Tulare County Office of Education

Jim Vidak, County Superintendent of Schools

Hands-On Strategies for Transformational Geometry

Grades 8-9

October 25, 2014

http://ccss.tcoe.org/math

Presented by

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TCOE CCSS Website: http://ccss.tcoe.org/

Supporting California's Common Core State Standards

Standards for Mathematical Practice (K-12)

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

www.corestandards.org

Transformations

Objectives

Students will be able to identify and compare the three congruence transformations, apply the three congruence transformations to coordinates of the vertices of figures, identify and apply dilations, and apply transformations to real-world situations.

Core Learning Goal

2.1.3 The student will use transformations to move figures, create designs and/or demonstrate geometric properties.

Materials Needed

Worksheets, protractor, ruler, patty paper, Mira™

Optional – Dynamic geometry software

Approximate Time

Four 45-minute lessons

Additional Resources

National Council of Teachers of Mathematics (NCTM). *Navigating Through Geometry in Grades 6-8*, 2002, <u>Chapter 3 -Transformations and Symmetry</u>, pp. 43-58.

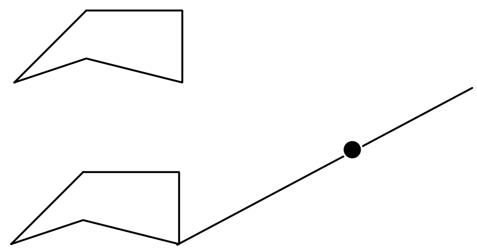
National Council of Teachers of Mathematics (NCTM). *Navigating Through Geometry in Grades 9-12*, 2001, <u>Chapter 1 - Transforming Our World</u>, pp. 9-26.

Dixon, Juli, *Movements in the Plane: Conjecturing about Properties of Transformations,* NCTM Math On-Line January 2003

Transformations

Translations

Example



1. a. Draw a polygon on patty paper.
   This polygon is called the pre-image.

   b. Draw a line segment from one vertex toward the edge of the paper.

   c. Mark a point on your line segment.

   d. Trace the polygon and line on a second sheet of patty paper.

   e. Place one copy under the other aligning the corresponding points. Now slide the top picture so that the point on the line on the bottom paper and the vertex of the polygon on the top paper coincide, keeping the lines on top of each other.

   f. Trace both polygon figures on to the same sheet of patty paper.

   g. Using a third sheet of patty paper, mark the length of the segment from the vertex of the original polygon to the point marked on the line.

   h. Draw segments connecting corresponding vertices of the pre-image polygon and the image polygon. Compare the lengths of each segment with the marked length.

2. a. How do the distances between the vertices of the pre-image and the vertices of the image compare?

   b. Write a statement about the distance between any point and its image in a translation.

3. How do the lengths of the corresponding sides and the measures of corresponding angles of the pre-image and the image compare?

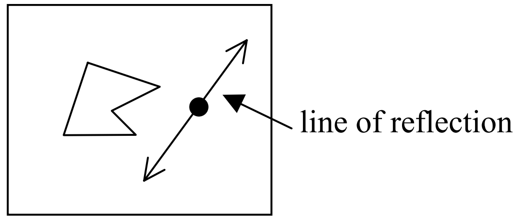
4. A translation is called a **rigid transformation** or an **isometry**. The word isometry can be broken into *iso* meaning the same, and *metry* meaning measure. Explain why a translation is an isometry.

Transformations

Reflections

1. a. Draw a simple polygon on a sheet of patty paper. Then draw a line on the patty paper outside the figure and place a point on the line.

Example



b. On a second sheet of patty paper, trace the polygon, the line of reflection and the point.

c. Flip the second piece of patty paper over and place it under the first so that the lines and the point coincide. Now trace the image onto the first patty paper.

d. Connect a point in the pre-image with its corresponding image point. Repeat this for two other points on the original polygon.

e. Compare the lengths of each segment connecting a point with its image. Are the segments all the same length?

f. Measure the angle formed by the line of reflection and each of the segments connecting a point and its image. What are the measures of these angles? How do the measures compare?

g. Fold the patty paper along the line of reflection. What can you say about the size of the image compared to the pre-image? Is a reflection an isometry?

h. The line of reflection divides each segment connecting a point and its image into two parts. Compare the lengths of the two parts of each segment.

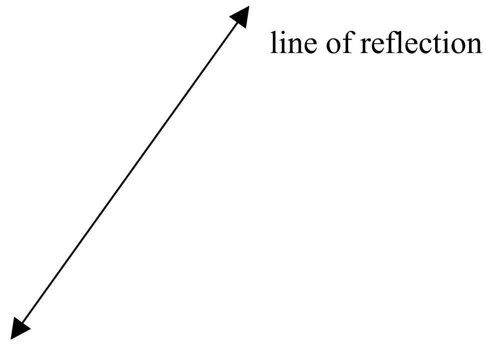
i. Complete the following statement:

The line of reflection is the ______________ _____________ of every segment connecting a point of the pre-image and its image.

Transformations

Reflections (Continued)

2. a. Draw a simple polygon on one side of the line below. Label the vertices, A, B, C, etc.

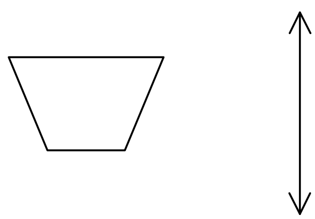


b. Place a Mira™ on the line of reflection and mark the vertices of the image. Then complete the image and label the corresponding vertices, A', B', C', etc.

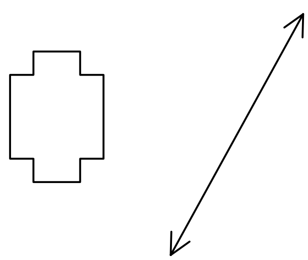
c. Write a statement comparing the size, position relative to the line of reflection, and orientation of the figure and its image.

3. Use your Mira™ to construct the image of each figure over the given line of reflection.

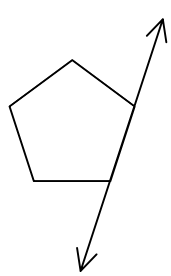
a.



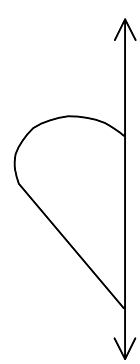
b.



c.



d.



Transformations

Rotations

1. a. On a piece of patty paper, draw a small polygon. Label vertices X, Y, etc. Make sure the figure is drawn toward the side of the patty paper.

   b. Mark a point P on the paper. Draw an acute angle APB with vertex at P. Determine the measure of $\angle APB$.

   c. On a second piece of patty paper, trace the polygon and point P. Also trace $\overrightarrow{PA}$.

   d. Stack the papers and align the polygon and point P. Holding point P aligned, turn the bottom paper until its $\overrightarrow{PA}$ is aligned with $\overrightarrow{PB}$ on the top paper. Trace the image onto the first piece of patty paper.

   e. Locate the vertex X of the original polygon and label its corresponding vertex on the image polygon as X'. Draw $\overline{XP}$ and $\overline{X'P}$. Measure the angle formed. Measure the length of each segment.

   f. Locate the vertex Y of the original polygon and label its corresponding vertex on the image polygon as Y'. Draw $\overline{YP}$ and $\overline{Y'P}$. Measure the angle formed. Measure the length of each segment.

   g. Measure XX' and YY'. Are the distances between points on the original polygon and their corresponding image points always the same?

   h. Correctly complete the following statement concerning rotations by circling the correct word in each pair of italicized words that makes a true statement about rotation.

Rotation:
When a figure is rotated around a point, the shape is *changed/not changed*, the orientation of the image in *changed/not changed* and the distance between points and their images are *the same/different* but the angle formed by a point and its image with the center of rotation is always *the same/different*.

TRANSFORMATIONS
Dilations Using Measurement

1.
   a) On a blank piece of paper place a point P in the upper right corner.

   b) Draw a scalene triangle ABC below and to the left of point P. Make sure the triangle is fairly ***small***.
      - Measure the length of each side.
        AB ________________ BC ______________ CA _____________
      - Measure each angle.
        ABC________________ BCA ____________ CAB _____________

   c) Draw three rays. The rays should have an endpoint at point P, go through each vertex of the triangle and extend to the edge of the paper.

   d) Measure the following.
      $\overline{PA}$ __________________ $\overline{PB}$ __________________ $\overline{PC}$ __________________

2. Create a second triangle by doing the following:
   a) Mark a point D on $\overrightarrow{PA}$ so that the distance from P to D is ***twice*** the distance from P to A.

   b) Mark a point E on $\overrightarrow{PB}$ so that the distance from P to E is ***twice*** the distance from P to B.

   c) Mark a point F on $\overrightarrow{PC}$ so that the distance from P to F is ***twice*** the distance from P to C.

   d) Connect the points D, E, and F to form triangle DEF.
      - Measure the length of each side.
        DE ________________ EF ______________ FD _____________
      - Measure each angle.
        DEF________________ EFD ____________ FDE _____________

   e) Compare triangle ABC and DEF. Write a statement about the two triangles.

TRANSFORMATIONS
Dilations Using Measurement

3.
a) On a second blank piece of paper place a point Q in the upper left corner.

b) Draw a scalene triangle ABC below and to the right of point Q. Make sure the triangle is fairly ***large***.
- Measure the length of each side.
  AB ________________ BC _______________ CA ______________
- Measure each angle.
  ABC________________ BCA ____________ CAB ______________

c) Draw three rays. The rays should have an endpoint at point Q, go through each vertex of the triangle and extend to the edge of the paper.

d) Measure the following.
$\overline{QA}$ __________________ $\overline{QB}$ __________________ $\overline{QC}$ ___________________

4. Create a second triangle by doing the following:
a) Mark a point X on $\overrightarrow{QA}$ so that the distance from Q to X is ***half*** the distance from Q to A.

b) Mark a point Y on $\overrightarrow{QB}$ so that the distance from Q to Y is ***half*** the distance from Q to B.

c) Mark a point Z on $\overrightarrow{QC}$ so that the distance from Q to Z is ***half*** the distance from Q to C.

d) Connect the points X, Y, and Z to form triangle XYZ.
- Measure the length of each side.
  XY ________________ YZ _______________ ZX ______________
- Measure each angle.
  XYZ________________ YZX ____________ ZXY ______________

e) Compare triangle ABC and XYZ. Write a statement about the two triangles.

5. In the above problems, you created a transformation called a **dilation.**
a) Does a dilation preserve lengths of sides and measures of angles?

b) Is a dilation an isometry? Use mathematics to justify your answer.

TRANSFORMATIONS
Dilations Using Measurement

6. In problems 1-2, point P was called the center of dilation and triangles DEF was the image of triangle ABC under a dilation.

a) Name the center of dilation in problems 3-4.

b) Name the image triangle under the dilation in problems 3-4.

7.

a) Place point T in the center of a third piece of paper.

b) Draw a large quadrilateral ABCD so that point T is in the interior of ABCD.

c) Draw four rays. The rays should have an endpoint at point T, go through each vertex of the quadrilateral and extend to the edge of the paper.

d) Measure the following.
$\overline{TA}$ _______________ $\overline{TB}$ _______________ $\overline{TC}$ _____________ $\overline{TD}$_____________

8.

a) Mark a point K on $\overrightarrow{TA}$ so that the distance from T to K is ***half*** the distance from T to A.

b) Mark a point L on $\overrightarrow{TB}$ so that the distance from T to L is ***half*** the distance from T to B.

c) Mark a point M on $\overrightarrow{TC}$ so that the distance from T to M is ***half*** the distance from T to C.

d) Mark a point N on $\overrightarrow{TD}$ so that the distance from T to N is ***half*** the distance from T to D.

e) Draw a new quadrilateral KLMN.

f) Are the two quadrilaterals similar? Use mathematics to justify your answer.

g) A scale factor for a dilation is the factor by which a figure is enlarged or reduced. What is the scale factor for the dilation that starts with quadrilateral ABCD and ends with quadrilateral KLMN?

h) What is the center of the dilation for this transformation?

Adapted by Tulare COE from: http://mdk12.org/instruction/curriculum/pdfs/clg2activity001.pdf

Task Model 2

Response Type: Matching Tables

DOK Level 2

8.G.2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Evidence Required:
2. The student describes sequences of rotations, reflections, translations, and dilations that can verify whether two-dimensional figures are similar or congruent to each other.

Tools: Calculator

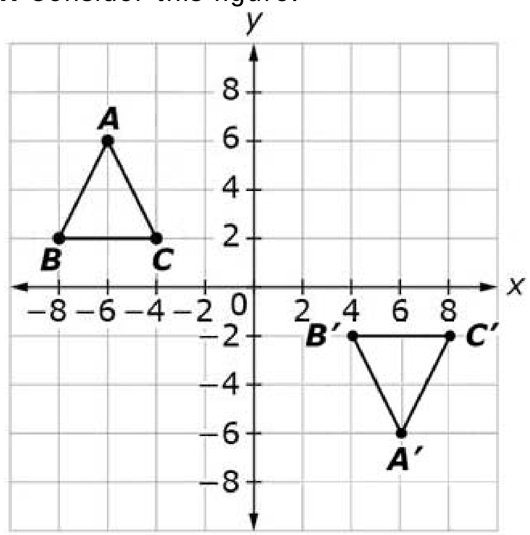
Prompt Features: The student is prompted to verify that two figures are similar or congruent by describing a sequence of rotations, reflections, translations, and dilations that exhibit the similarity or congruence between two given figures.

Stimulus Guidelines:

- A figure will contain no more than eight vertices.
- Item difficulty can be adjusted via these example methods:
    - o Varying the type and number of transformations
    - o Inclusion of dilations.

TM2
Stimulus: Transformations will include rotation, reflection, dilation, and/or translation.

Example Stem: Consider this figure.



Consider the statements in the table shown. Select True or False for each statement about the sequences of transformations that can verify that triangle *ABC* is congruent to triangle *A′B′C′*.

Statement	True	False
Triangle *ABC* is translated 12 units to the right, followed by a reflection across the *x*–axis.		
Triangle *ABC* is a reflected across the *y*–axis, followed by a translation 12 units down.		
Triangle *ABC* is reflected across the *x*–axis, followed by a translation 12 units to the right.		

Rubric: (1 point) The student selects True or False for the correct sequence of transformations for the figure (e.g., T, F, T).

Response Type: Matching Tables

Task Model 2

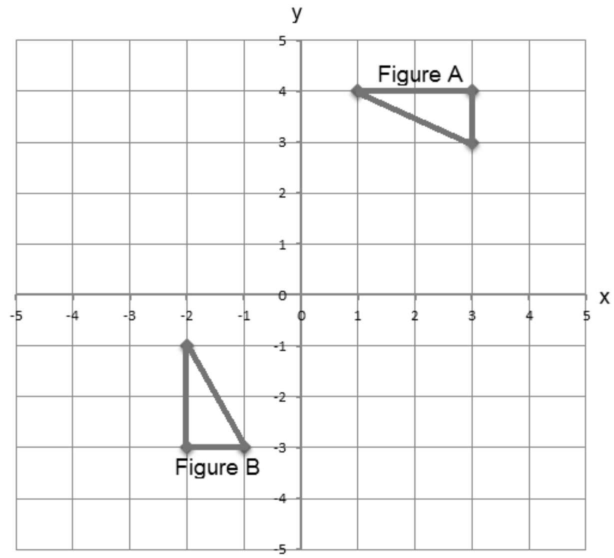
DOK Levels 3, 4

Target B:

Construct, autonomously, chains of reasoning that will justify or refute conjectures

Example Item 2 (Grade 8):
Primary Target 3B (Content Domain G), Secondary Target 1G (CCSS 8.G.2), Tertiary Target 3F

Two figures are shown on the coordinate grid.



Prove that Figure A and Figure B are congruent.

Describe three single transformations that, when performed, would transform Figure A to Figure B. In your response, be sure to identify the transformations in the order they are performed.

Rubric:
(2 points) The student describes three transformations with sufficient detail to prove that Figure A and Figure B are congruent (e.g., see exemplars).
(1 point) The student either describes all three transformations in general terms, without the degree of precision necessary to prove congruency (e.g., rotation, reflection, and translation) or correctly describes two out of three transformations and incorrectly describes the third (e.g., states the rotation is 180° instead of 90° or translates in the wrong direction or an incorrect number of units).
Exemplars[4]:
1st Transformation is to reflect over the *y*-axis. 2nd Transformation is to rotate 90° counter-clockwise about the origin. 3rd Transformation is to translate right by 2 units.
1st Transformation is to reflect over the *x*-axis. 2nd Transformation is to rotate 90° clockwise about the origin. 3rd Transformation is to translate right by 2 units.

Response Type: Short Text (hand scored)

[4] Exemplars only represent possible solutions. Typically, many other solutions/responses may receive full credit. The full range of acceptable responses is determined during rangefinding and/or scoring validation.

Task Model 7 **DOK Levels 2, 3** **Target G: Determine conditions under which an argument does and does not apply**	**Example Item 2 (Grade 8):** Primary Target 3G (Content Domain G), Secondary Target 1G (CCSS 8.G.3) A sequence of transformations is applied to a polygon. Identify each sequence of transformations where the resulting polygon has a greater area than the original polygon. A. Reflect over the *x*-axis, dilate about the origin by a scale factor of $\frac{1}{2}$, translate up 5 units B. Rotate 90° counterclockwise around the origin, dilate about the origin by a scale factor of $\frac{3}{2}$ C. Dilate about the origin by a scale factor of $\frac{2}{3}$, rotate 180° clockwise around the origin, translate down 2 units D. Dilate about the origin by a scale factor of 2, reflect over the *y*-axis, dilate about the origin by a scale factor of $\frac{2}{3}$ **Rubric:** (1 point) The student identifies all the correct conditions that make the argument true (e.g., B, D) **Response Type:** Multiple Choice, multiple correct response

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Task Model 2

DOK Levels 3, 4

Construct, autonomously, chains of reasoning that will justify or refute conjectures

Target B

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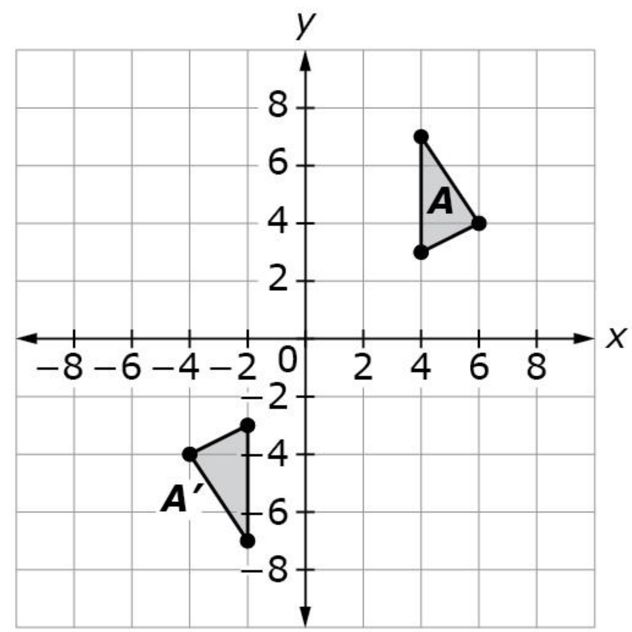
Task Expectations: The student is presented with a mathematical phenomenon and a conjecture. Student is asked to identify or construct reasoning that justifies or refutes a conjecture.

Example Item 1:

Primary Target 3B (Content Domain G-CO), Secondary Target 1X (CCSS G-CO.6), Tertiary Target 3C

Jose and Tina are writing a program for a computer game. They need to move Triangle *A* to Triangle *A′*.



To move Triangle *A* to Triangle *A′*,
Jose thinks:

- a sequence of three transformations must be performed, and
- there is only one possible way to do it.

Tina thinks:

- there are other sequences of transformations that will work, and
- it can be done using fewer than three transformations.

***Part A*:**
Describe a sequence of **three** transformations that maps Triangle *A* onto Triangle *A′* to support Jose's thinking.

***Part B*:**
If possible, support Tina's thinking by describing a sequence of **fewer than three** transformations that maps Triangle *A* onto Triangle *A′*.

Enter your response to Part A and Part B into the response box. Be sure to **number** the transformations in the order they should be performed (e.g., 1, 2, 3). **Label** each part of your response with "Part A" and "Part B."

Rubric: (2 points) The student is able to generate three correct translations to support Jose's thinking and one or two transformations to support Tina's thinking.
(1 point) The student is able to generate correct transformation(s) to support either Jose's thinking or Tina's, but not both.

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