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Jim Vidak, County Superintendent of Schools

DEVELOPING UNDERSTANDING OF FRACTIONS WITH THE COMMON CORE
GRADES 3 - 5

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GOALS/AGENDA

- Develop an understanding of the standards for fractions outlined in the Common Core State Standards.

- Develop strategies for supporting students in modeling fractions and understanding fractions conceptually.

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8 MATHEMATICAL PRACTICE STANDARDS

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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CCSS - FRACTIONS

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Fractions – Grades 1-6

A CLOSER LOOK AT THE FRACTION STANDARDS

Read 3.NF.1-3
Develop Understanding of Fractions as Numbers

Fractions – Grades 1-6

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PROGRESSION:
THE MEANING OF FRACTIONS

- In Grades 1 and 2, students use fraction language to describe partitions of shapes in to equal shares. In Grade 3 they start to develop the idea of a fraction more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle, a line segment, or any one finite entity susceptible to subdivision and measurement. In Grade 4, this is extended to include wholes that are collections of objects.

Fraction Progression page 2

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FRACTIONS PROGRESSION

- Understanding the arithmetic of fractions draws upon four prior progressions that informed the CCSS:
  - equal partitioning,
  - unitizing,
  - number line,
  - and operations.

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Phil Daro

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3.NF.1

3.NF.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

- "Students start with unit fractions (fractions with numerator 1)...."
- "Next, students build fractions from unit fractions, seeing the numerator 3 or ¾ as saying that ¾ is the quantity you get by putting 3 of the ¼'s together."
- "They read any fraction this way, and in particular there is no need to introduce "proper fractions" and "improper fractions" initially; 5/3 is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts."

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"3-5 Number and Operations – Fractions" Progression, page 2

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UNITIZING

One is One…or is it?

http://ed.ted.com/lessons/one-is-one-or-is-it

Discuss:

Why is the idea of the unit important in fractions?

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"UNIT (ONE)", A SIMPLE BUT POWERFUL CONCEPT

The following quotations are from Sheldon's Complete Arithmetic (1886)

Quotation 1

- A **unit** is *a single thing or one*; as one apple, one dollar, one hour, one.

Quotation 2

- *Like numbers* are numbers whose **units** are the same; as $7 and $9.
- *Unlike numbers* are numbers whose **units** are different; as 8 lb. and 12 cents.

  Can you add 8 cents and 7 cents? What kind of numbers are they?

  Can you add $5 and 5lb.? What kind of numbers are they?

Principle: *Only like numbers can be added and subtracted.*

Why do we need to line numbers up when we do addition ?

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WITH MULTIPLICATION AND DIVISION, THE CONCEPT OF "UNIT" IS EXPANDED:

Quotation 1

- A **unit** is *a single thing or one*.

Quotation 2

- A *group* of things if considered as a single thing or one is also a **unit**; as one class, one dozen, one group of 5 students.

  There are 3 plates each with 5 apples in it. How many apples are there in all?

  What is the unit (the "one")?

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WITH FRACTIONS, THE CONCEPT OF "UNIT" IS EXPANDED ONE MORE TIME:

Quotation 1

- A **unit** is *a single thing or one*.

Quotation 2

- A unit, however, may be divided into equal parts, and each of these parts becomes a single thing or a **unit**.

Quotation 3

- In order to distinguish between these two kinds of units, the first is called an *integral unit*, and the second a *fractional unit*.

  What is the fractional unit of 3/4 ? of 2/3?

Principle: *Only like numbers can be added and subtracted.*

Computing 3/4 + 2/3, Why do we need to turn the fractions into fractions with common denominator?

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UNITS ARE THINGS YOU COUNT

- Objects
- Groups of objects
- 1
- 10
- 100
- ¼   unit fractions
- Numbers represented as expressions

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PRINCIPLE:
ADDING AND SUBTRACTING "UNITS"

- 3 pennies + 5 pennies = 8 pennies
- 3 ones + 5 ones = 8 ones
- 3 tens + 5 tens = 8 tens
- 3 inches + 5 inches = 8 inches
- 3 "¼ inches" + 5 "¼ inches" = 8 "¼ inches"
- 3/4 + 5/4 = 8/4
- 3(x + 1) + 5(x + 1) = 8(x + 1)

  You can compute the sums and differences of *like terms*

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PRINCIPLE:
ADDING AND SUBTRACTING "UNITS"

- 2 dimes + 3 quarters
- 27 inches + 2 feet
- 8 ones + 9 tens
- 12 seconds + 1 minute
- 1/4 + 1/3
- What must happen before we can compute the sum or difference?
- Unlike Terms may be added and subtracted, but computing the sum or difference is only possible when we have Like Terms (Common Units)

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FRACTION STRIPS

- Materials
  - Six different colors of paper cut into 1" x 8" strips. Each child will need 6 strips, one of each color.
- Task Description
  - Give each student six strips of paper, one of each color.
  - Specify one color. Tell students that this strip will represent the whole. Have students write "one whole" on the fraction strip.
  - Specify a different color and have students fold it into two equal pieces. Have students draw a line on the fold. Ask students what they think each of these strips should be called. Discuss how we write fractions. Have students label their strips using both the word and the fractional representation – 1/2 and one-half.
  - Repeat this process for thirds, fourths, sixths, and eighths.
  - Students should have 6 fraction strips.

16 Adapted from Georgia Department of Education, Grade 3 Unit 6: Reasoning and Comparing Fractions

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EXPLORING FRACTIONS WITH FRACTION STRIPS

Small Groups – Discuss and Record
- What observations do you have about the fractions strips?

Group Discussion
- How many halves does it take to make a whole strip?
- How many thirds does it take to equal one whole?
- How many fourths, sixths, eighths?
- What patterns do you notice?
- What does the numerator represent?
- What does the denominator represent?
- If you made a 1/9 fraction strip, how many ninths would it take to make a whole?

17 Adapted from Georgia Department of Education, Grade 3 Unit 5: Reasoning and Comparing Fractions

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UNIT FRACTIONS

- "The goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers; just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions."

18 "3-5 Number and Operations – Fractions" Progression, page 3

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COUNTING BY FRACTIONS

- "Students should come to think of counting fractional parts in much the same way as they might count apples or any other objects."

- Example: " ...tell students what type of piece is being shown and simply count them together: "*one*-fourth, *two*-fourths, *three*-fourths, *four*-fourths, *five*-fourths." Ask, "If we have five-fourths, is that more than one whole, less than one whole, or the same as one whole?"

19   Teaching Student-Centered Mathematics: Grades 5-8. Van de Walle and Lovin, page 67.

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3.NF.3

3.NF.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <, =, or <, and justify the conclusions, e.g., by using a visual fractions model.

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COMPARING FRACTIONS

Materials:
- Fraction Strips: Cut along the folds
- Sandwich bags or envelopes to store the strips.

I. Small Groups – Discuss and Record
- What observations do you have about the separated Fraction Strips?
- Do you see any special relationships among the different colored strips?

Additional Questions
- Place a 1/2 strip on your desk. How many strips or combinations of strips are the same size as 1/2?
- When fractions are the same size, they are called equivalent. What other equivalent fractions can you create?

21   Adapted from Georgia Department of Education, Grade 3 Unit 5: Reasoning and Comparing Fractions

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COMPARING FRACTIONS
(CONTINUED)

II. Whole Group Discussion:
- What relationships did you discover about fractions?
- What equivalent groups of fractions did you discover?

Adapted from Georgia Department of Education, Grade 3 Unit 5: Reasoning
22 and Comparing Fractions

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COMPARING FRACTIONS
(CONTINUED)

III. The friends below are playing red light-green light. Who is winning? Explain your reasoning.

Use your fraction strips to determine how far each friend has moved.

Mary – 3/4
Harry – 1/2
Larry – 5/6
Sam – 5/8
Angie – 2/3

Adapted from Georgia Department of Education, Grade 3 Unit 5: Reasoning
23 and Comparing Fractions

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3.NF.2

Develop understanding of fractions as numbers

3.NF.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.
a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

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LOCATING ½ ON A NUMBER LINE

Scheme:
Unit (whole)
Unitizing (parts)

Why do students respond this way?

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FRACTION STRIPS ON A NUMBER LINE

Materials:
- Fraction Strips
- Sentence Strip marked into three 8" units.

Task:
- Use your fractions strips to mark off your number line in thirds.

- What observations can you make?

The number line

0   1   2   3   4   5   6 etc.

The number line marked off in thirds

0   1   2   3   4

$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$ $\frac{9}{3}$ $\frac{10}{3}$ $\frac{11}{3}$ $\frac{12}{3}$

Number line representation of $\frac{5}{3}$

One part of a division of the unit interval into 3 parts of equal length

0   1   2   3   4

5 parts

the point $\frac{5}{3}$ on the number line

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26   Diagrams from Fraction Progression, page 3

EQUIVALENT FRACTIONS ON A NUMBER LINE

Using the number line and fraction strips to see fraction equivalence

0          $\frac{1}{2}$ $\frac{2}{2}=1$

$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}=1$

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

$\frac{1}{2}$

- Use your fractions strips to mark off your number line in fourths.
- Now mark off your number line in halves.
- What observations can you make?
- What fractions are equivalent?

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27   Diagrams from Fraction Progression, page 4

4.NF.3

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

UNITIZING LINKS FRACTIONS TO WHOLE NUMBER ARITHMETIC

- Students' expertise in whole number arithmetic is the most reliable expertise they have in mathematics
- It makes sense to students
- If we can connect difficult topics like fractions and algebraic expressions to whole number arithmetic, these difficult topics can have a solid foundation for students

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ADDING AND SUBTRACTING FRACTIONS:
VIA COMPOSITION AND DECOMPOSITION

- Let's start with the number 5

- What do we have 5 of (What are we counting)?

- How else can we write 5?

Why do we use 5, rather than 3+2, or 1+1+1+1+1?

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ADDING AND SUBTRACTING FRACTIONS:
VIA COMPOSITION AND DECOMPOSITION

- So when working with the unit "ones", we can express that like this:

1 + 1 + 1 + 1 + 1 = 5

What unit would we be working with here?

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

Show what this expression looks like using your fraction strips.

31   Adapted from Fraction Progression, page 6

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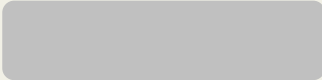*Jim Vidak, County Superintendent of Schools*

ADDING AND SUBTRACTING FRACTIONS:
VIA COMPOSITION AND DECOMPOSITION

- How else can we express this?

$$= \frac{5}{8}$$

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ADDING AND SUBTRACTING FRACTIONS:
VIA COMPOSITION AND DECOMPOSITION

5/6 + 3/6

How many ways can we do this?

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CONSTRUCTING TASK: FRACTION ADDITION

Part 1 - In pairs:

- Using the paper fraction strips, demonstrate a model of the following expressions.

  - 1/4 + 1/4

  - 1/3 + 1/3

  - 2/3 + 2/3

- Even if you know the "answer," prove it with the model. Can the answer be written in other ways?

- Show the same expressions on your number line.

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34   Adapted from Georgia Department of Education, Grade 5 Unit 4   *Jim Vidak, County Superintendent of Schools*

CONSTRUCTING TASK: FRACTION ADDITION

- In groups:

  - How do we know that our answers are correct?

  - How do we know that our points on the number line are labeled correctly?

  - Are there other names for some of the points marked on our number line?

  - Are there any patterns or rules for adding these types of fractions? Explain what you have discovered.

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35   Adapted from Georgia Department of Education, Grade 5 Unit 4   *Jim Vidak, County Superintendent of Schools*

CONSTRUCTING TASK: FRACTION ADDITION AND SUBTRACTION

Part 2 - In pairs:

- Using the paper fraction strips, demonstrate a model of the following expressions.

  - 1/2 + 1/3

  - 2/3 – 1/2

  - 2/3 + 1/2

- Even if you know the "answer," prove it with the model. Can the answer be written in other ways?

- Show the same expressions on your number line.

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36   Adapted from Georgia Department of Education, Grade 5 Unit 4   *Jim Vidak, County Superintendent of Schools*

CONSTRUCTING TASK: FRACTION ADDITION AND SUBTRACTION

- In groups:
  - How do we know that our answers are correct?

  - How do we know that our points on the number line are labeled correctly?

  - How can we change these problems to make them easier, like in part I?

  - Is there one fraction name for your equation points on the number line?

  - Did you identify any patterns or rules for adding and subtracting these kinds of fractions? Explain what you have found.

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37   Adapted from Georgia Department of Education, Grade 5 Unit 4

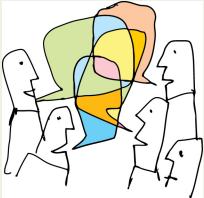## DISCUSS

How is this similar and how is this different from how your curriculum currently addresses fractions?

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RESOURCES

- Common Core Connect
  http://commoncore.tcoe.org

- E-mail
  - Julie Joseph– jjoseph@ers.tcoe.org

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