Tulare County
Office of Education
Jim Vidak, County Superintendent of Schools

Mapping Out Tasks to Engage Students in Productive Struggle

Grades 3 – 5

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What is productive struggle?

Initial Definition	Revised Definition

arise. Thinking about these in advance allows teachers to plan ways to support students productively without removing the opportunities for students to develop deeper understanding of the mathematics.

Mathematics classrooms that embrace productive struggle necessitate rethinking on the part of both students and teachers. Students must rethink what it means to be a successful learner of mathematics, and teachers must rethink what it means to be an effective teacher of mathematics. Figure 20 summarizes one such effort to redefine success in the mathematics classroom (Smith 2000), including expectations for students in regard to what it means to know and do mathematics, and actions for teachers with respect to what they can do to support students' learning, including acknowledging and using struggles as opportunities to learn.

Expectations for students	Teacher actions to support students	Classroom-based indicators of success
Most tasks that promote reasoning and problem solving take time to solve, and frustration may occur, but perseverance in the face of initial difficulty is important.	Use tasks that promote reasoning and problem solving; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students are engaged in the tasks and do not give up. The teacher supports students when they are "stuck" but does so in a way that keeps the thinking and reasoning at a high level.
Correct solutions are important, but so is being able to explain and discuss how one thought about and solved particular tasks.	Ask students to explain and justify how they solved a task. Value the quality of the explanation as much as the final solution.	Students explain how they solved a task and provide mathematical justifications for their reasoning.
Everyone has a responsibility and an obligation to make sense of mathematics by asking questions of peers and the teacher when he or she does not understand.	Give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.	Students question and critique the reasoning of their peers and reflect on their own understanding.
Diagrams, sketches, and hands-on materials are important tools to use in making sense of tasks.	Give students access to tools that will support their thinking processes.	Students are able to use tools to solve tasks that they cannot solve without them.
Communicating about one's thinking during a task makes it possible for others to help that person make progress on the task.	Ask students to explain their thinking and pose questions that are based on students' reasoning, rather than on the way that the teacher is thinking about the task.	Students explain their thinking about a task to their peers and the teacher. The teacher asks probing questions based on the students' thinking.

Fig. 20. Redefining student and teacher success. Adapted from Smith (2000, p. 382).

CARD SET A: CLUES

A – This number will make a rectangle 3 tiles wide.	**F – This number is a multiple of 5.**
B – This number has exactly 8 factors.	**G – This number has an odd number of factors.**
C – 1 factor of this number is 4.	**H – This number is a square number.**
D – This number is not a multiple of 5 or 7.	**I – This number is not even and is less than 50.**
E – Add the digits of this number and the sum is odd.	**J – The product of the digits of this number is greater than 20.**
Blank Clue Card	

CARD SET B: TASK CARDS

NUMBER PUZZLE 1: **Clue 1 – A** **Clue 3 – C** **Clue 2 – B** **Clue 4 – D**	**NUMBER PUZZLE 5:** **Clue 1 – E** **Clue 3 – G** **Clue 2 – F** **Clue 4 – H**
NUMBER PUZZLE 2: **Clue 1 – A** **Clue 3 – I** **Clue 2 – E** **Clue 4 – F**	**NUMBER PUZZLE 6:** **Clue 1 – E** **Clue 3 – J** **Clue 2 – I** **Clue 4 – D**
NUMBER PUZZLE 3: **Clue 1 – A** **Clue 3 – C** **Clue 2 – H** **Clue 4 – G**	**NUMBER PUZZLE 7:** **Clue 1 – A** **Clue 3 – B** **Clue 2 – E** **Clue 4 – F**
NUMBER PUZZLE 4: **Clue 1 – A** **Clue 3 – C** **Clue 2 – E** **Clue 4 – D**	**NUMBER PUZZLE 8:** **Clue 1 – D** **Clue 3 – E** **Clue 2 – I** **Clue 4 – Blank (write a fourth clue)**

CARD SET C: MYSTERY NUMBERS

36	45
24	30
72	47
100	29

What's the Secret Code?
Green Group

1. Use the clues to find the code number:
   - It is between 8,500 and 8,800.
   - When multiplied by 8, the result is a whole number.
   - The digit in the hundreds place is $\frac{3}{4}$ the digit in the thousands place.
   - The sum of all digits in the number is 26.
   - The digit in the hundredths place is 200% of the digit in the tenths place.
   - There are no zeros in the decimal places.

2. What code numbers fit these clues?

3. Explain how you used all of these clues to find these possibilities.

4. Write one more clue so that there is only one possible code number.

From *Math for All: Differentiating Instruction, Grades 6-8* by Linda Dacey & Karen Gartland (Sausalito, CA: Math Solutions), pp. 257. Reprinted with permission.

Structure for Supporting Student Understanding and Solving of a Task/Word Problem

1. Introduce the task to the students.
2. Read the problem – Have students read the task/word problem independently without marking the text and turn over the task/word problem when they are done reading.
3. Group Share – The teacher will ask all students to turn over the task and share something they remember about the task in partners or in small groups.
4. Read the problem – Have students reread the task/word problem independently and mark the text without beginning the problem.
5. Group Share – The teacher will ask all students to turn over the task and share something they remember about the task in partners or in small groups.
6. Independent Work Time – Ask the students to begin solving the problem independently. Tell them that they will have a group check in time after x number of minutes.
7. Group Check In – Ask students to take turns sharing something that they tried that worked or did not work and what they learned from it. They may not share a solution at this point.
8. Independent Work Time – Ask students to continue solving the problem independently. Tell them that they will have another group check in time after x number of minutes.
9. Group Check In – Ask students to share possible solutions and how they determined their possible solutions. They may work together collaboratively at this point to help all students in the group reach a solution.
10. Group Share Out – Have 1 person from each group list a possible solution on the board.
11. Teacher asks students to look at the possible solutions and share with their table groups to determine if they agree or disagree with the solutions posted.
12. Group Share Out – Have groups share out a solution that they agree or disagree with and why.
13. Class Conversation – The teacher facilitates a whole class conversation based on what he/she noticed during group discussions and group share outs.

Read and flip over structure adapted from Max Ray's NCSM 2014 Presentation: *Does That Make Sense in the Story? Front-loading Formative Assessment During Problem Solving*, http://mathforum.org/nctm/2014/ncsmpresent.html

Illustration

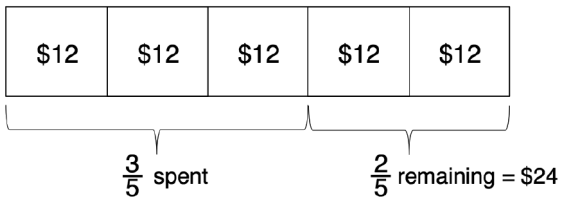
Figure 21 illustrates how two teachers, Ms. Flahive and Ms. Ramirez, present a real-world task related to fractions to two classes of fifth-grade students. In both classrooms, some students are immediately at a loss, upset, and vocal about their feeling that they don't know what to do. The two teachers respond to their students' discomfort in different ways.

Ms. Flahive and Ms. Ramirez teach fifth grade and plan their lessons collaboratively. Their current instructional unit focuses on fractions. They have selected the Shopping Trip task shown below because they think it will be accessible to their students yet provoke some struggle and challenge, since a solution pathway is not straightforward. The mathematics goal for students is to draw on and apply their understanding of how to build non-unit fractions from unit fractions and to use visual representations to solve a multi-step word problem:

> **Shopping Trip Task**
>
> Joseph went to the mall with his friends to spend the money that he had received for his birthday. When he got home, he had $24 remaining. He had spent $^3/_5$ of his birthday money at the mall on video games and food. How much money did he spend? How much money had he received for his birthday?

When Ms. Flahive and Ms. Ramirez present the problem in their classrooms, both teachers see students struggling to get started. Some students in both classrooms immediately raise their hands, saying, "I don't get it," or "I don't know what to do."

Ms. Flahive is very directive in her response to her students. She tells them to draw a rectangle and shows them how to divide it into fifths to represent what Joseph had spent and what he had left. She then guides her students step by step until they have labeled each one-fifth of the rectangle as worth $12, as shown below. Finally, she tells the students to use the information in the diagram to figure out the answers to the questions.



Ms. Ramirez approaches her students' struggles very differently. After she sees them struggling, she has them stop working on the problem and asks all the students to write down two things that they know about the problem and one thing that they wish they knew because it would help them make progress in solving the problem. Then Ms. Ramirez initiates a short class discussion in which several ideas are offered for what to do next. Suggestions include drawing a tape diagram or number line showing fifths, or just picking a number, such as $50 and proceeding through trial and error. Ms. Ramirez encourages the students to consider the various ideas that have been shared as they continue working on the task.

Fig. 21. Two teachers' responses to students' struggles to solve a multi-step word problem involving fractions

Support productive struggle in learning mathematics Teacher and student actions	
What are teachers doing?	What are students doing?
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.	Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.	Persevering in solving problems and realizing that it is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Helping one another without telling their classmates what the answer is or how to solve the problem.