



Tulare County  
Office of Education

Jim Vidak, County Superintendent of Schools

## 59th CMC-South Annual Mathematics Conference

California Mathematics Council - Southern Section  
Palm Springs Convention Center • Renaissance • Hilton • Zoso  
Friday, November 2 and Saturday, November 3, 2018



*Let's Make Sense of Word Problems*

Kim Webb & Arcy Alafa  
November 2, 2018

# Sense-Making through Questioning



# The Power of Questioning

## Read norms 3 & 4

- Share one learning and/or wondering from each of the norms.

### 3. Questions are really important.

Tell your students that you love questions about math and that they are really important. Research shows us that question asking is linked to high achievement – yet as students move through school they ask fewer and fewer questions, for fear of being thought clueless. You don't need to be able to answer every question that students may come up with, sometimes it is good to say that you don't know but you will find out, or ask other students if someone would like to answer the question.

Some suggestions for encouraging questions:

1. When good questions are asked, write them in large colored letters onto posters that you post around the room, to celebrate them. Show questions from a range of students.
2. Tell students they have 2 responsibilities in your classroom. One is to always ask a question if they have one, and the other is to always answer a question from classmates if asked.
3. Encourage students to ask questions – from you, other students and themselves, such as: why does that work? why does that make sense? Can I draw that? How does that method connect to another?
4. Encourage students to ask their own math questions. Instead of asking questions for them, give them interesting mathematical situations and see what questions arise for them.

In studies, student question asking has been shown to steadily decline as students go through the grades in the US, showing this relationship:



### 4. Math is about creativity and making sense.

The key to understanding math is making sense of it. Many students believe that math is a set of formulas that have to be remembered - this belief is associated with low achievement. Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

PISA data from 15 million 15-year olds worldwide shows that the lowest achieving students in the world are those who believe that mathematical success comes from memorization. The USA and UK are countries where the highest numbers of students believe this.

Some methods for encouraging sense making and creative math:

1. Always ask students - why does that make sense? Ask this whether their answers are correct or incorrect.
2. Encourage visual mathematics. Ask students to draw their solutions. Ask them to think about how they see math. In this video (<http://youtu.be/1EoX-gs5Qg>) Cathy Humphreys asks students to make sense of 1 divided by 23 by drawing their solutions.
3. Show mathematical ideas through visual representations. All mathematics can be represented visually, and visual representations give many more students access to understanding. We have many examples of visual mathematics on youcubed and in the classroom video above.
4. Use number talks that value students' different ways of seeing math and solving problems. This video teaching number talks also shows visual solutions. <http://yocubed.org/teachers/2014/from-stand-ones-how-to-learn-math-for-teachers-and-parents-number-talks/>
5. When students finish questions, ask them to think of new, harder questions. These could be questions to give to other students. This is a really good strategy for differentiation.



# Powerful Questions

- How do you see that?
- Can you prove it visually?
- Why does that answer make sense?

Building a Mathematical Mindset Community	
<p><b>Teachers and students believe everyone can learn maths at HIGH LEVELS.</b></p> <ul style="list-style-type: none"> <li>Students are not tracked or grouped by achievement</li> <li>All students are offered high level work</li> <li>"I know you can do this" "I believe in you"</li> <li>Praise effort and ideas, not the person</li> <li>Students vocalize self-belief and confidence</li> </ul>	<p><b>Communication and connections are valued.</b></p> <ul style="list-style-type: none"> <li>Students work in groups sharing ideas and visuals</li> <li>Students relate ideas to previous lessons or topics</li> <li>Students connect their ideas to their peers' ideas, visuals, and representations.</li> <li>Teachers create opportunities for students to see connections.</li> <li>Students relate ideas to events in their lives and the world.</li> </ul>
<p><b>The maths is VISUAL.</b></p> <ul style="list-style-type: none"> <li>Teachers ask students to draw their ideas</li> <li>Tasks are posed with a visual component</li> <li>Students draw for each other when they explain</li> <li>Students gesture to illustrate their thinking</li> </ul>	<p><b>The maths is OPEN.</b></p> <ul style="list-style-type: none"> <li>Students are invited to see maths differently</li> <li>Students are encouraged to use and share different ideas, methods, and perspectives</li> <li>Creativity is valued and modeled.</li> <li>Students' work looks different from each other</li> <li>Students use ownership words - "my method", "my idea"</li> </ul>
<p><b>The environment is filled with WONDER and CURIOSITY.</b></p> <ul style="list-style-type: none"> <li>Students extend their work and investigate</li> <li>Teacher invites curiosity when posing tasks</li> <li>Students see maths as an unexplored puzzle</li> <li>Students freely ask and pose questions</li> <li>Students seek important information</li> <li>"I've never thought of it like that before."</li> </ul>	<p><b>The classroom is a risk-taking, MISTAKE VALUING environment</b></p> <ul style="list-style-type: none"> <li>Students share ideas even when they are wrong</li> <li>Peers seek to understand rather than correct</li> <li>Students feel comfortable when they are stuck or wrong</li> <li>Teachers and students work together when stuck</li> <li>Tasks are low floor/high ceiling</li> <li>Students disagree with each other and the teacher</li> </ul>

Developed by Jo Butler/Youcubed.org and Tulare County Office of Education

Recommendations for Task/Lesson Design	Powerful Questions to develop a deep level of understanding
Open the task to encourage multiple methods, pathways and representations.	How do you see that idea?
Pose a problem before teaching the method.	Why does that answer make sense?
Design a task that allows all learners to contribute to the learning and have room for extension.	Why does that method work?
Make opportunities for students to authentically share their thinking with peers.	How is that method connected to others?
Add a visual component.	How can that idea be represented in different ways?
Add the requirement to convince and reason, be skeptical.	Can you prove it?
	Can you prove it visually?
	Can you justify your thinking?
	Can you predict what would happen if....?
	Did you make any interesting mistakes?

Developed by Jo Butler/Youcubed.org and Tulare County Office of Education

# Saving Money

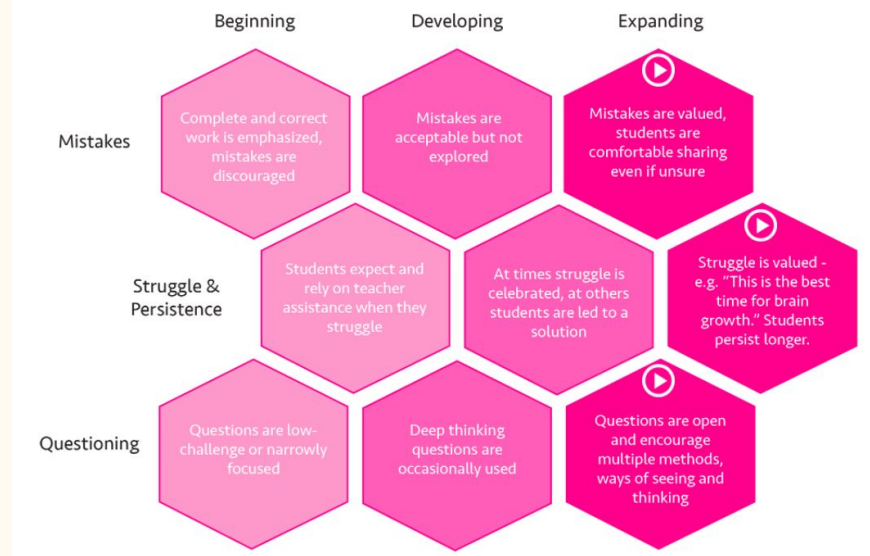
- Engage and complete task as a learner.
- Groups of three.
  - One group member is the facilitator using the [Powerful Questions](#) while the other two engage in completing the activity.

Louis wants to give \$15 to help kids who need school supplies. He also wants to buy a pair of shoes for \$39.

If Louis gets \$1 every day for his allowance, how many days will it take him to save enough money for both? Explain how you know.

# Reflect

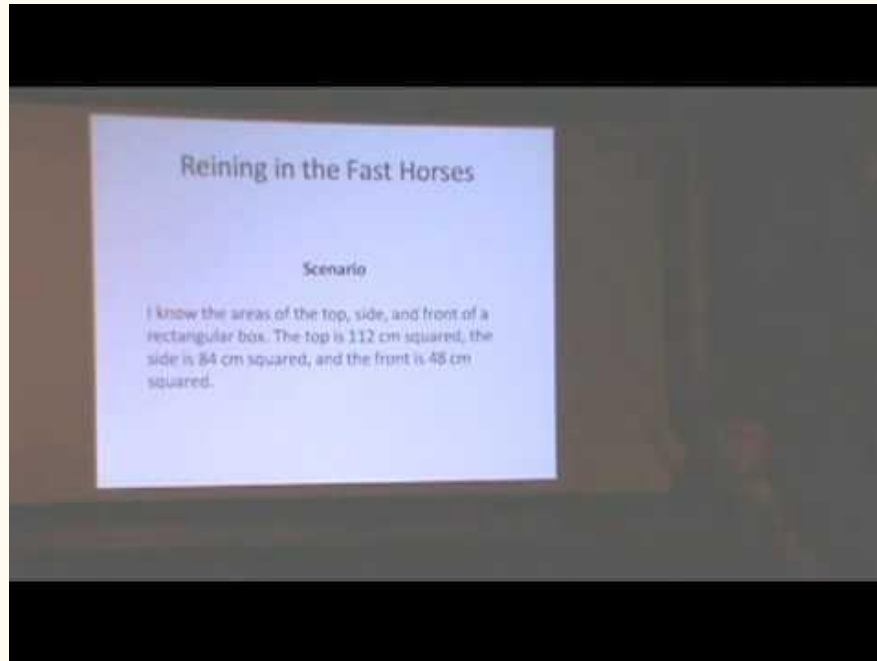
- Based on your group's experience, where would you identify yourself based upon the Mathematical Mindset Reflection Tool?
- What other wonderings do you have?



# Purposeful Questioning

It is important to recognize the power of purposeful questions while allowing students to have the freedom to creatively show their sense-making of a math process.

# The Power of Notice & Wonder





# Discuss

- In what ways did you connect with the video?
- What ideas resonated with you that you might want to try with your students?



# Why Notice and Wonder?

- Creates a safe environment
- Engages students and allows them to share without pressure to solve the problem
- Provides access into the task and allows students time to make sense of the task
- Serves as a strategy where teachers can learn about student thinking and make connections between ideas and the math students are learning

# Access for All Students

Notice and Wonder provides access for all students into the task and serves as a strategy for teachers that illuminates student thinking and provides opportunities to connect student ideas.

Intuition comes from **noticing**, **thinking** and **questioning**.

How do you make these part of everything you do?

# Notice and Wonder Resources

- [TCOE Common Core Connect](#)
- [Images in CA Math Frameworks](#)
- [Estimation 180](#)
- [Robert Kaplinsky's Tasks](#)
- [Math4Love - Free Lesson Library](#)
- [\*Eyes on Math: A Visual Approach to Teaching Math Concepts\*](#) by Marian Small
- Your curriculum, google images, your camera



# Sense-Making Strategies



1

**The florist has some roses. She is going to use some of the roses in each bouquet she makes.**

What are you picturing in your mind when you read this story?

What color roses are you picturing in your mind?

How many roses are you picturing in each bouquet?



<https://bstockus.wordpress.com/numberless-word-problems/>

1

**The florist has 24 roses. She is going to use some of the roses in each bouquet she makes.**

What changed? What did we learn from this new information?  
How many bouquets do you think she can make with 24 roses?  
What do we know about each bouquet?



1

**The florist has 24 roses. She is going to use 6 of the roses in each bouquet she makes.**

What changed? What did we learn from this new information?  
What question could we ask about this situation?

<https://bstockus.wordpress.com/numberless-word-problems/>

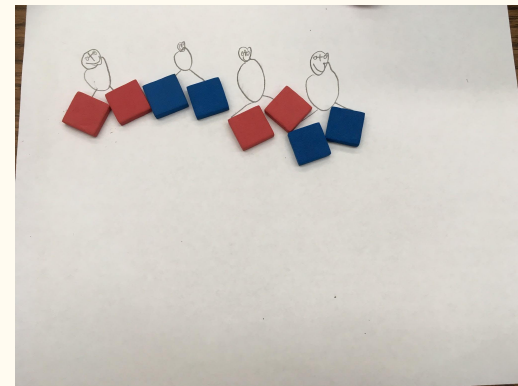
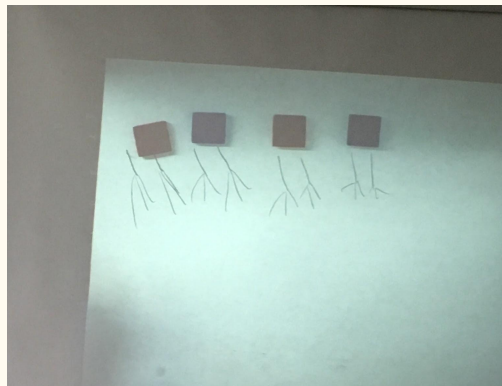
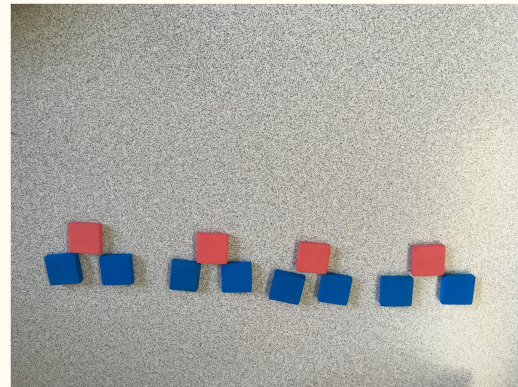
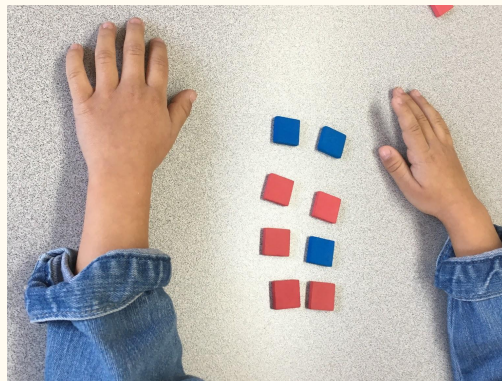
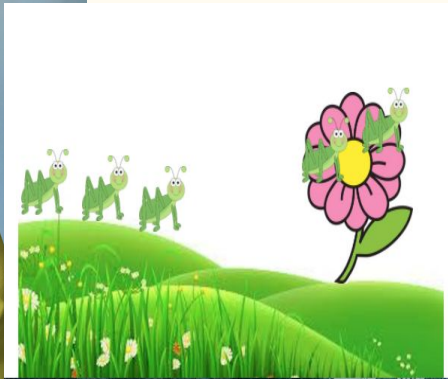
1

**The florist has 24 roses. She is going to use 6 of the roses in each bouquet she makes. How many bouquets can she make using all of the roses?**

Do you think she can make a lot of bouquets or just a few? Why do you say that? How can you use objects, pictures, or numbers to answer this question?

# Kinder Numberless WPs

My math story is about a sad grasshopper...



Numberless problems allow students the opportunity to access the problem situation:

- use their **reading strategies** as they notice and wonder about the language;
- use their **visual processing** when they notice and wonder as they view any charts, diagrams, graphs, and/or equations;
- use their **own wonderings and curiosities** as they begin to **think mathematically** to understand the relationships in the problem before any specific quantity is considered.

# Flipping the I do, We do, You do Approach

Mike  
**FLYNN**

Exploring  
**MATHEMATICAL  
PRACTICES** with  
Young Children

**BEYOND  
ANSWERS**

**Stenhouse**  
PUBLISHER

*Professional Resources by Teachers for Teachers*

[www.stenhouse.com](http://www.stenhouse.com)

**Mike Flynn**

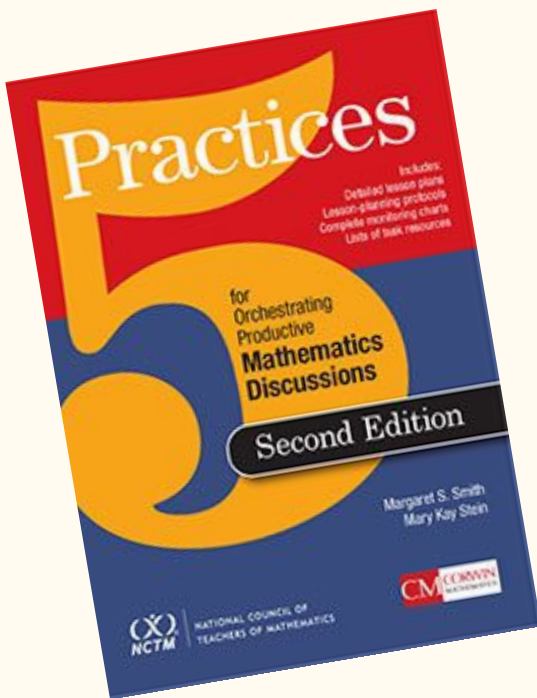
@MikeFlynn55  
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POWERED BY  
**Mount Holyoke**

02:15 / 00:11 by Deborah Schifter  
@MikeFlynn55  
mathleadership.org  
mflynn@mtholyoke.edu

<https://vimeo.com/289651220>

# 5 Practices



## 5 Practices for Orchestrating Productive Math Discussions

Margaret S. Smith & Mary Kay Stein, NCTM & Corwin Press, 2011 [www.nctm.org](http://www.nctm.org)

### 1. Anticipating

- Do the problem yourself
- What are students likely to produce?
- Which problems will most likely be the most useful in addressing the mathematics?

### 2. Monitoring

- Listen, observe, identify key strategies
- Keep track of approaches
- Ask questions of students to get them back on track or to think more deeply

### 3. Selecting

- **CRUCIAL STEP** – what do you want to highlight?
- Purposefully select those that will advance mathematical ideas

### 4. Sequencing

- In what order do you want to present the student work samples?
- Do you want the most common? Present misconceptions first?
- How will students share their work? Draw on board? Put under doc cam?

### 5. Connecting

- Craft questions to make the mathematics visible.
- Compare and contrast 2 or 3 students' work – what are the mathematical relationships?
- What do parts of student's work represent in the original problem? The solution? Work done in the past?

# Notice and Wonder



# What Did You Hear?

## Grade 4 - Claim 2

Greg has 76 marbles. He gives an equal number of marbles to 9 people. He keeps the remaining marbles. How many marbles does Greg keep?



# Anticipated Work

1

XXXX  
left over  
to keep

9 groups  
1 at a time

2

45 + 18 + 9 = 72

76 - 72 = 4

3

10 10 10 10 10 10 10 10 10 = 90 not enough

5 5 5 5 5 5 5 5 5 = 45  
2 2 2 2 2 2 2 2 2 = 18  
1 1 1 1 1 1 1 1 1 = 9

45 + 18 + 9 = 72

76 - 72 = 4

Greg keeps 4 marbles

4

because this can be  $\begin{array}{r} 9 \\ \overline{) 76} \\ \underline{45} \\ 31 \end{array}$

$45 \div 9 = 5$

$\begin{array}{r} 9 \\ \overline{) 31} \\ \underline{27} \\ 4 \end{array}$  — left over for Greg

$27 \div 9 = 3$

#5

$9 \times ? = 76$

$9 \times 8 = 72$

$76 - 72 = 4$  for Greg

# Read and Flip

## Grade 3 - Claim 4

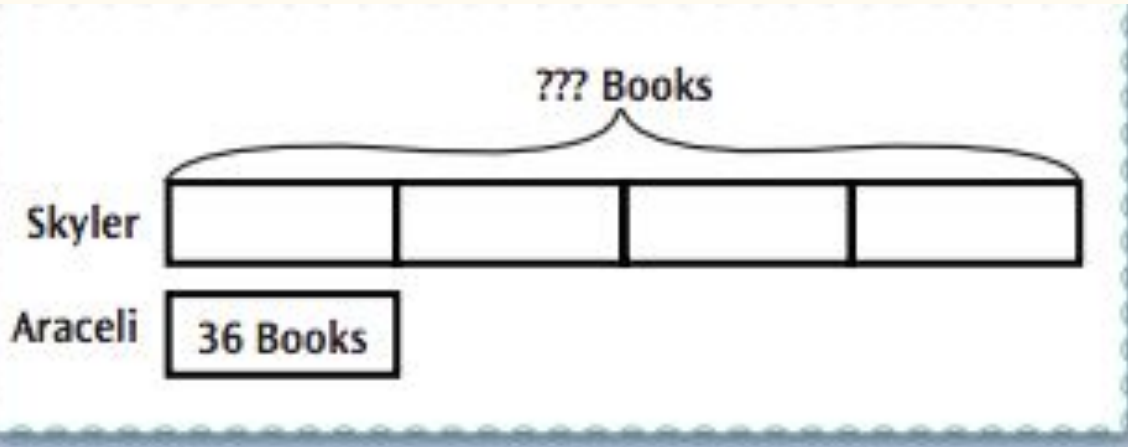
Jenny went to the store to buy 15 bottles of water.

- The bags at the store can each hold 6 kilograms.
- The bottles of water each weigh 2 kilograms.
- Jenny bought 15 bottles of water.

What is the fewest number of bags that Jenny needs to hold all 15 water bottles?

# Tape Diagrams

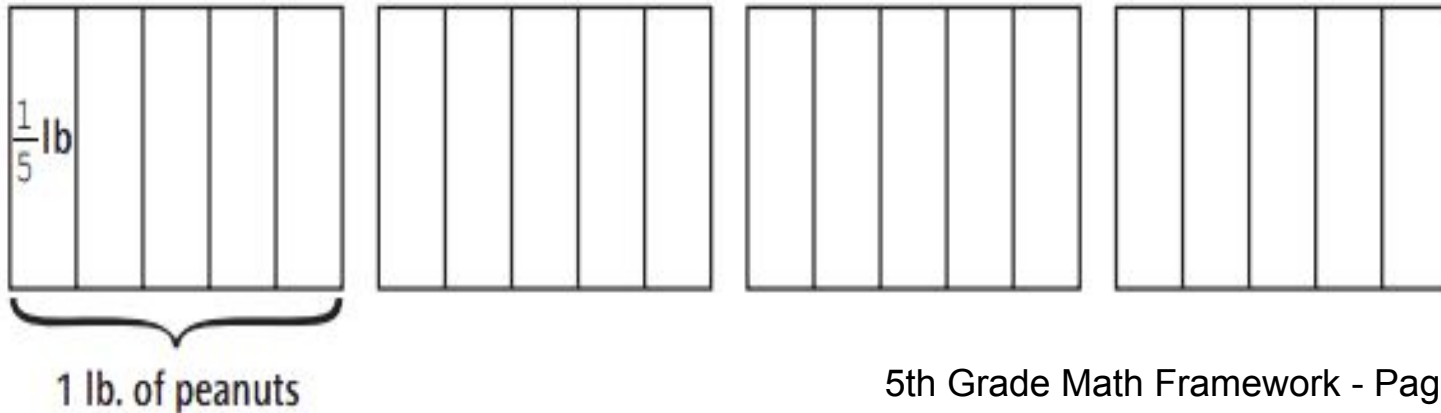
“Skyler has 4 times as many books as Araceli. If Araceli has 36 books, how many books does Skyler have?”



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# Tape Diagrams

Angelo has 4 pounds of peanuts. He wants to give each of his friends of a pound. How many friends can receive of a pound of peanuts?



# Reflections

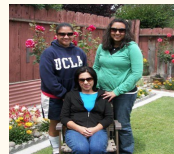


What did you find valuable/useful?

Write or tweet **@CAMathCouncil**  
and hashtag **#cmcmath**.



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# Feedback - Resources

We value your feedback!

Resources:

[https://drive.google.com/drive/folders/16\\_rSCOa5HxJ-NbUPcz6EGtd0ggoM8LPO?usp=sharing](https://drive.google.com/drive/folders/16_rSCOa5HxJ-NbUPcz6EGtd0ggoM8LPO?usp=sharing)

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