.A Investigate patterns of association in bivariate data.

8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Essential Skills and Concepts:

- ☐ Understand patterns and frequency/relative frequency of categorical data.
- ☐ Calculate relative frequency
- ☐ Construct and understand a two-way table.
- ☐ Summarize data

Question Stems and Prompts:

- ✓ Is there an association between _______ and _____? How do you know?
- ✓ What is the relative frequency between ____ and ______?
- ✓ Collect data about _____ and ______. Use the results to complete a table. Is there an association? Explain.

Vocabulary	**Spanish Cognates**
Tier 2	
• frequency	frecuencia
• association	asociación
• rows	
• columns	columnas
Tier 3	
• two-way tables	
• bivariate data	datos bivariado
• relative frequencies	frecuencia relativa
• categorical variables	variable categórica

8.SP.4 Example:

Example: Two-Way Tables for Categorical Data 8.SP.4

The table at right illustrates the results when 100 students were asked these survey questions: (1) Do you have a curfew? (2) Do you have assigned chores? Students can examine the survey results to determine if there is evidence that those who have a curfew also tend to have chores.

		Curfew	
		Yes	No
Chores	Yes	40	10
	No	10	40

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there seems to be a positive correlation between having a curfew and having chores: it appears that most students with chores have a curfew and most students without chores do not have a curfew.

Adapted from CDE 2012d, ADE 2010, and NCDPI 2013b.

8th Grade – CCSS for Mathematics

8.SP.A Investigate patterns of association in bivariate data.

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Essential Skills and Concepts:
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☐ Construct and understand a two-way table.
☐ Summarize data

Question Stems and Prompts:
✓ Is there an association between _______ and _____? How do you know?
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Vocabulary / Spanish Cognates
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• association — asociación
• rows
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Tier 3
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8.SP.A.4

Standard Explanation

Students learn to see patterns of association in bivariate categorical data in a two-way table (8.SP.4). They construct and interpret a two-way table that summarizes data on two categorical variables collected from the same subjects. The two-way table displays frequencies and relative frequencies. Students use relative frequencies calculated from rows or columns to describe a possible association between the two variables. For example, students collect data from their classmates about whether they have a curfew and whether they do chores at home. The two-way table allows students to easily see if students who have a curfew also tend to do chores at home (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Building on experience with decimals and percent, and the ideas of association between measurement variables, students now take a more careful look at possible association between categorical variables. "Is there a difference between sixth graders and eighth graders with regard to their preference for rock, rap, or country music?" Data from a random sample of sixth graders and another random sample of eighth graders are summarized by frequency counts in each cell in a two-way table of preferred music type by grade. The proportions of favored music type for the sixth graders are then compared to the proportions for eighth graders. If the two proportions for each music type are about the same, there is little or no association between the grade and music preference because both grades have about the same preferences. If the two proportions differ, there is some evidence of association because grade level seems to make a difference in music preferences. The nature of the association should then be described in more detail (Common Core Standards Writing Team, Draft Progressions 6-8 Statistics and Probability).

8.SP.4 Illustrative Task:

- What's Your Favorite Subject?, https://www.illustrativemathematics.org/content-standards/8/SP/A/4/tasks/973

All the students at a middle school were asked to identify their favorite academic subject and whether they were in 7th grade or 8th grade. Here are the results:

Favorite Subject by Grade

Grade	English	History	Math/Science	Other	Totals
7th Grade	38	36	28	14	116
8th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Is there an association between favorite academic subject and grade for students at this school? Support your answer by calculating appropriate relative frequencies using the given data.

8th Grade – CCSS for Mathematics

8.SP.A.4

Standard Explanation

Students learn to see patterns of association in bivariate categorical data in a two-way table (8.SP.4). They construct and interpret a two-way table that summarizes data on two categorical variables collected from the same subjects. The two-way table displays frequencies and relative frequencies. Students use relative frequencies calculated from rows or columns to describe a possible association between the two variables. For example, students collect data from their classmates about whether they have a curfew and whether they do chores at home. The two-way table allows students to easily see if students who have a curfew also tend to do chores at home (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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8.SP.4 Illustrative Task:

- What's Your Favorite Subject?, https://www.illustrativemathematics.org/content-standards/8/SP/A/4/tasks/973

All the students at a middle school were asked to identify their favorite academic subject and whether they were in 7th grade or 8th grade. Here are the results:

Favorite Subject by Grade

Grade	English	History	Math/Science	Other	Totals
7th Grade	38	36	28	14	116
8th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Is there an association between favorite academic subject and grade for students at this school? Support your answer by calculating appropriate relative frequencies using the given data.

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Resources for the CCSS 8th Grade Bookmarks

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics Tucson, AZ: Institute for Mathematics and Education, University of Arizona (Drafts)

- The Number System, 6 – 8 (2013, July 9)
- 6 – 8, Expressions and Equations (2011, April 22)
- Grade 8, High School, Functions, (2012, December 3)
- 6 – 8, Statistics and Probability (2011, December 26)
- 6 – 7, Ratios and Proportional Relationships (2011, December 26)

Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, http://www.illustrativemathematics.org/

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North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, http://www.ncpublicschools.org/acre/standards/common-core-tools/ - unmath

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) http://www.katm.org/baker/pages/common-core-resources.php

8th Grade – CCSS for Mathematics

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Arizona's College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona's College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication, http://www.azed.gov/azccrs/mathstandards/

Howard County Public School System, Elementary Mathematics Office, Standards for Mathematical Practice for Parents, Draft 2011, https://grade3commoncoremath.wikispaces.hcpss.org/file/view/SFMP for Parents.docx/286906254/SFMP for Parents.docx

Howard County Public School System, Elementary and Secondary Mathematics Offices, Wiki Content and Resources, Elementary by grade level https://grade5commoncoremath.wikispaces.hcpss.org/home, and Secondary https://secondarymathcommoncore.wikispaces.hcpss.org

Long Beach Unified School District, Math Cognates, retrieved on 7/14/14, http://www.lbschools.net/Main_Offices/Curriculum/Areas/Mathematics/XCD/ListOfMathCognates.pdf

Smarter Balanced Assessment Consortium: Mathematics Item Specifications Grades 6-8, Developed by Measured Progress/ETS Collaborative April 2012, Item specifications by target retrieved from zip file, http://www.smarterbalanced.org/smarter-balanced-assessments/

A Graph of the Content Standards, Jason Zimba, June 7, 2012, http://tinyurl.com/ccssmgraph

Arizona's College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona's College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication, http://www.azed.gov/azccrs/mathstandards/

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