

**Tulare County**  
**Office of Education**

*Jim Vidak, County Superintendent of Schools*

# **BUILDING MULTIPLICATION FACT FLUENCY**

**Presented by Julie Joseph  
Tulare County Office of Education  
November 7, 2015**

# GOALS

- Develop all students' fluency with multiplication and division facts through well orchestrated instructional strategies.
- Learn how students develop understanding and fluency through activities and games to use in your classroom throughout the year.

# AGENDA

- Research
- Games and Activities
- Assessment



# WHAT IS FLUENCY?

**IN WHAT WAYS DO YOU DEVELOP AND ASSESS  
STUDENT FLUENCY?**

## STANDARD 3.OA.7

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

# WHAT IS FLUENCY IN MATHEMATICS?

“According to CCSSM, fluency is “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSO 2010, p. 6). Thus, far from just being a measure of speed, fluency with multiplication facts involves flexibly and accurately using an appropriate strategy to find the answer efficiently.”

*“Three Steps to Mastering Multiplication Facts”, Gina Kling and Jennifer Bay-Williams, Teaching Children Mathematics, May 2015, Vol.21, issue 9,  
<http://www.nctm.org/Publications/Teaching-Children-Mathematics/2015/Vol21/Issue9/Three-Steps-to-Mastering-Multiplication-Facts/>*

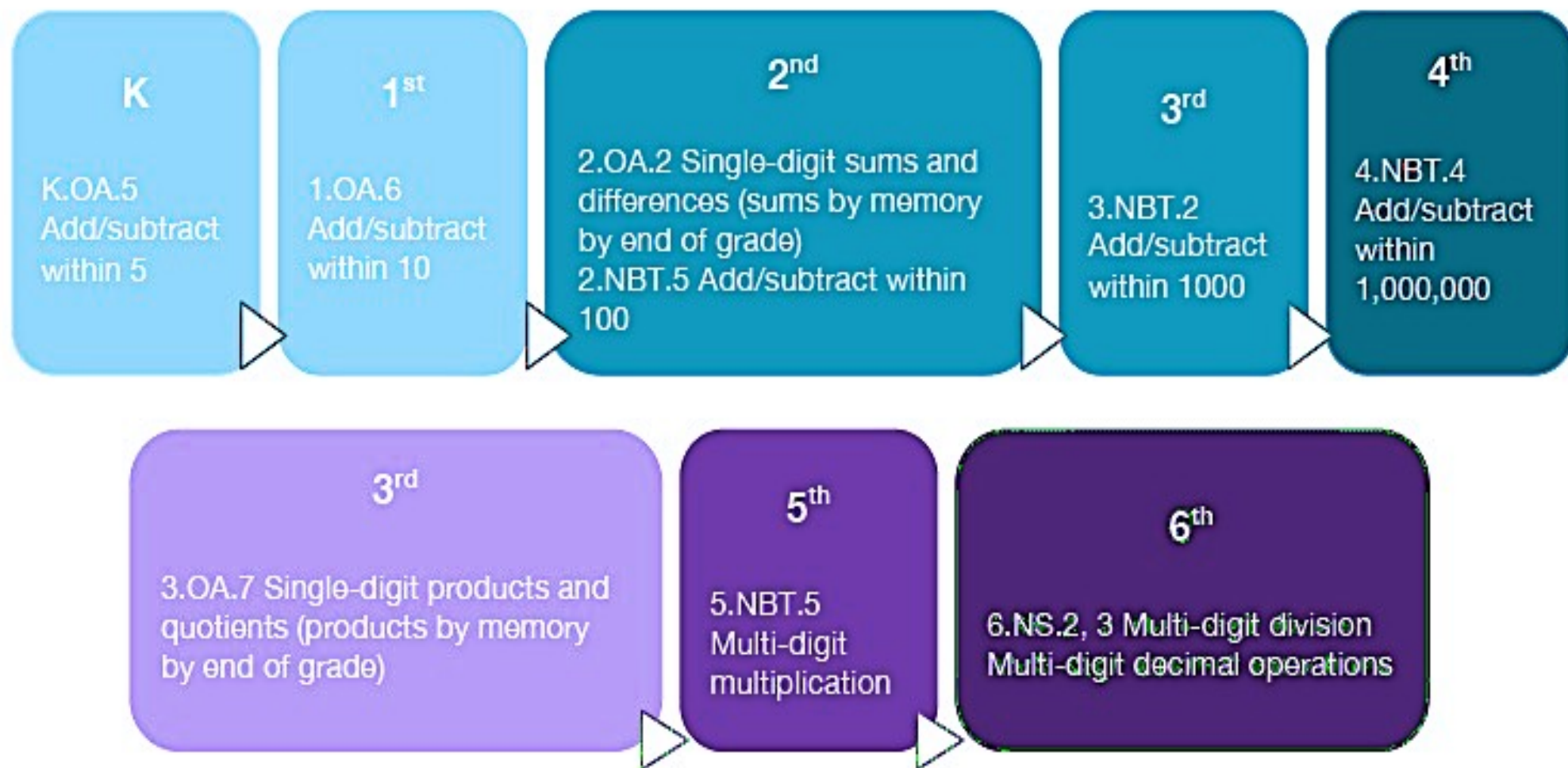
# Procedural Fluency

**Knowing  
from  
Memory**



**Memorization**

## CCSSM Standards for Fluency





# RESOURCES



Updated January 28, 2014

## Fluency Without Fear: Research Evidence on the Best Way to Learn Math Facts

By Jo Boaler  
Professor of Mathematics Education, co-founder youcubed  
with the help of Cathy Williams, co-founder youcubed, & Amanda Conner, co-founder youcubed, Stanford University.

### Introduction

A few years ago a British politician, Stephen Byers, made a harmless error in an interview. A notable minister was asked to give the answer to  $7 \times 8$  and he gave the answer of 54. His error prompted widespread ridicule in the national media, accompanied by a focus on 'times table' memorization in schools. This past September the Conservative Party for England, a man with no education experience, insisted that all students in England memorize times tables up to  $12 \times 12$  by the age of 9. This requirement has now been placed into the national curriculum and will result, I predict, in rising levels of math anxiety and students' performance in record numbers. The US is moving in the opposite direction, as the Common Core Standards (CCSS) de-emphasize the rote memorization of math facts. Unfortunately, the meaning of the word 'fluency' in the CCSS are commonplace and published as rote memorization, encouraging the persistence of damaging classroom practices.

Mathematics facts are important but the memorization of math facts through rote practice and timed testing is unnecessary and damaging. The English minister who asked  $7 \times 8$  prompted calls for more memorization. This was ironic as his minister was asked  $7 \times 8$  prompted calls for more memorization. People with number sense are those who can work out  $7 \times 8$  without having to rely on a distant memory. Math facts are best learned through the use of numbers in real-world contexts and they are best learned through the use of numbers in real-world contexts. Unfortunately many classrooms focus on math facts in unproductive ways. We remove them from classrooms, as they play a large role in the production of affected students.

It is useful to hold some math facts in memory. I don't stop and think about them. I know that math fact. But I learned math facts through using them in real-world contexts and being tested on them. I grew up in the product of many schools focused on the 'whole child' and I was not presented with multiplication facts to memorize in school. This has never held me back.

# THREE STEPS TO MASTERING BASIC FACTS

First understand what fluency is, then use these games and a sequence of strategies to help your students develop facility and confidence.

By Gina Kling and Jennifer M. Bay-Williams

## "That was the day I decided I was bad at math."

Countless times, we have heard preservice and in-service teachers make statements such as this after sharing vivid memories of learning multiplication facts. Timed tests; public competitive games, such as Around the World; and visible displays of who has and has not mastered groups of facts still resonate as experiences that led them to doubt their mathematical abilities. Others who appeared to be successful with such activities have shared such statements as these: "We learned a song for every fact. I can find any fact quickly, but I still need to sing the song first" and "I use the nines finger trick but have no idea how or why it works." Are these people truly fluent with their multiplication facts?

## Assessing Basic Fact Fluency

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.

By Gina Kling and Jennifer M. Bay-Williams

Think about how you assess reading fluency. Does your assessment plan involve listening and observing as children read as well as asking reading comprehension questions? Now imagine what you might learn about students' reading fluency if you used only timed quizzes. How would your confidence in your assessment change? Formative assessments—including observations, interviews, performance tasks, and journaling—have become common practice in many classrooms, with a recognition that by using different ways to assess children, we gain a more comprehensive, accurate picture of what they know, what they do not know, and their misconceptions. These data are then used to design instruction accordingly (William 2011). Yet, in spite of this trend in other areas of education, timed, skill-based assessments continue to be the prevalent measure of basic mathematics facts achievement. As a result, many rich opportunities for assessing basic fact fluency are lost. In this article, we share a variety of ways to formatively assess basic fact fluency. We define fluency, raise some issues related to timed testing, and then share a collection of classroom-tested ideas for authentic fact fluency assessment.

# Two Ways to Learn Math Facts

Strategies

Memorization

# RESEARCH

“...research evidence points in one direction:  
The best way to develop fluency with  
numbers is to develop number sense and to  
work with numbers in different ways, not to  
blindly memorize without number sense.”

– Boaler, Page 3

# RESEARCH FINDINGS

Study of students learning math facts in two ways - through *strategies* or *memorization*.

“Importantly the study...found that those who learned through strategies achieved ‘superior performance’ over those who memorized, they solved problems at the same speed, and showed better transfer to new problems.” (Delazer et al, 2005)

# RESEARCH FINDINGS

Data from 13 million 15-year olds on International PISA mathematics test.

“...the *lowest* achieving students are those who focus on memorization and who believe that memorizing is important when studying mathematics. The *highest* achievers in the world are those who focus on big ideas in mathematics and connections between ideas.



# DEVELOPING FLUENCY

**IN WHAT WAYS DO YOU DEVELOP  
STUDENT FLUENCY?**

# Number Talk

- If your friend was having trouble remembering this fact, what strategy would you suggest to him or her?
- $8 \times 7$

# PHASES OF BASIC FACT MASTERY

**Phase 1: Modeling and/or Counting**  
(Counts with objects mentally)

**Phase 2: Deriving answers using  
reasoning strategies based on known  
facts**

**Phase 3; Mastery**  
(efficient production of answers)

Adapted from Baroody, 2006

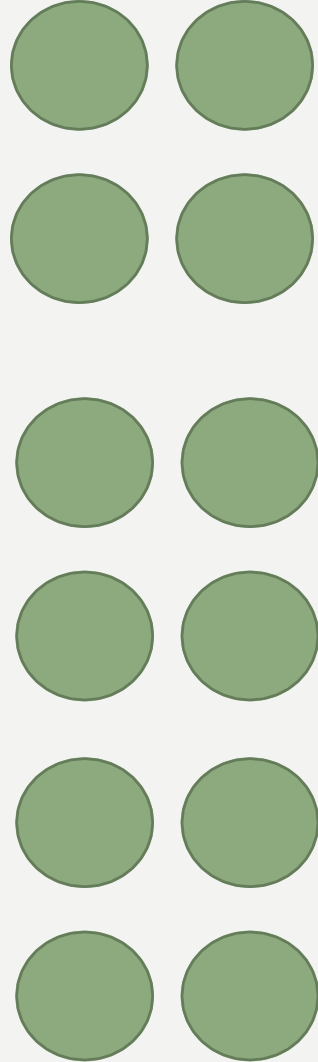
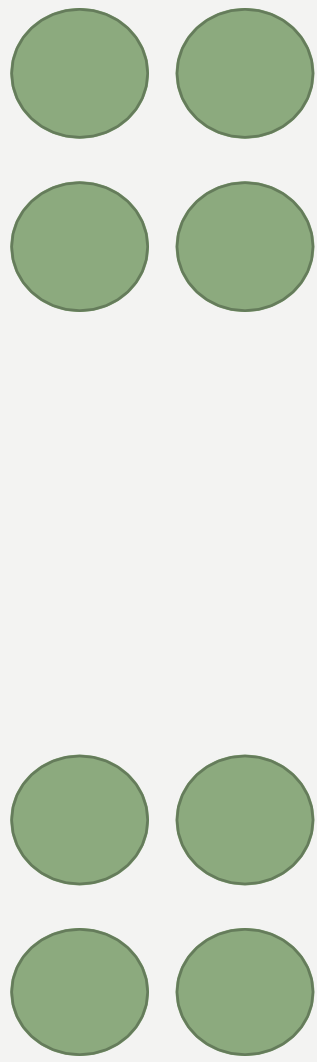
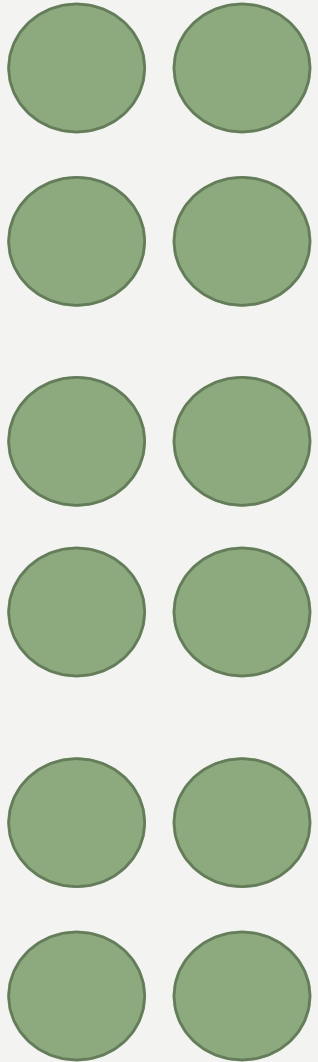


# NUMBER TALKS

- Intentional focus on specific strategies
- Develop connections between strategies
- Compare the effectiveness of strategies for given problems
- Opportunities to hear about and utilize new strategies

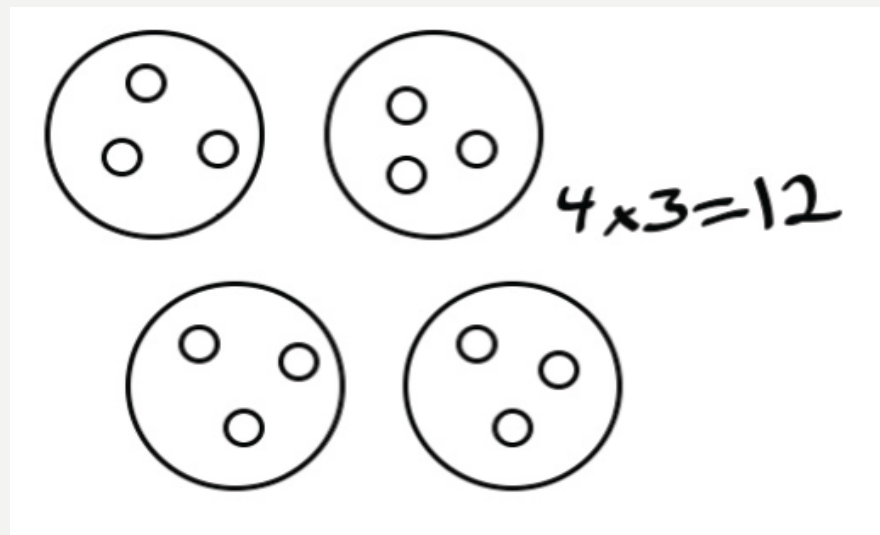
# QUICK IMAGE

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Phase I

# PEPPERONI PIZZA



**Directions:**

- I. Roll a dice twice and draw pizzas.
  - a. The first roll tells how many pizzas to draw.
  - b. The second roll tells how many pepperonis to put on EACH pizza.
  
- I. Write the number sentence that matches your picture.
  
- I. **How many pepperonis in all?**

# MASTERING BASIC FACTS

**Phase 1: Counting**  
(Counts with objects mentally)

**Phase 2: Deriving**  
(uses reasoning strategies based on  
known facts)

**Phase 3; Mastery**  
(efficient production of answers)

Adapted from Baroody, 2006

# DEVELOPING STRATEGIES

## Foundational Facts

- 2, 5, 10
- 0, 1 (use contexts)

## Derived Fact Strategies

- Adding or subtracting a group
- Halving and doubling
- Using a square product
- Decomposing a factor

# CONTEXT



- How many peeps?
- How does  $3 \times 16$  appear in this picture?
- How does  $4 \times 12$  appear in this picture?
- How does  $4 \times (3 \times 4)$  appear in this picture?



# VARIATIONS

## **Split a factor**

- As above, but on each roll the player is allowed to split one of their factors and fill in two arrays. For example, if  $5 \times 6$  would not fit on the board, they could split it into  $2 \times 6$  and  $3 \times 6$ . They would then outline these two arrays and two products, claiming both areas.

## **Change the numbers**

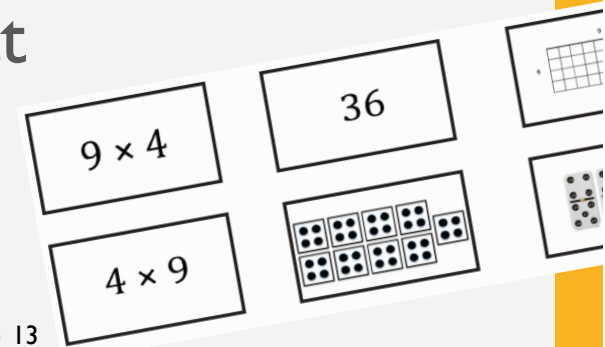
- Use cards instead of dice. Remove face cards.
- Practice one factor at a time and roll the other. For example, if the 6 times tables are being focused on, one factor is always 6.
- Use various sided dice (10 sided, 12 sided, 20 sided)



## Phase 2

# Math Cards

1. Lay all of the cards down on a table.
2. Have students take turns picking them. They can pick as many as they can find with the same answer (shown through any representation.)
3. Students explain how they know that the different cards are equivalent.



# MASTERING BASIC FACTS

**Phase 1: Counting**  
(Counts with objects mentally)

**Phase 2: Deriving**  
(uses reasoning strategies based on  
known facts)

**Phase 3; Mastery**  
(efficient production of answers)

Adapted from Baroody, 2006

## Phase 3

# TOP IT

- Place students in pairs and give each pair a deck of cards (omitting face cards and using aces as 1).
- Have each student take half of the deck.
- Both players turn over two cards and say the product of the two cards.
- Whoever has the larger product wins the cards.
- Play continues until time is called. Whoever has the most cards wins.
- *Differentiation: Use only specific numbers for the deck rather than using all factors 0-10.*



## Phase 3

# SALUTE!



- Place students in groups of 3, and give each group a deck of cards (omitting face cards and using aces = 1).
- Two students draw a card without looking at it and place it on their forehead facing outward (so others can see it).
- The student with no card tell the product. The other 2 players determine the value of their cards.
- Once both players have done so, they look at their cards and then students rotate roles.

## Phase 3

# CAROLINA CLIP-IT

GRADE 4 • NC DEPARTMENT OF PUBLIC INSTRUCTION | 5

### Carolina Clip-It

**Building Fluency:** multiplication facts

**Materials:** gameboard, 2 paper clips, game markers (approximately 15 of one color per player)

**Number of Players:** 2

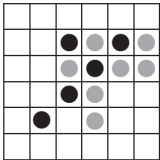
**Directions:**

1. Player one places paper clips on two numbers at the bottom of the page.
2. Then multiply the two numbers and place a marker on the correct product.
3. Player two can move only one of the paper clips at the bottom of the page.
4. Then multiply the two numbers and place a marker on the correct product.
5. Both paper clips may be placed on the same number.
6. Play continues until one player has 4 markers in a row, horizontally, vertically or diagonally.

**Variation/Extension:** Students share strategies of how they learned the more difficult multiplication facts.

1	7	15	25	36	54
2	8	16	27	40	56
3	9	18	28	42	63
4	10	20	30	45	64
5	12	21	32	48	72
6	14	24	35	49	81

1 2 3 4 5 6 7 8 9



### Directions:

1. Player one places paper clips on two numbers at the bottom of the page.
2. They multiply the two numbers and place a marker on the correct product.
3. Player two can move only one of the paper clips at the bottom of the page.
4. They multiply the two numbers and place a marker on the correct product.
5. Both paper clips may be placed on the same number.
6. Play continues until one player has 4 markers in a row, horizontally, vertically or diagonally.

## [HTTP://GREGTANGMATH.COM/GAMES.HTML](http://gregtangmath.com/games.html)

### Phase 2 Games

- *Breakapart*

### Phase 3 Games

- *Kakooma*
- *Missing*

The screenshot shows the GregTangMath website interface. At the top, there are navigation icons for Home, Kakooma, Games, and Books, along with the site logo 'GregTangMath'. Below the navigation is a social media bar with icons for Facebook, Twitter, Google+, and Pinterest. The main content area is divided into two sections: 'GAMES' and 'TEACHING GAMES'. The 'GAMES' section features three large game tiles: 'TEN FRAME MANIA', 'MATH LIMBO', and 'KAKOOMA'. The 'TEACHING GAMES' section features eight smaller game tiles: 'BREAKAPART', 'EXPRESSO', 'NUMSKILL', 'MISSING', 'FUNNY NUMBERS', 'PLACE VALUE', 'STD ALGORITHMS', and 'SAT FRACTION'. On the left side of the page, there is a sidebar with a navigation menu for 'All Games' listing 'Ten Frame Mania', 'Math Limbo', and 'Kakooma'. Below the menu is a large advertisement for 'metromile' insurance, featuring a cartoon character and the text 'PER-MILE INSURANCE, FOR PEOPLE WHO HARDLY DRIVE. Get a quote'.

# WHY USE GAMES?

Games:

- Are engaging.
- Provide opportunities for strategy discussion and assessment.
- Should be sequenced developmentally.
- Can be targeted practice or general practice.
- Lead to differentiation.



# ASSESSING FLUENCY

**WHAT CAN WE LEARN FROM THIS  
ASSESSMENT IN REGARDS TO STUDENT  
FLEXIBILITY, ACCURACY, EFFICIENCY, AND  
APPROPRIATE STRATEGY USE?**



My kids don't  
know their  
basic facts . . .



Instead ask:  
Which kids?  
And  
Which facts?

In order to know which kids and which facts,  
we need to assess and monitor.

# ASSESSING FLUENCY

- Observation
- Interviews
- Writing prompts
- Strategy quizzes
- Self-assessment

# OBSERVATION

Use questions such as the following to encourage good mathematical thinking during game play:

- How did you figure it out?
- Can you say out loud how you thought about it in your head?
- Is there another way you could figure it out?
- Can you think of another fact that strategy would work well for?
- If someone didn't know the answer to\_\_\_\_, how would you tell them to figure it out?
- What are you hoping for next?
- What are all the possibilities?



# INTERVIEWS

1. Write  $7 \times 8$  on a card. (point at card) What does  $7 \times 8$  mean?
2. What is your solution to  $7 \times 8$ ?
3. How did you find your solution? Can you find it another way?
4. If your friend was having trouble remembering this fact, what strategy might you suggest to him/her?

# STRATEGY QUIZZES

- Students solve problems and indicate how they solved them.
- Add the information to your observation log.
- Connect to student self assessment.

Kling, Gina and Jennifer M. Bay-Williams. 2014. Assessing Basic Fact Fluency. Teaching Children Mathematics, Volume 20, Number 8, 488-497.

FIGURE 3

Quizzes that focus on fluency are alternatives to timed tests.

(a) Quiz questions can be used to see if students “just know” foundational facts.

Solve these problems and tell how you solved them.

$4 \times 5 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$10 \times 6 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$6 \times 2 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$5 \times 3 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$2 \times 9 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$3 \times 10 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$5 \times 7 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____
$8 \times 10 =$ _____	Check one: _____	I used this strategy: _____
		I just knew. _____

(b) A quiz assesses if students recognize the commutativity of addition for one-more-than facts. Notice that these examples are shorter, not timed, and also focus on strategies.

On completion, say to class, “Circle the row that was easier for you to solve. If they were both the same, write ‘same’.”

Solve these addition problems.

		8			6
ROW A:	$9 + 1 =$	$\pm 1$	$5 + 1 =$	$3 + 1 =$	$\pm 1$
			1	1	
ROW B:	$1 + 8 =$	$1 + 7 =$	$\pm 4$	$\pm 2$	$1 + 9 =$

# FACT ASSESSMENTS

- Use Traditional Fact Assessments as strategy quizzes.
- For example,
  - Solve all the facts you know in your mind. Skip the others. 😊
  - Solve only the facts you need to work out.
  - Solve only the products that are greater than 36.

# DISCUSS

- How will you support and develop fluency in the classroom?
- What ideas or activities are you most excited to try with your students? Why?



# RESOURCES

- Developing and Assessing Fact Fluency, Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, NCTM 2015
- Bay-Williams, Jennifer M. and Gina Kling. 2014. Enriching Addition and Subtraction Fact Mastery Through Games. Teaching Children Mathematics, Volume 21, Number 4, 238-247.
- Boaler, Jo. 2015. Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts. <https://www.youcubed.org/fluency-without-fear/>
- Kling, Gina and Jennifer M. Bay-Williams. 2014. Assessing Basic Fact Fluency. Teaching Children Mathematics, Volume 20, Number 8, 488-497.
- Kling, Gina and Jennifer M. Bay-Williams. 2015. Three Steps to Mastering Multiplication Facts. Teaching Children Mathematics, Volume 21, Number 9, 548-559.

# CONTACT INFORMATION

- TCOE Common Core Connect, <http://ccss.tcoe.org/>
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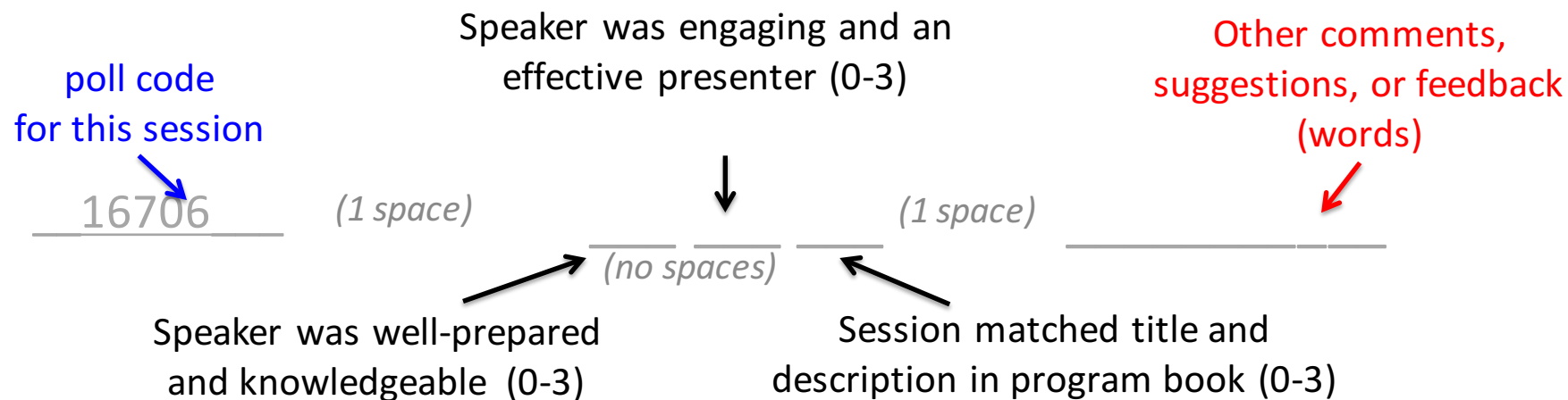
Strongly  
Disagree  
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Disagree  
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Agree  
2

Strongly  
Agree  
3

Send your text message to this Phone Number: 37607



Example: 16706 323 Inspiring, good content

Non-Example: 16706 3 2 3 Inspiring, good content

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