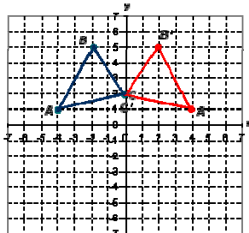Tulare County
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Hands-On Strategies for Transformational Geometry

Facilitated by Julie Joseph

CMC - South

October 25, 2014

Supporting California's
Common Core State Standards

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Goals

- Build understanding of **congruence** and **similarity** through the use of **translations**, **reflections**, **rotations**, and **dilations**.

- Utilize **hands-on** tools and the **mathematical practices** to develop understanding and solve problems.

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Timed-Round Robin

What do you know about Transformational Geometry?

What do you hope to learn?

What is different about CCSS Geometry?

- Challenges include attention to precision and language about transformations.

- Grade 8 students get hands on experience with rotations, reflections, translations, and dilations, observing their effects on two-dimensional figures on the coordinate plane.[8.G.3] This prepares for work in high school on formulating precise definitions of these transformations.

- Congruence and Similarity is approached through transformations.

Gearing Up for the Common Core Standards in Mathematics,
ime.math.arizona.edu/2010-11/0401_workshop.html

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Grade 8 and High School Standards

Standards

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Standards

Grade 8 – 8.G.1	High School – G-CO.2
Understand congruence and similarity using physical models, transparencies, or geometry software.	**Experiment with transformations in the plane**
1. Verify experimentally the properties of rotations, reflections, and translations:	G-CO.2.Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
a. Lines are taken to lines, and line segments to line segments of the same length.	
b. Angles are taken to angles of the same measure.	
c. Parallel lines are taken to parallel lines.	

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Transformations

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- Use Patty Paper and Miras to explore the following Transformations:
  ◦ Translations
  ◦ Reflections
  ◦ Rotations

- Be prepared to share your strategies and your observations.

Pages 4-6

http://mdk12.org/instruction/curriculum/pdfs/clg2activity001.pdf

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Standards for Mathematical Practice

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- What standards for mathematical practice did you use in this task?

- How would you need to structure it so that students engage in the standards for mathematical practice?

Page 1

Standards for Mathematical Practice

Mathematically proficient students:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

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Standards - Congruence

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Grade 8	High School
Understand congruence and similarity using physical models, transparencies, or geometry software.	**Experiment with transformations in the plane.**
8.G.2 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	G-CO.5.Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
8.G.3 - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	**Understand congruence in terms of rigid motions.** [Build on rigid motions as a familiar starting point for development of concept of geometric proof.]
	G-CO.6.Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

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Dilations and Similarity

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Grade 8	High School – Math 2
Understand congruence and similarity using physical models, transparencies, or geometry software.	**Understand similarity in terms of similarity transformations.**
8.G.3 - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	G-SRT.1.Verify experimentally the properties of dilations given by a center and a scale factor:
	a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
8.G.4 - Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
	G-SRT.2.Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

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Is it Dilated?

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- For each pair, explain why you think it is dilated or not?

http://njcore.org/sites/default/files/Lesson%20Plan%208.G.A.3.docx

Dilations Using Measurement

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- Use blank paper and patty paper (or a protractor) to complete the following:
  ◦ Person A: Problems 1 to 2.
  ◦ Person B: Problems 3 to 4.
  ◦ Person C: Problems 7 to 8.

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Discuss:

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Question 5

a. Does a dilation preserve lengths of sides and measures of angles?

b. Is a dilation an isometry? Use mathematics to justify your answer.

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SBAC Item Specifications

Grade 8

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Achievement Level Descriptors

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Level 1 Students should be able to identify reflections, rotations, and translations and show the result of these rigid motions on figures.

Level 2 Students should be able to construct reflections and translations of figures in a coordinate plane and identify dilations and the results of dilations on figures.

Level 3 Students should be able to understand and describe the impact of a transformation on a figure and its component parts with or without coordinates. They should be able to use or describe a sequence of transformations to determine or exhibit the congruence of two figures. They should also be able to construct rotations and dilations of figures in a coordinate plane.

Level 4 Students should be able to describe a sequence that exhibits the similarity between two shapes and understand that the angle measures are unchanged.

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Vocabulary

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- Angle
- Transformation
- Translation
- Translate
- Rotation
- Rotate
- Reflection
- Reflect
- Dilation
- Dilate
- Line segment
- Similar
- Congruent
- Parallel
- Transversal
- Exterior angle
- Interior angle
- Angle-angle criterion
- Scale factor
- Adjacent angle
- Supplementary angles
- Complementary angles

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Target-Specific Attributes

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Rotations are only multiples of 90 degrees about the origin.
Reflections are only over the *x*- and *y*-axes.
Dilations are only with the origin as the center.
There are at most three transformations in a sequence of transformations.
Transformations are limited to the coordinate plane.

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Slide 1

Sample Items

Pages 10-12

Example Stem: Consider this figure.

Try the Sample Items.

What are students expected to know and be able to do?

Consider the statements in the table shown. Select True or False for each statement about the sequences of transformations that can verify that triangle *ABC* is congruent to triangle *A'B'C'*.

Statement	True	False
Triangle *ABC* is translated 12 units to the right, followed by a reflection across the x-axis.		
Triangle *ABC* is a reflected across the y-axis, followed by a translation 12 units down.		
Triangle *ABC* is reflected across the x-axis, followed by a translation 12 units to the right.		

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Slide 2

High School
Claim 3 – Communicating Reasoning

Development Notes	• Items and task assessing Claim 3 may involve application of more than one standard. The focus is on communicating reasoning rather than demonstrating mathematical concepts or simple applications of mathematical procedures. • Targeted content standards for Claim 3 should belong to the major work of the grade (reference table of standards shown below). • Claim 1 Specifications that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 3. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 3. • Claim 3 items that require any degree of hand scoring must be written to primarily assess Target B.

N-RN.A	A-REI.C	G-CO.A
N-RN.B	A-REI.10	G-CO.B
N-RN.3	A-REI.11	G-CO.C
A-SSE.2	F-IF.1	G-CO.9
A-APR.1	F-IF.5	G-CO.10
A-APR.B	F-IF.9	G-CO.11
A-APR.4	F-BF.3	G.SRT.A
A-APR.6	F-BF.4a	G.SRT.B
A-REI.A	F-TF.1	
A-REI.1	F-TF.2	
A-REI.2	F-TF.B	

High School Claim 3

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Slide 3

Task Model 2	**Task Expectations:** The student is presented with a mathematical phenomenon and a conjecture. Student is asked to identify or construct reasoning that justifies or refutes a conjecture.
DOK Levels 3, 4	**Example Item 1:** Primary Target 3B (Content Domain G-CO), Secondary Target 1X (CCSS G-CO.6), Tertiary Target 3C
Construct, autonomously, chains of reasoning that will justify or refute conjectures	Jose and Tina are writing a program for a computer game. They need to move Triangle *A* to Triangle *A'*. To move Triangle *A* to Triangle *A'*, Jose thinks: • a sequence of three transformations must be performed, and • there is only one possible way to do it.
Target B	Tina thinks: • there are other sequences of transformations that will work, and • it can be done using fewer than three transformations.

Part A:
Describe a sequence of **three** transformations that maps Triangle *A* onto Triangle *A'* to support Jose's thinking.

Part B:
If possible, support Tina's thinking by describing a sequence of **fewer than three** transformations that maps Triangle *A* onto Triangle *A'*.

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Enter your response to Part A and Part B into the response box. Be sure to **number** the transformations in the order they should be performed (e.g., 1, 2, 3). **Label** each part of your response with "Part A" and "Part B."

Rubric: (2 points) The student is able to generate three correct translations to support Jose's thinking and one or two transformations to support Tina's thinking.
(1 point) The student is able to generate correct transformation(s) to support either Jose's thinking or Tina's, but not both.

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Resources

- Common Core Connect
  - http://commoncore.tcoe.org
    - Search by Standard
    - Search by "Transformational Geometry"

E-mail
- Julie Joseph
  - jjoseph@ers.tcoe.org

Strongly Disagree 0	Disagree 1	Agree 2	Strongly Agree 3

Send your text message to this Phone Number: 37607

5 digit poll code for this session

Speaker was engaging and an effective presenter (0-3)

Other comments, suggestions, or feedback (words)

_45059_ *(1 space)* ____ ____ ____ *(1 space)* _____

(no spaces)

Speaker was well-prepared and knowledgeable (0-3)

Session matched title and description in program book (0-3)

Example: **45059** **323** **Great session!**
Non-Example: 45059 3 2 3 Great session!
Non-Example: 450593-2-3Great session!