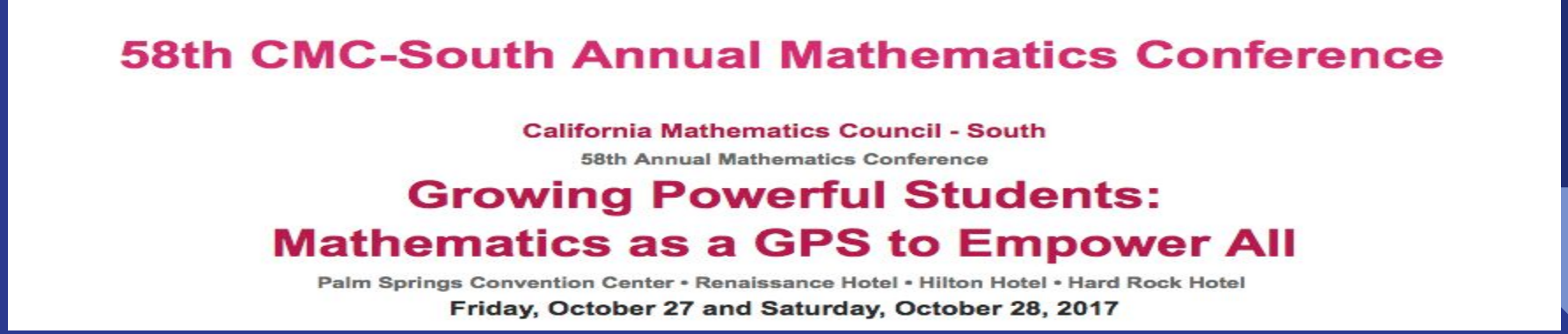

Making Sense of Word Problems with a Mathematical Mindset

Presented by:

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How Old is the Shepherd?

Standards for

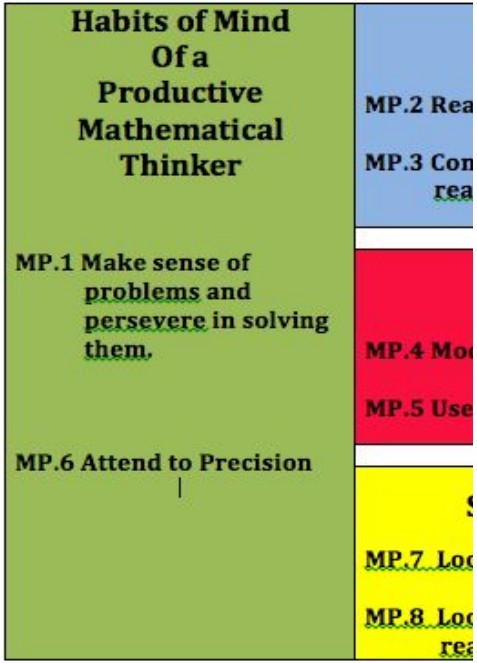


Table 5-2. Standards for Mathematical Practice—Explanation and Examples for Grade Five

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade five, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions that include mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. For example, "Sonia had $2\frac{1}{3}$ sticks of gum. She promised her brother that she would give him $\frac{1}{2}$ of a stick of gum. How much will she have left after she gives her brother the amount she promised?" Teachers can encourage students to check their thinking by having students ask themselves questions such as these: "What is the most efficient way to solve the problem?""Does this make sense?""Can I solve the problem in a different way?"
MP.2 Reason abstractly and quantitatively.	Students recognize that a number represents a specific quantity. They connect quantities to written symbols and create logical representations of problems, considering appropriate units and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Teachers can support student reasoning by asking questions such as these: "What do the numbers in the problem represent?""What is the relationship of the quantities?" Students write simple expressions that record calculations with numbers and represent or round numbers using place-value concepts. For example, students use abstract and quantitative thinking to recognize, without calculating the quotient, that $0.5 \times (300 \div 15)$ is $\frac{1}{2}$ of $(300 \div 15)$.
MP.3 Construct viable arguments and critique the reasoning of others.	In grade five, students may construct arguments by using visual models such as objects and drawings. They explain calculations based upon models, properties of operations, and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. Students use various strategies to solve problems, and they defend and justify their work to others. For example: "Two after-school clubs are having pizza parties. The teacher will order 3 pizzas for every 5 students in the math club and 5 equally sized pizzas for every 8 students on the student council. How much pizza will each student get at the respective parties? If a student wants to attend the party where she will get the most pizza (assuming the pizza is divided equally among the students at the parties), which party should she attend?"

Problem Types

Table 2-3 *(continued)*

	Total Unknown	Addend Unknown	Both Addends Unknown
Put together/ Take apart	There are 30 red apples and 20 green apples on the table. How many apples are on the table? 30 + 20 = ?	Roger puts 24 apples in a fruit basket. Nine (9) are red and the rest are green. How many are green?" There is no direct or implied action. The problem involves a set and its subsets. It may be modeled by 24 – 9 = ☐ or 9 + ☐ = 24. This type of problem provides students with opportunities to understand subtraction as an unknown-addend problem.	Grandma has 5 flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	Pat has 19 peaches. Lynda has 14 peaches. How many more peaches does Pat have than Lynda? "Compare" problems involve relationships between quantities. Although most adults might use subtraction to solve this type of problem (19 – 14 = ☐), students will often solve this problem as an unknown-addend problem (14 + ☐ = 19) by using a counting-up or matching strategy. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context—not the representation separated from its context.	**("More" version):** Theo has 23 action figures. Rosa has 2 more action figures than Theo. How many action figures does Rosa have? This problem can be modeled by 23 + 2 = ☐. **("Fewer" version):** Lucy has 28 apples. She has 2 fewer apples than Marcus. How many apples does Marcus have? This problem can be modeled as 28 + 2 =☐. The misleading language form "fewer" may lead students to choose the wrong operation.	**("More" version):** David has 27 more bunnies than Keisha. David has 28 bunnies. How many bunnies does Keisha have? This problem can be modeled by 28 – 27 = ☐. The misleading language form "more" may lead students to choose the wrong operation. **("Fewer" version):** Bill has 24 stamps. Lisa has 2 fewer stamps than Bill. How many stamps does Lisa have? This problem can be modeled as 24 – 2 = ☐.

Note: Further examples are provided in table GL-4 of the glossary.

Table 3-3. Types of Multiplication and Division Problems (Grade Three)

	Unknown Product	Group Size Unknown[2]	Number of Groups Unknown[3]
	3×6=?	3×?=18 and 18÷3=?	? × 6 = 18 and 18÷6=?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there altogether? **Measurement example** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally and packed into 3 bags, then how many plums will be in each bag? **Measurement example** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed, with 6 plums to a bag, then how many bags are needed? **Measurement example** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example** What is the area of a rectangle that measures 3 centimeters by 6 centimeters?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example** A rectangle has an area of 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?.	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example** A rectangle has an area of 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?
Compare	Grade-three students do not solve multiplicative "compare" problems; these problems are introduced in grade four (with whole-number values) and also appear in grade five (with unit fractions).		
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

https://www.cde.ca.gov/ci/ma/cf/documents/mathgrade3fwlmg2.pdf

Annie Fetter - Sense-Making: It Isn’t Just for Literacy Anymore

Math Stories - Read and Flip

Math Stories

Materials:

- Chart paper or white board
- Markers
- Student copy of word problem

Procedures:

1. Tell students they will be reading a "math story."
2. Each student will have a math story face down on their desk (no pencils).
3. First Read: Students will have 30 seconds to read the math story. As they read, students will need to remember as much information from the story as they can. If they finish reading the story early, students can read the problem again. After 30 seconds, students will turn the paper face down again.
4. Teacher will ask students to share everything that they remember from the story.
5. Teacher will scribe on chart paper or the whiteboard the details students remember about the math story.
6. Second Read: Students will have 20 seconds to read the math story again. Students will reread to check the story details and gain more information. After 20 seconds, students will turn the paper face down again.
7. Teacher will make any changes to the details written on the chart paper or whiteboard.
8. Third Read: Students will turn their paper over and keep the story facing up. Check the information and clarify the details of the story.
9. Have students underline the question from the problem. Teacher and students will write an answer statement to the question in the story (I had ____ markers before my friend gave me 12 more.)
10. Students will have 30 seconds to draw a quick sketch to solve the math story question. Students may not use any numbers or math equation symbols (+, −, ×, ÷) in their sketch. Later on with larger numbers, students may use numbers in their sketches (tape diagrams).
11. Students will share their sketch with their partner and to critique and justify their answers.
12. Students will make a number sentence to match their drawing.
13. Students will share aloud their number sentences and answers to the question.
14. Students and teacher complete their answer statement.

Read and Flip

...Focus on the CONTEXT before they get to the math!

READ AND FLIP Process:

- Students read the problem multiple times in order to make sense of the context.
- Students share their ideas and teacher records them.
- Teacher facilitates student understanding and helps them VISUALIZE how the problem in behaving. The teacher may even ask students to act out what is happening.
- Students problem solve while assessing the reasonableness of their process and solutions.

Sense Making Questions

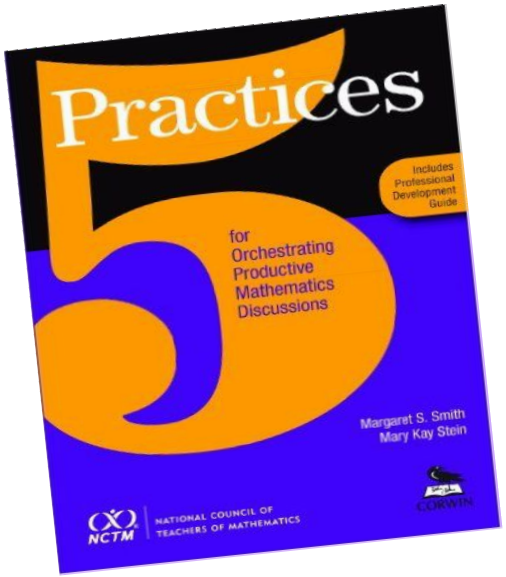
- What are the details of what you read?
- Can you retell this in your own words?
- Can you see this as a movie in your head?
- Where is the evidence in the text?
- What are we trying to figure out?
- What do you predict?
- Can you draw a picture of how this looks?

Jenny’s Video

Debriefing

- The length of the problem and wording is less a factor than we think.
- Teaching ***keywords*** can **actually interfere** with students' ability to solve problems.
- **The most important factor is whether students can visualize what the problem is asking.**

5 Practices for Orchestrating Productive Mathematical Discussions



0 - Selecting the Task

1 - Anticipating

2 - Monitoring

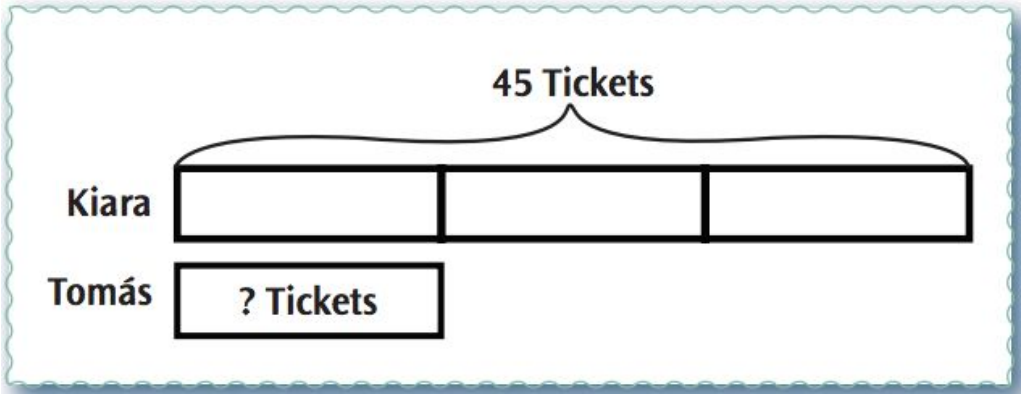
3 - Selecting

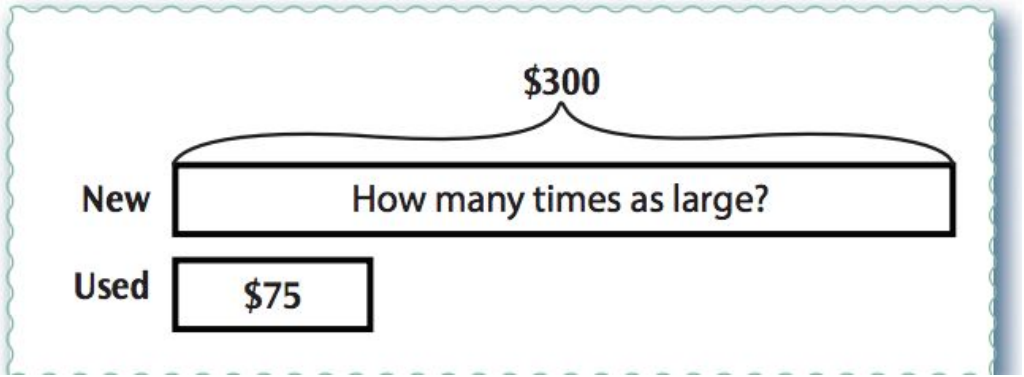
4 - Sequencing

5 - Connecting

- ***Anticipating*** student responses to challenging mathematical tasks;
- ***Monitoring*** students' work on and engagement with the tasks;
- ***Selecting*** particular students to present their mathematical work;
- ***Sequencing*** the student responses that will be displayed in a specific order and
- ***Connecting*** different students' responses and connecting the responses to key mathematical ideas

Notice and Wonder





Math Story

Molly the zookeeper has 24 bananas to feed the monkeys. Each monkey needs to eat 4 bananas. How many monkeys can Molly feed?

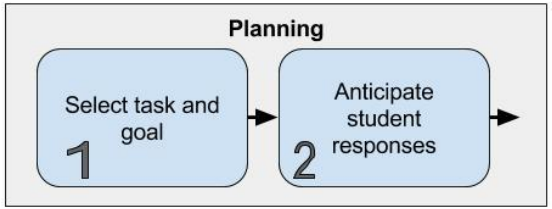
CA Mathematics Framework Grade 3 p.165

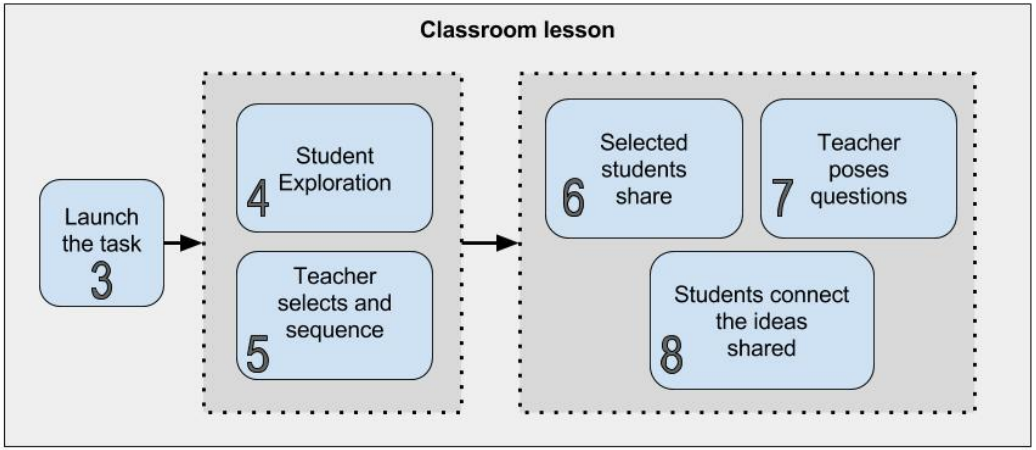
Student Sharing

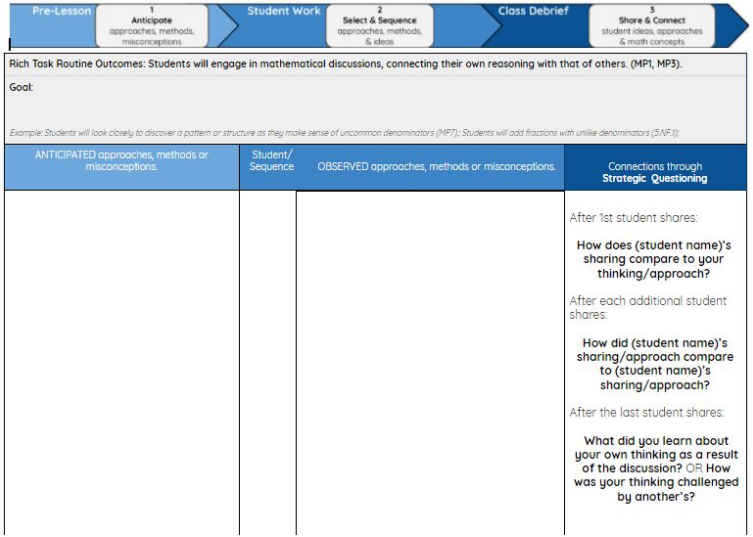
Think about how you have students share and why.

This lesson structure provides opportunities and supports for students to make sense of problems and hear how others made sense of the problem too.

Rich Teaching Routine

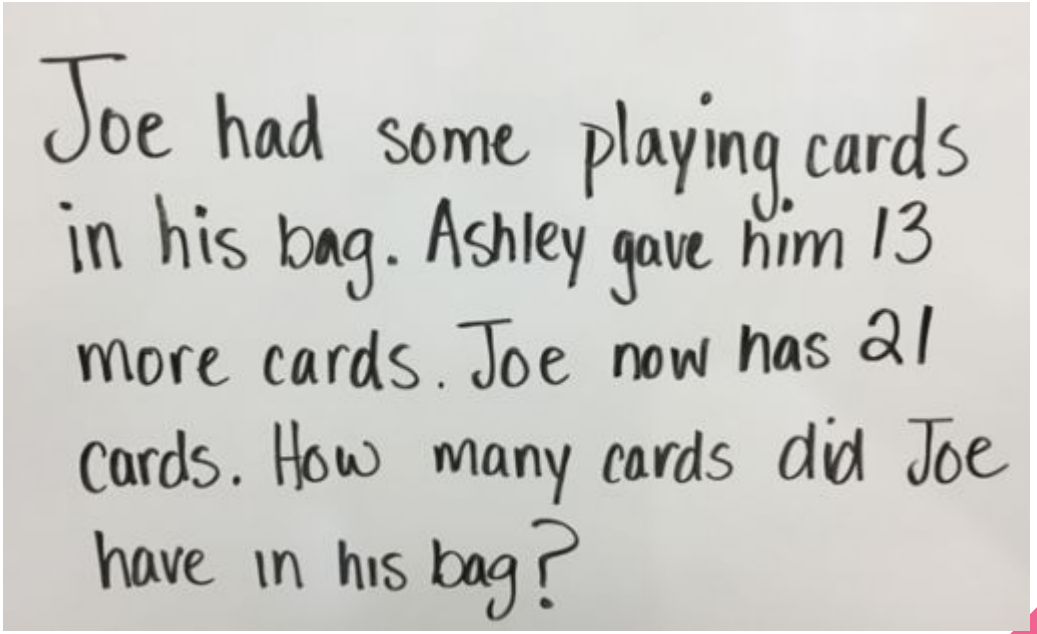


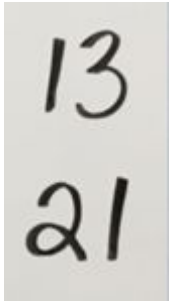


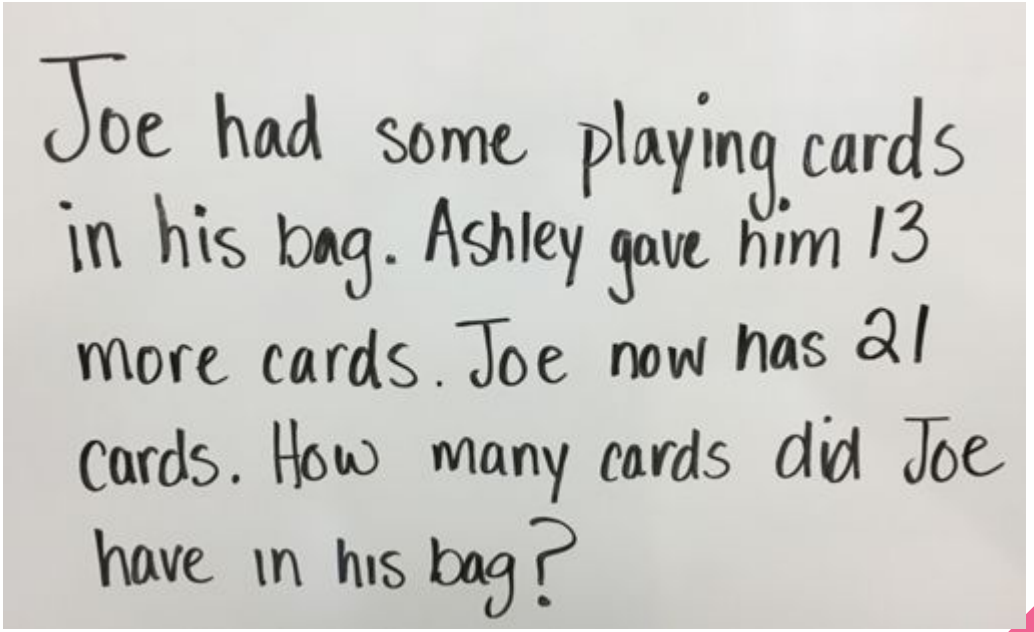


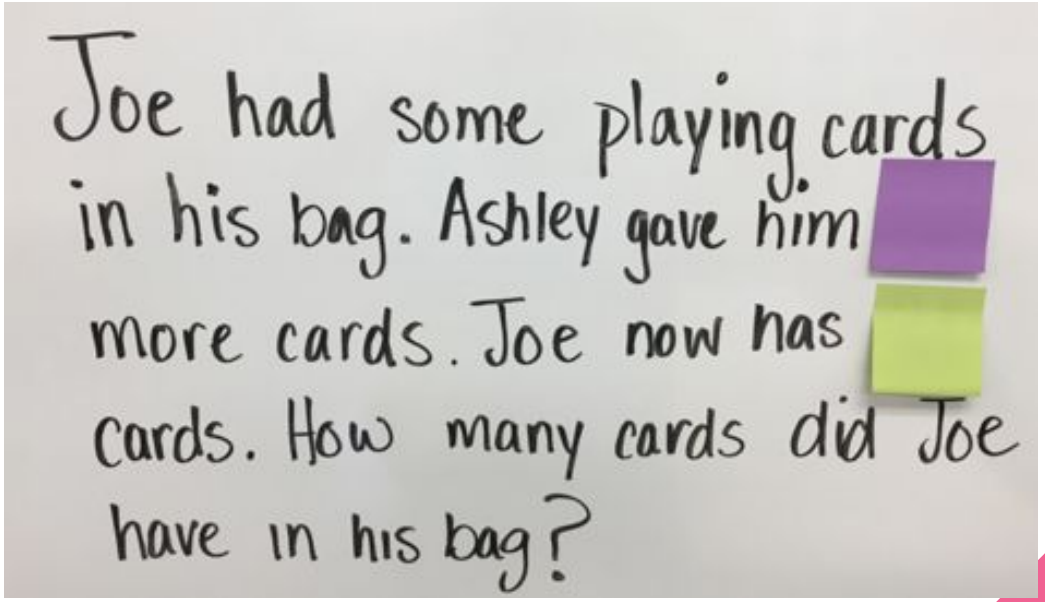
Teacher Reflection

Numberless Word Problems




13
21


Joe had some playing cards in his bag. Ashley gave him 13 more cards. Joe now has 21 cards. How many cards did Joe have in his bag?


Joe had some playing cards in his bag. Ashley gave him
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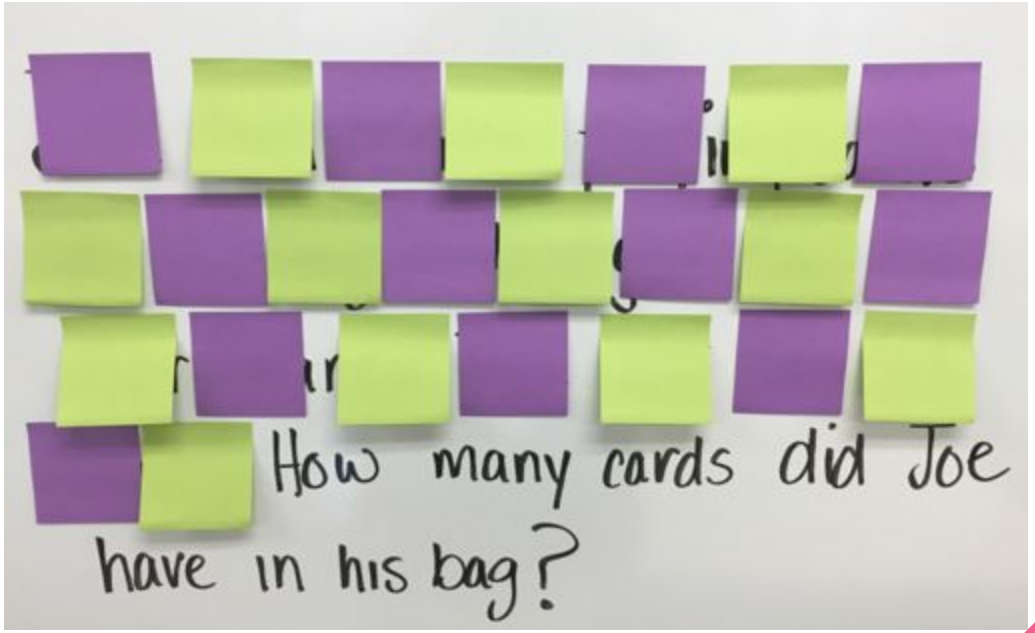

How many cards did Joe have in his bag?

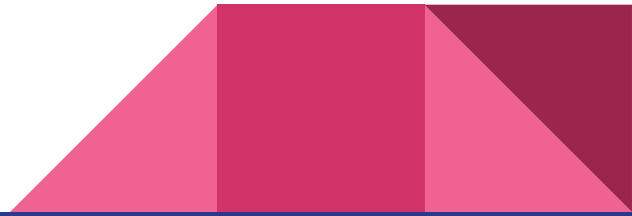
Numberless Word Problem

There were some friends reading books. There were more friends reading books than playing board games.

https://bstockus.wordpress.com/numberless-word-problems/

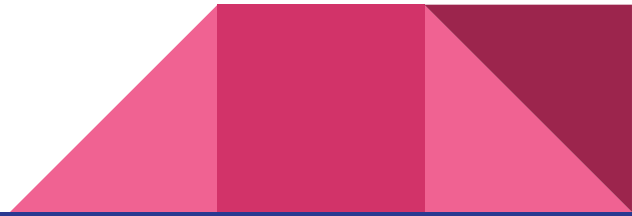
Numberless Word Problem

There were 10 friends reading books. There were more friends reading books than playing board games.



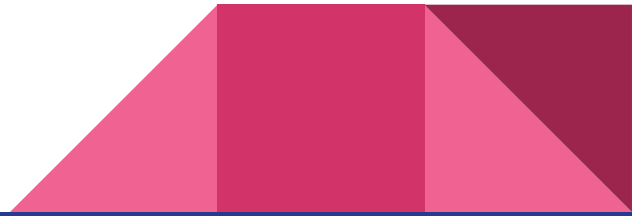
Numberless Word Problem

There were 10 friends reading books. There were 3 more friends reading books than playing board games.



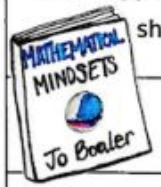
Numberless Word Problem

There were 10 friends reading books. There were 3 more friends reading books than playing board games. How many friends were playing board games?



Math Mindset

Recommendations for Task/Lesson Design
Open the task to encourage multiple methods, pathways and representations.
Pose a problem before teaching the method.
Design a task that allows all learners to contribute to the learning and have room for extension.
Make opportunities for students to authentically share their thinking with peers.
Add a visual component.
Add the requirement to convince and reason, be skeptical.



Powerful Questions to develop a deep level of understanding
How do you see that idea?
Why does that answer make sense?
Why does that method work?
How is that method connected to others?
How can that idea be represented in different ways?
Can you prove it?
Can you prove it visually?
Can you justify your thinking?
Can you predict what would happen if....?
Did you make any interesting mistakes?

Developed by Jo Boaler/Youcubed.org and Tulare County Office of Education

Tape Diagrams

↗Juan has 3 more seashells than Kim. Juan and Kim have 15 seashells altogether. Find the number of seashells that Juan has.

↗Joe has 3 times as much money as Ming. Joe and Ming have $120 altogether. Find the amount of money Joe has.

What did you find valuable/useful?

Write or tweet (@CAMathCouncil and hashtag#cmcmath).

Please be prepared to share out.

Resources

Sense-Making: It isn't Just for Literacy Anymore: Annie Fetter
https://www.youtube.com/watch?v=Iz7aELjGy_Y

Numberless Word Problems:
https://bstockus.wordpress.com/numberless-word-problems/

How Old is the Shepherd: https://www.youtube.com/watch?v=kibaFBgaPx4

California Math Framework: Grades 2 & 3
https://www.cde.ca.gov/ci/ma/cf/mathfwchapters.asp

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