High School Mathematics Item Specification Claim 3	
This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, and explanation, or counter-example. (Math Content Specifications, p.63) Communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate.	
Primary Claim 3: Communicating Reasoning: Students clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.	
Secondary Claim(s): Most items/tasks written to assess Claim 3 will involve some Claim 1 content targets and related Claim 1 targets should be listed below the Claim 3 targets in the item form[1]. If Claim 2 or Claim 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including Grade 11 within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the N and S domains in the high school grades can be used to construct higher difficulty items for the adaptive pool. The integration of the A, F, and G domains with N or S allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels Target(s)	2, 3, 4
Allowable Response Types	**Response Types:** Multiple-Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA); Fill-in Table (TI) No more than six choices in MS and MA items. Short Text - Performance tasks and Target B only **Scoring:** Scoring rules and answer choices will focus students' ability to use the appropriate reasoning. For some problems, multiple correct responses are possible. • MC will be scored as correct/incorrect (1 point)

[1] The full scope of Claim 3 items for high school often involves standards that are not specifically assessed in Claim 1. Be sure to include the standards and/or clusters in the metadata for all Claim 3 items in order to capture this information.

	• If MS and MA items require two skills, scored as: o All correct choices (2 points); at least ½ but less than all correct choices (1 point) o Justification[2] for more than 1 point **must be** clear in the scoring rules o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly • EQ, GI, and TI items will be scored as: o Single requirement items will be scored as correct/incorrect (1 point) o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) o Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to all or most students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 5 minutes to solve; Claim 3 items that are part of a performance task may take 3 to 10 minutes to solve.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[3] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context

[2] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[3] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[4]
Development Notes	• Items and task assessing Claim 3 may involve application of more than one standard. The focus is on communicating reasoning rather than demonstrating mathematical concepts or simple applications of mathematical procedures. • Targeted content standards for Claim 3 should belong to the major work of the grade (reference table of standards shown below). • Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 3. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 3. • Claim 3 items that require any degree of hand scoring must be written to primarily assess Target B. At least 80% of the items written to Claim 3 should primarily assess the standards and clusters listed in the table that follows.

N-RN.A N-RN.B N-RN.B.3 A-SSE.A.2 A-APR.A.1 A-APR.B A-APR.C.4 A-APR.D.6 A-REI.A A-REI.A.1 A-REI.A.2	A-REI.C A-REI.D.10 A-REI.D.11 F-IF.A.1 F-IF.B.5 F-IF.C.9 F-BF.B.3 F-BF.B.4 F-TF.A.1 F-TF.A.2 F-TF.C.8	G-CO.A G-CO.B G-CO.C G-CO.C.9 G-CO.C.10 G-CO.C.11 G.SRT.A G.SRT.B

[4] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Assessment Targets: Any given task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the task.
Target A: Test propositions or conjectures with specific examples. (DOK 2) Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item might begin, "Provide 3 examples to show why/how…"). **Target B: Construct, autonomously[5], chains of reasoning that will justify or refute propositions or conjectures[6]. (DOK 3, 4)** Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone. Some tasks for this target will ask students to formulate and justify a conjecture. **Target C: State logical assumptions being used. (DOK 2, 3)** Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)** Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both. **Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)** Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning. **Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)** In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents will often support generalizations as part of the justification rather than constituting the entire expected response.

[5] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

[6] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)

Tasks used to assess this target will ask students to determine whether a proposition or conjecture always applies, sometimes applies, or never applies and provide justification to support their conclusions. Targets A, B, C, and D will likely be included also in tasks that collect evidence for Target G.

High School standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in High School Claim 3 items:** **The Real Number System (RN)** **N-RN.A Extend the properties of exponents to rational exponents** **N-RN.B Use properties of rational and irrational numbers** **N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **Algebra (A)** **A-SSE Interpret the structure of expressions** **A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. **A-APR Arithmetic with Polynomials and Rational Expressions** **A-APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A-APR.B Understand the relationship between zeros and factors of polynomials** **A-APR.C Use Polynomial identities to solve problems** **A-APR.C.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
High School standards that lend themselves to communicating reasoning	**A-APR.D Rewrite rational expressions** **A-APR.D.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **A-REI Reasoning with Equations and Inequalities** **A-REI.A Understand solving equations as a process of reasoning and explain the reasoning** **A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A-REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how

High School standards that lend themselves to communicating reasoning	extraneous solutions may arise. **A-REI.C Solve systems of equations** **A-REI.D: Represent and solve equations and inequalities graphically** **A-REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A-REI.D.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ **F-IF Interpreting Functions** **F-IF.A Understand the concept of a function and use function notation** **F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. **F-IF.B Interpret functions that arise in applications in terms of the context** **F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **F-IF.C Analyze functions using different representations** **F-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **F-BF Building Functions** **F-BF.B Build new functions from existing functions** **F-BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-BF.B.4** Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

High School standards that lend themselves to communicating reasoning	**F-TF Trigonometric Functions** **F-TF.A Extend the domain of trigonometric functions using the unit circle** **F-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.C Prove and apply trigonometric identities** **F-TF.C.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$,or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. **G-CO Geometry Congruence** **G-CO.A Experiment with transformations in the plane** **G-CO.B Understand congruence in terms of rigid motions** **G-CO.C Prove geometric theorems** **G-CO.C.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G-CO.C.10** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **G-CO.C.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **G-SRT Similarity, Right Triangles, and Trigonometry** **G-SRT.A Understand similarity in terms of similarity transformations** **G-SRT.A.1** Verify experimentally the properties of dilations given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G-SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

	G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **G-SRT.B Prove theorems involving similarity** **G-SRT.B.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G-SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Target 3A: Test propositions or conjectures with specific examples.

General Task Model Expectations for Target 3A

- Items for this target should focus on the core mathematical work that students are doing around the real number system, algebra, functions, and geometry.
- In response to a claim or conjecture, the student should:
    - Find a counterexample if the claim is false,
    - Find examples and non-examples if the claim is sometimes true, or
    - Provide supporting examples for a claim that is always true without concluding that the examples establish that truth, unless there are only a finite number of cases and all of them are established one-by-one. The main role for using specific examples in this case is for students to develop a hypothesis that the conjecture or claim is true, setting students up for work described in Claim 3B.
- False or partially true claims that students are asked to find counterexamples for should draw upon frequently held mathematical misconceptions whenever possible.
- Note: Use appropriate mathematical language in asking students for a single example. While a single example can be used to refute a conjecture, it cannot be used to prove one is always true unless that is the one and only case.

Task Model 3A.1

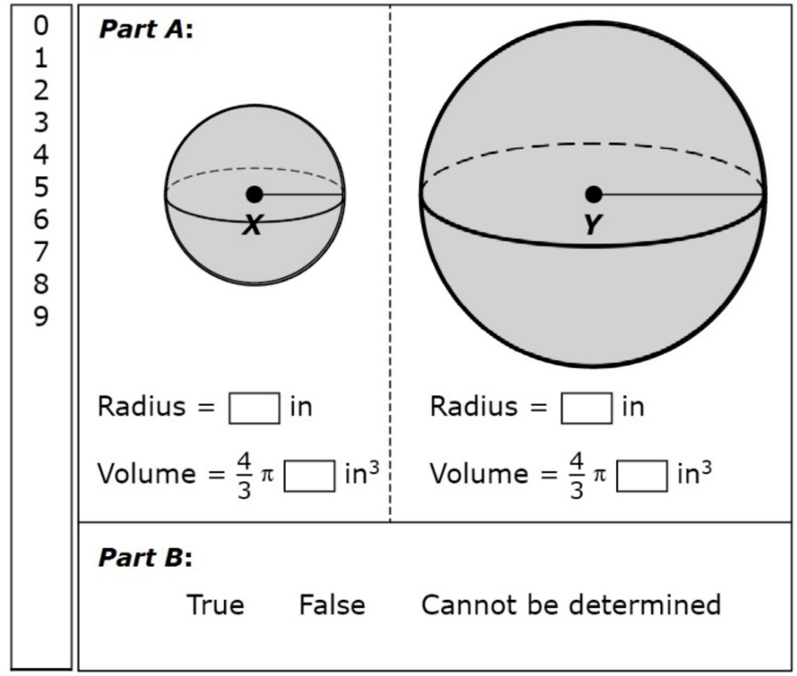
- The student is presented with a proposition or conjecture and asked to give
    - A counterexample if the claim is false,
    - Examples and non-examples if the claim is sometimes true, or
    - One or more supporting examples for a claim that is always true without concluding that the examples establish that truth unless there are only a finite number of cases and the set of examples addresses them all.

Example Item 3A.1a
Primary Target 3A (Content Domain G), Secondary Target 1I (CCSS 8.G.C), Tertiary Target 3F

The radius of sphere *Y* is twice the radius of sphere *X*. A student claims that the volume of sphere *Y* must be exactly twice the volume of sphere *X*.

Part A:
Drag numbers into the boxes to create one example to evaluate the student's claim.

Part B:
Decide whether the student's claim is True, False, or whether it Cannot be determined. Select the correct option.



Rubric: (1 point) The student supplies an example of two radii and missing numbers for the volumes that makes the conjecture false (e.g., radiuses are 1 and 2, missing numbers are 1 and 8; radii are 2 and 4, missing numbers are 8 and 64; radiuses are 1.5 and 3, missing numbers are 3.375 and 27; etc.) and responds with "False" in part B.

Response Type: Drag and Drop

Example Item 3A.1b
Primary Target 3A (Content Domain F-IF), Secondary Target 1K (CCSS F-IF.A)

The equation of a circle in the coordinate plane with center (0, 0) and radius 5 is shown:

$x^2 + y^2 = 25$

Fill in the table to show an example of two ordered pairs that show this equation does **not** define y as a function of x.

x	***y***

Rubric: (1 point) The student enters the coordinates of two points that have the same x-coordinates and different y-coordinates and that lie on the circle in the response box ($-5 \le x \le 5$ and $y = \pm\sqrt{25 - x^2}$).

Response Type: Fill-in Table

Example Item 3A.1c
Primary Target 3A (Content Domain N-RN), Secondary Target 1B (CCSS N-RN.3), Tertiary Target 3D

Consider the two numbers a and b as well as their sum and product.

Drag values for a and b into the boxes to make the paired statements true for both $a \cdot b$ and $a + b$.

If none of the values make both statements true, leave the boxes empty for that pair of statements.

Values
3
5
$\sqrt{3}$
$\sqrt{5}$
$3\sqrt{5}$
$5 - \sqrt{3}$

Statements	Example
$a \cdot b$ is an irrational number $a + b$ is a rational number	$a =$ ☐ $b =$ ☐
$a \cdot b$ is a rational number $a + b$ is an irrational number	$a =$ ☐ $b =$ ☐
$a \cdot b$ is an irrational number $a + b$ is an irrational number	$a =$ ☐ $b =$ ☐

Rubric: (1 point) The student drags correct combinations of values that result in the indicated rational and irrational perimeters and areas of the rectangle. (e.g., see one possible solution at right). Other correct responses are possible.

Response Type: Drag and Drop

Statements	Example
$a \cdot b$ is an irrational number $a + b$ is a rational number	$a = \sqrt{3}$ $b = 5 - \sqrt{3}$
$a \cdot b$ is a rational number $a + b$ is an irrational number	$a = \sqrt{5}$ $b = 3\sqrt{5}$
$a \cdot b$ is an irrational number $a + b$ is an irrational number	$a = \sqrt{5}$ $b = \sqrt{3}$

Example Item 3A.1d
Primary Target 3A (Content Domain G-CO), Secondary Target 1X (CCSS G-CO.9), Tertiary Target 3G

A geometry student made this claim:

If any two lines are cut by a transversal, then alternate interior angles are always congruent.

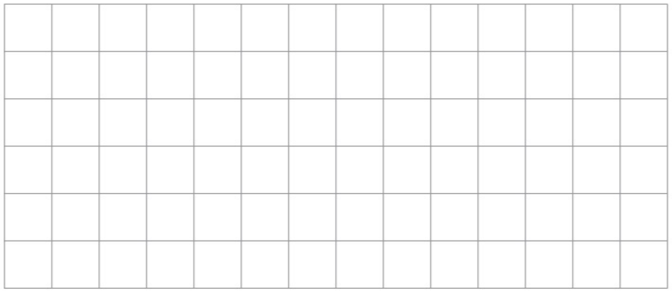
***Part A*:**
Draw a diagram that shows two lines cut by a transversal with alternate interior angles that **are** congruent **or** select **None** if there is not a situation that supports the student's claim.

***Part B*:**
Draw a diagram that shows two lines cut by a transversal with alternate interior angles that are **not** congruent **or** select **None** if the student's claim is always true.

***Part A*:**

None

***Part B*:**



None

Rubric: (1 point) The student draws a transversal through two parallel lines to create congruent alternate interior angles in Part A and through two non-parallel lines for Part B.

Response Type: Graphing and Hot Spot

Target 3B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

General Task Model Expectations for Target 3B

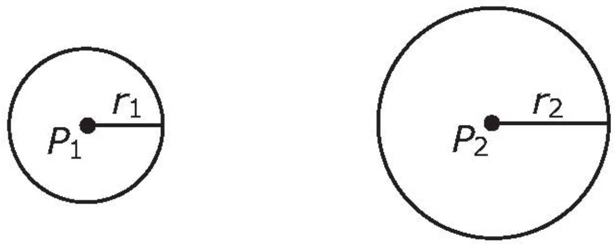
- Items for this target should focus on the core mathematical work that students are doing around the real number system, algebra, functions, and geometry.
- Items for this target can require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context. The difference between items for Claim 2A and Claim 3B is that the focus in 3B is on communicating the reasoning process in addition to getting the correct answer.
- Many machine-scorable items for these task models can be adapted to increase the autonomy of student's reasoning process but would require hand-scoring.

Task Model 3B.1

- The student is presented with a proposition or conjecture. The student is asked to identify or construct reasoning that justifies or refutes the proposition or conjecture.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.1a
Primary Target 3B (Content Domain G-C), Secondary Target 1X (CCSS G-C.A), Tertiary Target 3?

Proposition. All circles are similar.



The four statements below can be completed and ordered to outline an argument that proves this proposition.

1.	Translate Circle [drop-down choices: 1, 2] from [drop-down choices: P_1 , P_2] to [drop-down choices: P_1 , P_2] so that the two circles have the same center.
2.	Dilate Circle [drop-down choices: 1, 2] with center [drop-down choices: P_1 , P_2] by a factor of [drop-down choices: r_1, r_2 , r_1/r_2, r_2/r_1.
3.	The circles now coincide, showing they are similar.
4.	Given Circle 1 with center P_1 and radius r_1 and Circle 2 with center P_2 and radius r_2.

Rubric: (2 points) The student completes and orders the four statements that support the claim in a logical order (Examples below. Note that the order of 2 and 3 can be reversed but 1 must be first and 4 must be last. Also note that the ordering of 2 and 3 changes the correct center of dilation).
(1 point) The student completes and orders the four statements that support the claim in a logical order but either chooses some of the options from the drop-down menus incorrectly or chooses them in a sensible way if the order were correct, but the order is not correct.

Example 1

1. Given Circle 1 with center P_1 and radius r_1 and Circle 2 with center P_2 and radius r_2.
2. Translate Circle 2 from P_2 to P_1 so that the two circles have the same center.
3. Dilate Circle 2 with center P_1 by a factor of r_1/r_2.
4. The circles now coincide, showing they are similar.

Example 2

1. Given Circle 1 with center P_1 and radius r_1 and Circle 2 with center P_2 and radius r_2.
2. Dilate Circle 1 with center P_1 by a factor of r_2/r_1.
3. Translate Circle 1 from P_1 to P_2 so that the two circles have the same center.
4. The circles now coincide, showing they are similar.

Response Type: Drag and Drop, Drop-down Menu[7]

[7] Drop-Down Menu response type is not yet available in the Smarter Balanced item authoring tool, but it is a scheduled enhancement by 2017.

Example Item 3B.1b

Primary Target 3B (Content Domain A-APR), Secondary Target 1? (CCSS A-APR.A), Tertiary Target 3?

Kiera claimed that the sum of two linear polynomials with rational coefficients is always a linear polynomial with rational coefficients.

Drag the six statements into a logical sequence to outline an argument that proves this claim.

1.	$p(x) + q(x) = (ax+cx) + (b+d)$
2.	$p(x) + q(x) = (ax+b) + (cx+d)$
3.	Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are rational numbers.
4.	$(a+c)$ and $(b+d)$ are rational numbers
5.	So $p(x) + q(x)$ is a linear polynomial with rational coefficients.
6.	$p(x) + q(x) = (a+c)x + (b+d)$

Rubric: (2 points) The student completes and orders the four statements that support the claim in a logical order (Example below. Note that 1 must come first, 6 must come last, the order of 2, 3, and 4 must be preserved but 5 can go anywhere in between 1 and 6).
(1 point) The student completes and orders the four statements that support the claim in a logical order but either chooses some of the options from the drop-down menus incorrectly or chooses them in a sensible way if the order were correct, but the order is not correct.

<u>Example</u>

1. Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are rational numbers.
2. $p(x) + q(x) = (ax+b) + (cx+d)$
3. $p(x) + q(x) = (ax+cx) + (b+d)$
4. $p(x) + q(x) = (a+c)x + (b+d)$
5. $(a+c)$ and $(b+d)$ are rational numbers
6. So $p(x) + q(x)$ is a linear polynomial with rational coefficients.

Response Type: Drag and Drop

Example Item 3B.1c
Primary Target 3B (Content Domain F-TF), Secondary Target 1? (CCSS F-TF.C), Tertiary Target 3?

Suppose θ is an acute angle. Then $\sin^2(\theta) + \cos^2(\theta) = 1$.

Drag the six statements into a logical sequence to outline an argument that proves this claim.

1.	Construct a right triangle that includes θ as one of its acute angles.
2.	So $\sin^2(\theta) + \cos^2(\theta) = 1$.
3.	Give the label a to the side that is adjacent to θ, the label b to the side that is opposite θ, and the label c to the hypotenuse.
4.	$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$ if we divide both sides by c^2.
5.	$a^2 + b^2 = c^2$ by the Pythagorean Theorem.
6.	$\sin(\theta) = \frac{b}{c}$ and $\cos(\theta) = \frac{a}{c}$ by the definition of $\sin(\theta)$ and $\cos(\theta)$.

Rubric: (1 point) The student completes and orders the four statements that support the claim in a logical order (example below. Note that 1 and 2 must come first, 5 and 6 must come last, but the order of 3 and 4 can be switched).

<u>Example</u>

1. Construct a right triangle that includes θ as one of its acute angles.
2. Label the side adjacent to θ a, the side opposite θ b, and the hypotenuse c.
3. $\sin(\theta) = b/c$ and $\cos(\theta) = a/c$ by the definition of $\sin(\theta)$ and $\cos(\theta)$.
4. $a^2 + b^2 = c^2$ by the Pythagorean Theorem.
5. $a^2/c^2 + b^2/c^2 = 1$ if we divide both sides by c^2.
6. So $\sin^2(\theta) + \cos^2(\theta) = 1$.

Response Type: Drag and Drop

Task Model 3B.2

- The student is asked a mathematical question and is asked to identify or construct reasoning that justifies his or her answer.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.2a
Primary Target 3B (Content Domain A-APR), Secondary Target 1F (CCSS A-APR.B)

$P(x)$ is a 4th degree polynomial. The graph of $y = P(x)$ has exactly three distinct x-intercepts. Which polynomial could be $P(x)$?

A. $x^3(x-3)$

B. $x^2(x-2)(x-1)$

C. $(x-3)(x-2)(x-1)$

D. $x(x-3)(x-2)(x-1)$

For **one** of the polynomials above, explain why it could **not** be $P(x)$.

The graph of y = [drop-down choices: the four polynomials listed in the table] has exactly [drop-down choices: 0, 1, 2, 3, 4] distinct x-intercepts at

x = [0] [1] [2] [3] [4]

Click on all of the distinct x-intercepts

The degree of this polynomial is [drop-down choices: 0, 1, 2, 3, 4].

Rubric: (2 points) The student clicks on the correct polynomial (B) and selects one of the polynomials, identifies the correct number of distinct x-intercepts, correctly identifies the x-intercepts, and correctly identifies the degree of the polynomial. (See Example below. This is just one possibility.)
(1 point) The student clicks on the polynomials that could be $P(x)$ or fills out the information about one of the polynomials correctly.

Example:
The graph of $y = x^3(x-3)$ has exactly 2 distinct x-intercepts at

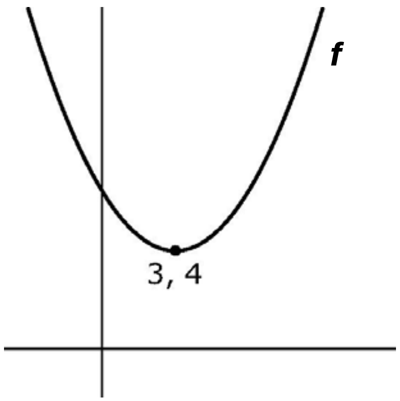
- x = [0] [1] [2] [3] [4]

The degree of this polynomial is 4.

Response Type: Multiple Choice, single correct response and Drop Down Menu

Example Item 3B.2b
Primary Target 3B (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.B), Tertiary Target 3E

The graph of a quadratic function f is shown and the vertex is labeled with its coordinates.
If $g(x) = f(x-1)+2$, what is the minimum value of g?



The minimum value of g is [1, 2, 3, 4, 5, 6] because the minimum value for f is [1, 2, 3, 4, 5, 6] and the graph of g is shifted [up, down, left, right] from the graph of f by [1, 2, 3, 4, 5, 6] units (in addition to the other shift).

Rubric: (1 points) The student selects the correct choices from the drop-down menus (6; 4; up; 2 or 6; 4; right; 1).

Response Type: Drop Down Menu

Note: The functionality for this item does not currently exist, but it could be implemented as a multiple choice or hotspot currently. Drop down choices are given in the brackets.

Example Item3B.2c
Primary Target 3B (Content Domain F-IF), Secondary Target 1L (CCSS F-IF.B), Tertiary Target 3F

A student examines two graphs representing the functions $f(x) = x + 5$ and $g(x) = x^2 + 5$. The student notices that the graphs both have a y-intercept at the point (0, 5). The student makes the following claim:

“For any real number c, the y-intercepts for the graphs of $y = c \bullet f(x)$ and $y = c \bullet g(x)$ are the same.”

Is this true or false?

If it is **true**, enter the y-coordinate of the y-intercept in terms of c. (0, [ ])

If it is **false**, enter the y-coordinates of the y-intercepts of the two graphs that are a counter-example. (0, [ ]); (0, [ ])

Rubric: (1 point) The student is able to identify the correct y-coordinate of the y-intercept and enter it into the first response box ($5c$).

Response Type: Equation/Numeric

Task Model 3B.3

- Items for this target require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context.
- The difference between Claim 2 task models and this task model is that the student needs to provide some evidence of his/her reasoning. The difference between Claim 4 task models and this task model is that the problem is completely well posed and no extraneous information is given.

Example Item 3B.3a
Primary Target 3B (Content Domain N-Q), Secondary Target 1C (CCSS N-Q.A), Tertiary Target 4F

- 1 hour = 60 minutes
- 1 kilometer = 100 meters
- 1 mile ≈ 1.6 kilometers

Calvin biked 24 miles in 2 hours. What is his approximate average speed in meters per minute?

Explain or show clear steps for how you determined your answer.

Rubric: (2 points) The student enters the correct numeric value in the response (32) and enters a coherent, complete explanation or sequence of computations that shows where this comes from (see Examples).
(1 point) The student enters the correct numeric value in the response but does not provide a coherent explanation OR the student enters a different number in the response but includes an explanation that shows an understanding of how the answer could be found, but with some computational errors or a small misstep in reasoning.

<u>Example 1</u>
24 miles in 2 hours is 12 mi/hr.
12 mi/hr * 60 min/hr = 1/5 mi/min.
1 km is 100 meters and 1 mile is 1.6 km, so 1 mile is 160 meters.
1/5 mi/min *160 m/mi = 32 meters per minute.

<u>Example 2</u>
Going 24 miles in 2 hours is the same as going 12 miles per hour.
There are 100 meters in a km and 1.6 km in a mile, so there are 160 m in a mile. There are 60 minutes in an hour.
1 mile per hour is 160 meters per 60 minutes which is 8/3 meters per minute, so
12 miles per hour is 12*8/3=32 meters per minute.

Response Type: Short Text (handscored)

Target 3C: State logical assumptions being used.

General Task Model Expectations for Target 3C

- Items for this target should focus on the core mathematical work that students are doing around the real number system, algebra, functions, and geometry.
- For some items, the student must explicitly identify assumptions that
    - Make a problem well-posed, or
    - Make a particular solution method viable.
- When possible, items in this target should focus on assumptions that are commonly made implicitly and can cause confusion when left implicit.
- For some items, the student will be given a definition and be asked to reason from that definition.

Task Model 3C.1

- The student is asked to identify an unstated assumption that would make the problem well-posed or allow them to solve a problem using a given method.

Example Item 3C.1

Primary Target 3C (Content Domain A-REI), Secondary Target 1H, Tertiary Target 3G

Beth is solving this equation: $\frac{1}{x} + 3 = \frac{3}{x}$.

She says "I can multiply both sides by x and get the linear equation $1 + 3x = 3$, whose solution is $x = \frac{2}{3}$."
Which of the following statements makes this a correct argument, or shows that it is incorrect? Select **all** that apply.

A. You can assume $x \neq 0$ because both sides are undefined if $x = 0$.
B. After multiplying both sides by x you need to subtract 1 from both sides.
C. You cannot multiply both sides by x because you do not know what x is.
D. The equation is not linear, so you cannot use the methods normally used for solving linear equations.

Rubric: (1 point) The students selects A, or A and B.

Response type: Multiple Choice, multiple correct response

Task Model 3C.2

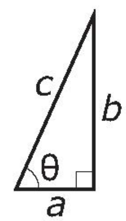
- The student will be given one or more definitions or assumptions and will be asked to reason from that set of definitions and assumptions.

Example Item 3C.2a

Primary Target 3C (Content Domain F-TF), Secondary Target 1O (CCSS G-SRT.C)

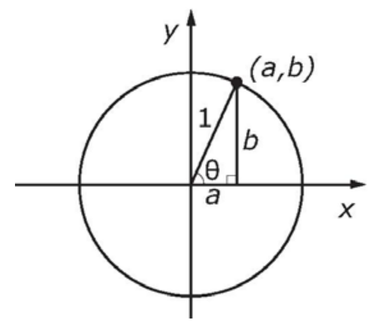
For an acute angle θ, $\sin(\theta)$ can be defined in terms of the side-lengths of a right triangle that includes angle θ. Here is the definition:

Given a right-triangle with side-lengths a and b and hypotenuse c, if θ is the angle opposite b, then $\sin(\theta) = \frac{b}{c}$.



Part A:

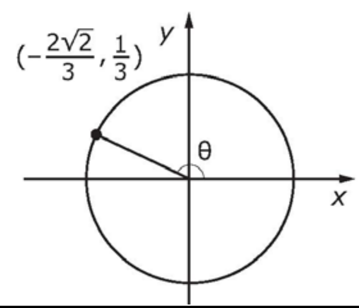
In the figure, angle θ has a vertex at the origin, its initial side corresponds to the positive x-axis, and the terminal side intersects the unit circle at the point (a, b).



What is $\sin(\theta)$ in terms of a and b according to the definition given?
Enter your answer in the first response box.

Part B:

For angles that are not acute, the definition of $\sin(\theta)$ is given in terms of the unit circle.



If angle θ has a vertex at the origin, its initial side corresponds to the positive x-axis, and the terminal side intersects the unit circle at the point (a, b), then $\sin(\theta) = b$.

In the figure shown, what is $\sin(\theta)$?
Enter your answer in the second response box.

Rubric: (1 point) The student enters the correct value for $\sin(\theta)$ according to the given definition (b; 1/3).

Response type: Equation/Numeric

Note: It would be best to implement this so students answer Part A before they go on to Part B, once this functionality is installed. They should still be able to go back and change their answer to Part A.

Example Item 3C.2b
Primary Target 3B (Content Domain A-APR), Secondary Target 1X (CCSS A-APR.A)

Proposition 1 The sum of two linear polynomials with integer coefficients is always a linear polynomial with integer coefficients.

The Closure of the Integers Under Addition The sum of two integers is always an integer

The Commutative Property If A and B are real numbers, then $A + B = B + A$

The Associative Property If A, B, and C are real numbers, then $(A + B) + C = A + (A + C)$

The Distributive Property If A, B, and C are real numbers, then $A(B + C) = AB + AC$

The Any-Order Property of Addition The sum of two or more real numbers can be performed in any order or any grouping.

The outline of a proof of Proposition 1 is shown. Add one or more justifications for steps 3, 4, and 5 of the proof.

Statement	Justification
1. Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are integers.	Hypothesis
2. $p(x) + q(x) = (ax+b) + (cx+d)$	By Definition
3. $p(x) + q(x) = (ax+cx) + (b+d)$	
4. $p(x) + q(x) = (a+c)x + (b+d)$	
5. $(a+c)$ and $(b+d)$ are integers	
6. So $p(x) + q(x)$ is a linear polynomial with integer coefficients.	Conclusion

Rubric: (1 point) The student provides a correct justification for each step of the proof (Example below. Note that for step (3), students can either cite the any-order property of addition or cite the commutative and associative properties together).

Example

Statement	Justification
1. Given $p(x) = ax+b$ and $q(x) = cx+d$ where a, b, c, and d are integers.	Hypothesis
2. $p(x) + q(x) = (ax+b) + (cx+d)$	By Definition
3. $p(x) + q(x) = (ax+cx) + (b+d)$	**The Any-Order Property of Addition**
4. $p(x) + q(x) = (a+c)x + (b+d)$	**The Distributive Property**
5. $(a+c)$ and $(b+d)$ are integers	**The Closure of the Integers Under Addition**
6. So $p(x) + q(x)$ is a linear polynomial with rational coefficients.	Conclusion

Response Type: Drag and Drop

Example Item 3C.2c
Primary Target 3C (Content Domain A-APR), Secondary Target 1F (CCSS A-APR.C), Tertiary Target 3B

An **algebraic identity** is an equation that is always true for any value of the variables.

For example, $\mathbf{2(x+y) = 2x+2y}$ is an identity because of the distributive property.

For each equation, select "Yes" if the equation is an algebraic identity. Select "No" if it is not an algebraic identity.

Equation	Yes	No
$(x+3)^2 = x^2 + 3^2$		
$y^{-1} = \frac{1}{y}$ if $y \neq 0$		
$a^2 + b^2 = c^2$		

Rubric: (1 point) The student correctly indicates which equation is an identity and which is not (NYN).

Response Type: Matching Table

Target 3D: Use the technique of breaking an argument into cases.

General Task Model Expectations for Target 3D

- Items for this target should focus on the core mathematical work that students are doing around the real number system, algebra, functions, and geometry.
- The student is given
  - A problem that has a finite number of possible solutions, some of which work and some of which don't, or
  - A proposition that is true in some cases but not others.
- Items for Claim 3 Target D should either present an exhaustive set of cases to consider or expect students to consider all possible cases in turn in order to distinguish it from items in other targets.

Task Model 3D.1

- The student is given a problem that has a finite number of possible solutions that need to be treated on a case-by-case basis.

Example Item 3D.1a

Primary Target 3D (Content Domain F-IF), Secondary Target 1M (CCSS F-IF.C).

Consider the piecewise-defined function given by

$$f(x) = \begin{cases} 2x - 7 & \text{for } x < -1 \\ x^2 + 3 & \text{for } -1 \leq x < 3 \\ x - 1 & \text{for } x \geq 3 \end{cases}$$

For how many real values of x does $f(x) = 0$?

A. 0
B. 1
C. 2
D. 3

Rubric: (1 point) The student selects the correct number of zeros for the function (A).

Response Type: Multiple Choice, single correct response

Commentary: This could be implemented with an equation/numeric response type. By varying the functions defining each piece, you can cover a wide variety of content from the Functions domain.

Example Item 3D.1b
Primary Target 3D (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A), Tertiary Target 3C

An arithmetic sequence is sequence in which the difference between any two consecutive terms is the same. For example, the sequence 2, 9, 16, 23, 30, 37 is an arithmetic sequence because the difference between any two consecutive numbers is 7.

Suppose that an arithmetic sequence of integers starts with a 5 and also later includes an 11.

5, …?…. , 11, ____

Which number could **not** be the term that immediately follows 11 in the sequence?

A. 12
B. 13
C. 14
D. 15
E. 17

Rubric: (1 point) The student selects the number that could not be the next term in an arithmetic sequence (D).

Response Type: Multiple Choice, single correct response

Task Model 3D.2

- The student is given a proposition and asked to determine in which cases the proposition is true.

Example Item 3D.2a
Primary Target 3D (Content Domain A-SSE), Secondary Target 1D (CCSS A-SSE.A)

Mrs. Beno wrote this equation on the board and asked, "Is this a true equation?"

$(x+y)^2 = x^2+y^2$

Nigel said, "Sometimes it is, and sometimes it isn't."

For each case shown, determine whether this equation is true or false.

Case	True	False
$x \neq 0, y \neq 0$		
$x = 0, y \neq 0$		
$x \neq 0, y = 0$		
$x = 0, y = 0$		

Rubric: (1 point) The student correctly indicates which conditions make the equation true and which make it false (FTTT).

Response Type: Matching Table

Example Item 3D.2b:
Primary Target 3D (Content Domain N-RN), Secondary Target 3G**,** Tertiary Target 1X (CCSS N-RN.1),

Consider this inequality: $\sqrt[3]{m} \leq m$

Part A:
Determine the positive values of *m* for which the inequality is **true**.
Enter your response as an inequality in the first response box.

Part B:
Determine the positive values of *m* for which the inequality is **false**.
Enter your response as an inequality in the second response box.

Rubric: (2 points) The student provides the values that make the given inequality true ($m \geq 1$) and false ($0 < m < 1$).
(1 point) The student errors in the use of the inequality signs (uses > instead of ≥ or ≤ instead of <) but otherwise has the correct range of values. OR The student is able to provide an example of where the statement is true (e.g., a positive value greater than or equal to 1) and a number between 0 and 1 that makes the statement false.

Response Type: Equation/Numeric

Example Item 3D.2c:
Primary Target 3D (Content Domain A-REI), Secondary Target 3G, Tertiary Target 1X (CCSS A-REI.A),

Determine whether each statement is
- true for all values of *x*,
- true for some values of *x*, or
- not true for any value of *x*

Statement	True for all	True for some	Not true for any
If $x^2 = 9$, then $\sqrt{9} = x$			
If $x^3 = y$, then $\sqrt[3]{y} = x$			
If $(-x)^4 = y$, then $\sqrt[4]{y} = (-x)$			

Rubric: (1 point) The student correctly selects True for all, True for some, or Not true for any for each statement correctly (see below).

Statement	True for all	True for some	Not true for any
If $x^2 = 9$, then $\sqrt{9} = x$			
If $x^3 = y$, then $\sqrt[3]{y} = x$			
If $(-x)^4 = y$, then $\sqrt[4]{y} =$ (-x)			

Response Type: Matching Tables

Target 3E: Distinguish correct reasoning from flawed reasoning

General Task Model Expectations for Target 3E

- Items for this target should focus on the core mathematical work that students are doing around the real number system, algebra, functions, and geometry.
- The student is presented with valid or invalid reasoning and told it is flawed or asked to determine its validity. If the reasoning is flawed, the student identifies, explains, and/or corrects the error or flaw.
- The error should be more than just a computational error or an error in counting, and should reflect an actual error in reasoning.
- Analyzing faulty algorithms is acceptable so long as the algorithm is internally consistent and it isn't just a mechanical mistake executing a standard algorithm.

Task Model 3E.1

- Some flawed reasoning or student work is presented and the student identifies and/or corrects the error or flaw.
- The student is presented with valid or invalid reasoning and asked to determine its validity. If the reasoning is flawed, the student will explain or correct the flaw.

Example Item 3E.1a

Primary Target 3E (Content Domain A-REI), Secondary Target 1H (CCSS A-REI.A)

A student solves a quadratic equation this way:

Equation: $x^2 - 3x - 4 = 0$
Step 1: $x^2 - 3x = 4$
Step 2: $x(x - 3) = 4$
Step 3: $x = 2$ or $x - 3 = 2$
Step 4: $x = 2$ or $x = 5$

The solution is incorrect. Identify the step that does not follow logically from the previous step.

A. Step 1
B. Step 2
C. Step 3
D. Step 4

Rubric: (1 point) Student selects the incorrect step (C).

Response Type: Multiple Choice, single correct response

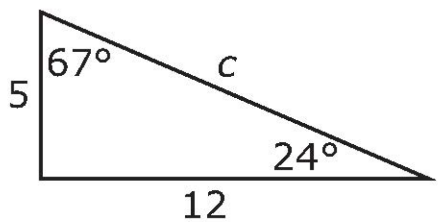
Commentary: Step 1 is a distractor because although it is procedurally unproductive, it does follow logically from the previous step. A variation on this task could ask the student say whether or not the solution is correct.

Example Item 3E.1b

Primary Target 3E (Content Domain Geometry) Secondary Target 1X (CCSS 8.G.B)

Brandon uses the following reasoning to find the length *c* of the third side in the triangle shown.

"The two legs of the triangle are 5 and 12. I know by the Pythagorean Theorem that $5^2 + 12^2 = c^2$.
I calculate $5^2 + 12^2 = 25 + 144 = 169$. My calculator says that the square root of 169 is 13, so *c* = 13."



Which statement is true about this reasoning?

A. Brandon's reasoning is correct.
B. The Pythagorean Theorem does not apply to this triangle.
C. The Pythagorean Theorem applies, but Brandon made a calculation error.
D. It is true that *c* = 13, but Brandon's reasoning is incorrect because he used a calculator.

Rubric: (1 point) Student selects the correct answer (B).

Response Type: Multiple Choice, single correct response

Task Model 3E.2

- Two or more approaches or chains of reasoning are given and the student is asked to identify the correct method and justification OR identify the incorrect method/reasoning and the justification.

Example Item 3E.2a

Primary Target 3E (Content Domain N-Q), Secondary Target 1C (CCSS N-Q.A), Tertiary Target Grade 7 1A

Sherry wants to enlarge a photograph to 300% of its original size.
The machine she is using can only make enlargements for the following percentages: 100%, 125%, 150%, and 200%.

Sherry thinks that she should enlarge the photograph by 100% and then by 200% to get a total enlargement of 300%.

Decide if Sherry is correct. If Sherry is correct, drag the 100 and 200 from the palette into the first two answer spaces.

%	Sequence of Enlargements
100	1: ☐ %
125	2: ☐ %
	3: ☐ %
150	4: ☐ %
200	5: ☐ %

If Sherry is incorrect, drag a sequence of enlargements she can use to get a total enlargement of 300%.

Each enlargement percentage may be used more than once. If a sequence is not needed, leave the box blank.

Rubric: The student drags the percentages into the appropriate boxes to show Sherry is incorrect (e.g., 200, 150. Other correct responses are possible).

Response Type: Drag and Drop

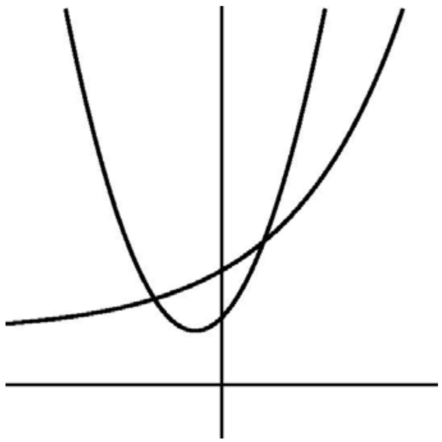
Target 3F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions

Task Model 3F.1

- The student uses concrete referents to help justify or refute an argument.

Example Item 3F.1a
Primary Target 3F (Content Domain A-REI), Secondary Target 1J (CCSS A-REI.D), Tertiary Target 3B

The two graphs shown represent the equations $y = P \bullet b^x$ and $y = a(x\text{-}h)^2 + k$, where a, $b>0$, and h, k, and P are rational numbers.



Which statement best describes the number of solutions the equation $P \bullet b^x = a(x\text{-}h)^2 + k$ has and why?

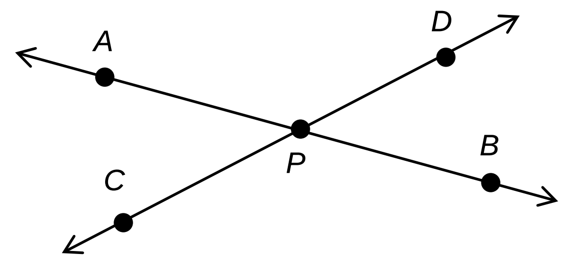
A. There is only one solution because b can't be negative.
B. There are no solutions to this equation because you can't solve it.
C. There are exactly two solutions because the graphs intersect twice.
D. There could be three solutions because the graphs might intersect at a third point.

Rubric: (1 point) Student selects the correct answer (D).

Response type: Multiple Choice, single correct response.

Example Item 3F.1b
Primary Target 3F (Content Domain G-CO), Secondary Target 1O (CCSS G-CO.C), Tertiary Target 3B

The line through *A* and *B* intersects the line through *C* and *D* at point *P*, as shown in the figure. Prove that angle *APC* is congruent to angle *BPD*.



Rubric: (2 points) The student gives a correct argument for the angle congruence (see Examples).
(1 point) The student gives a response that shows some understanding of a correct argument, but some details are missing or unclear.

Example 1
Since A and B lie on a line, a 180 degree rotation about point P will take ray PA to ray PB and vice versa.
Similarly, since C and D lie on a line, a 180 degree rotation about point P will take ray PC to ray PD and vice versa.
Thus a 180 degree rotation about point P will take angle APC to angle BPD, showing they are congruent.

Example 2
Angles APB and CPD are both 180 degrees.
So (measure angle CPA) + (measure angle APD) = 180 and
(measure angle APD) = 180 – (measure angle CPA).
Also, (measure angle APD) + (measure angle DPB) = 180.
Substituting for angle APD in the third equation, we get
180 – (measure angle CPA) + (measure angle DPB) = 180.
Subtracting the first two numbers in the left from both sides of the equation shows
(measure angle DPB) + (measure angle CPA).
If two angles have equal measures, they are congruent.

Response Type: Short text (handscored)

Target 3G: Determine conditions under which an argument does and does not apply

Target 3G is a closely related extension of the expectations in Targets 3A, 3B, 3C, and 3D, and as with those targets, is often a tertiary alignment for items in those targets. Students often test propositions and conjectures with specific examples (as described in Target 3A) for the purpose of formulating conjectures about the conditions under which an argument does and does not apply. Students then must explicitly describe those conditions (as in Target 3C). Expectations for Target 3D include determining conditions under which an argument is true given cases—the next step is articulating those cases autonomously.