Smarter Balanced
Assessment Consortium

High School Mathematics Item Specification Claim 2
Problem solving, which of course builds on a foundation of knowledge and procedural proficiency, sits at the core of *doing* mathematics. Proficiency at problem solving requires students to choose to use concepts and procedures from across the content domains and check their work using alternative methods. As problem solving skills develop, a student's understanding of and access to mathematical concepts becomes more deeply established. (*Math Content Specifications, p.56*)
Primary Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
Secondary Claim(s): Tasks written primarily to assess Claim 2 will necessarily involve some Claim 1 content targets and related Claim 1 targets should be listed below the Claim 2 targets in the item form[1]. If Claim 3 or Claim 4 targets are also directly related to the task, list those following the Claim 1 targets in order of prominence.
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.
Secondary Content Domain(s): While tasks developed to assess Claim 2 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the N and S domains in the high school grades can be used to construct higher difficulty items for the adaptive pool. The integration of the A, F, and G domains with N or S allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.

DOK Levels	1, 2, 3
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA); Fill-in Table (TI) No more than six choices in MS and MA items. Short Text – Performance tasks only **Scoring:** Scoring rules and answer choices will focus on students' ability to solve problems and/or to apply appropriate strategies to solve problems. For some problems, multiple correct responses and/or strategies are possible. • MC will be scored as correct/incorrect (1 point)

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[1] The full scope of Claim 2 items for high school often involves standards that are not specifically assessed in Claim 1. Be sure to include the standards and/or clusters in the metadata for all Claim 2 items in order to capture this information.

	• If MS and MA items require two skills, they will be scored as:
	   o All correct choices (2 points); at least ½ but less than all correct choices (1 point)
	   o Justification[2] for more than 1 point **must be** clear in the scoring rules
	   o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly.
	• EQ items will be scored as:
	   o Equation items: All numbers correct, correct operations, and correct solutions (2 points); error in numbers OR operations OR solution (1 point)
	   o Numeric items scored as correct/incorrect (1 point)
	• GI and TI items will be scored as:
	   o Single requirement items: will be scored as correct/incorrect (1 point)
	   o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point)
	   o Justification for more than 1 point **must be** clear in the scoring rules

Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 8 minutes to solve; Claim 2 items that are part of a performance task may take 5 to 12 minutes.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[3] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level

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[2] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[3] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Avoid ambiguous or obscure words, idioms, jargon, unusual names and references
	Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics
	Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology. [4]
Development Notes	Tasks generating evidence for Claim 2 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 Specifications that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 2. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 2. There are some other useful distinctions between Claim 1 and Claim 2 in high school that has supported the approach to alignment. The following points describe some attributes of items in Claim 2: • Multiple approaches are feasible or a range of responses is expected (e.g., if a student can solve a word problem by identifying a key word or words and selecting operations, then it is Claim 1). • The use of tools in Claim 2 is intended to support the problem solving process. In some cases, students may be asked to display their answer on the tool (e.g., by clicking the appropriate point or interval on a number line or ruler). • Assessing the reasonableness of answers to problems is a Claim 2 skill with items that align to Target C. For high school, 80% of the Claim 2 tasks should be written to support the standards and clusters shown in the table below.

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[4] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

High School, Claim 2

High School	
N-Q.A	F-IF.A
A-SSE.A	F-IF.B
A-SSE.B	F-IF.C
A-CED.A	F-BF.A
A-REI.2	G-SRT.C
A-REI.B	S-ID.C
A-REI.C	S-CP.A
A-REI.D	

Assessment Targets: Any given item/task should provide evidence for two or more Claim 2 assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace. (DOK 1, 2, 3)

Under Claim 2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools.

Target B: Select and use appropriate tools strategically. (DOK 1, 2)

Tasks used to assess this target should allow students to find and choose tools; for example, using a "Search" feature to call up a formula (as opposed to including the formula in the item stem) or using a protractor in physical space.

Target C: Interpret results in the context of a situation. (DOK 2)

Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem's context (e.g., understanding that the number of buses required for a given situation cannot be 32½, or that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context).

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)

For Claim 2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim 2 evidence.

> ### What sufficient evidence looks like for Claim 2 (Problem Solving)[5]:
>
> "Although items and tasks designed to provide evidence for this claim must primarily assess the student's ability to identify the problem and to arrive at an acceptable solution, mathematical problems nevertheless require students to apply mathematical concepts and procedures."
>
> **"Properties of items/tasks that assess Claim 2:** The assessment of many relatively discrete and/or single-step problems can be accomplished using short constructed-response items, or even computer-enhanced or selected-response items.
>
> More extensive constructed-response items can effectively assess multi-stage problem solving and can also indicate unique and elegant strategies used by some students to solve a given problem, and can illuminate flaws in student's approach to solving a problem. These tasks could:
>
> - Present non-routine[6] problems where a substantial part of the challenge is in deciding what to do, and which mathematical tools to use; and
>
> - Involve chains of autonomous[7] reasoning, in which some tasks may take a successful student 5 to 10 minutes, depending on the age of student and complexity of the task.
>
> "A distinctive feature of both single-step and multi-step items and tasks for Claim 2 is that they are "well-posed." That is, whether the problem deals with pure or applied contexts, the problem itself is completely formulated; the challenge is in identifying or using an appropriate solution path."

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[5] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 56-57).

[6] As noted earlier, by "non-routine" we mean that the student will not have been taught a closely similar problem, so will not expect to *remember* a solution path but to have to *adapt* or *extend* their earlier knowledge to find one.

[7] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

High School Content Combinations	**The following standards can be effectively used in various combinations in High School Claim 2 items:**
	Number and Quantities – Quantities (N-Q)
	N-Q.A: Reason quantitatively and use units to solve problems
	N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
	N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.
	N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	Algebra – Seeing Structure in Expressions (A-SSE)
	A-SSE.A: Interpret the structure of expressions
	A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★
	a. Interpret parts of an expression, such as terms, factors, and coefficients.
	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
	A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
	A-SSE.B: Write expressions in equivalent forms to solve problems
	A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
	a. Factor a quadratic expression to reveal the zeros of the function it defines.
	b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \square\square 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
	A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★
	Algebra – Creating Equations (A-CED)
	A-CED.A: Create equations that describe numbers or relationships

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Algebra – Reasoning with Equations and Inequalities (A-REI)

A-REI.A: Understand solving equations as a process of reasoning and explain the reasoning

A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.B: Solve equations and inequalities in one variable

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.B.4 Solve quadratic equations in one variable.
  a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A-REI.C: Solve systems of equations

A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

A-REI.D: Represent and solve equations and inequalities graphically

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions (F-IF)

F-IF.A: Understand the concept of a function and use function notation

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

F-IF.B: Interpret functions that arise in applications in terms of the context

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or

as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C: Analyze functions using different representations

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

  c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

  d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

  b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*

Functions — Building Functions (F-BF)

F-BF.A: Build a function that models a relationship between two quantities

F-BF.A.1 Write a function that describes a relationship between two quantities.★

  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

  b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

  c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Geometry — Similarity, Right Triangles, and Trigonometry (G-SRT)

G-SRT.C: Define trigonometric ratios and solve problems involving right triangles

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Statistics and Probability – Interpreting Categorical and Quantitative Data (S-ID)

S-ID.C: Interpret linear models

S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.C.8 Distinguish between correlation and causation.

Statistics and Probability – Conditional Probability & the Rules of Probability (S-CP)

S-CP.A: Understand independence and conditional probability and use them to interpret data

S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Range ALDs – Claim 2 High School	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a nonscaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to map, display, and identify relationships, use appropriate tools strategically, and apply mathematics accurately in everyday life, society, and the workplace. They should be able to interpret information and results in the context of an unfamiliar situation.
	Level 4 Students should be able to analyze and interpret the context of an unfamiliar situation for problems of increasing complexity and solve problems with optimal solutions.

Target 2A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 2A
- The student is asked to solve a well-posed problem arising in a purely mathematical context, in a thin context, which is defined to be a context that is nominally from outside mathematics but in reality serves a purely mathematical purpose, or in a context from everyday life, society, or the workplace.
- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- Solving the problem requires either using units, setting up and solving an equation or system of equations, building and interpreting equations or functions that represent relationships between quantities, finding or calculating geometric measures, or reasoning about geometric figures in the plane.
- The problem may require the integration of concepts and skills from multiple domains.
- The task does not indicate by key words or other scaffolding which operations, constructions, or transformations are to be performed or in what order.
- Difficulty of the task may be varied by varying (a) the difficulty of extracting information from the context, (b) the number of steps, or (c) the complexity of the expressions, equations, functions, or geometric figures used.
- Tasks have DOK Level 1, 2, or 3.
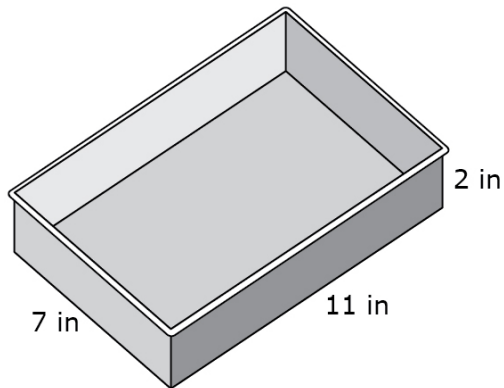
Task Model 2A.1

Expectations:
- The student is asked to solve a well-posed multi-step problem requiring careful attention to and use of units in a real-world context.
- Dimensions along which to vary the task include (a) varying the context, (b) varying the types of quantities, including measurement quantities and derived quantities, or (c) introducing a rate of flow and a time dimension (e.g. filling a pool).

Example Item 2A.1a

Primary Target 2A (Content Domain N-Q), Secondary Target 1C (CCSS N-Q.A), Tertiary Target 2D

A company that makes rectangular baking pans labels each pan with the dimensions, in inches, and the capacity in quarts. A company employee needs to label a rectangular pan with dimensions 7 inches by 11 inches by 2 inches.

1 quart = 57.75 cubic inches

What is the capacity, in quarts, of this pan? Round your answer to the nearest tenth.

Rubric: (1 point) The student correctly determines that the capacity in quarts of the pan (e.g., accept 2.66 - 2.7).

Response Type: Equation/Numeric

Commentary on Example Item 2A.1: Ways to vary this item to increase or decrease its complexity include (a) presenting the conversion from quarts to cubic inches in a less direct manner, e.g., by giving the dimensions of a cylindrical quart measure, or by saying that a certain number of quarts completely fills another vessel with different dimensions, or (b) varying the shape of the vessel, e.g., by making it a circular cake pan or a conical sieve. When the geometric work to be done is more complex, the item might fall more naturally within Task Model 2A.5. The difficulty can also be varied by varying the numerical complexity of the dimensions, but care should be taken not to do so in a way that merely increases the computational complexity. For example, if the dimensions are given as two-digit decimals, a calculator should be provided, since the target is problem solving, not computation. On the other hand, changing the dimensions to 7.5 x 10 x 2, with the expectation that students who look for structure will see that the 2 x 7.5 = 15 and 10 x 15 = 150, is a valid increase in the problem-solving demand of the task. Any version of this task would require students to identify quantities of interest and to map the relationship between them, and so draws on the skill set identified in Target 2D.

Task Model 2A.2

Expectations:
- The student sets up an equation in one variable given a real-world context and solves it to answer a question about the context.
- A task that both names the variable and scaffolds the equation to be set up through the use of key words or the order in which operations are described is not appropriate for Claim 2; however, tasks which do one but not the other of these things are acceptable. Tasks which do neither are at the more difficult end of the range
- Dimensions along which to vary the task include (a) varying the context, (b) naming the variable (easier) or expecting the student to identify the quantity of interest and choose a variable to represent it (harder), (c) intending an equation of the form "expression = constant" (easier) or "expression = expression" (harder, as in the example given here); having one or more constants represented by letters so that the solution is expressed in terms of those constants, not given numerically (hardest), or (d) varying the difficult of extracting the expressions involved in the equation from the context given.

Example Item 2A.2a
Primary Target 2A (Content Domain A-CED), Secondary Target 1G (CCSS A-CED.A), Tertiary Target 2D

The $1000 prize for a lottery is to be divided evenly among the winners. Initially, there are x winners.

However, one more winner comes forward, causing each winner to receive $50 less.

Enter an equation that represents the situation and can be used to solve for x, the initial number of winners.
Enter your equation in the response box.

Rubric: (1 point) The student creates a correct equation (e.g., $\frac{1000}{x} - 50 = \frac{1000}{x+1}$).

Response Type: Equation/Numeric

Task Model 2A.3

Expectations:
- The student sets up a system of linear equations given a real-world context and solves it to answer a question about the context. Most tasks should involve a system of two linear equations in two variables.
- A task that both names the variables and scaffolds the equations to be set up through the use of key words or the order in which operations are described is not appropriate for Claim 2; however, tasks which do one but not the other of these things are acceptable. Tasks which do neither are at the more difficult end of the range
- Dimensions along which to vary the task include (a) varying the context, (b) naming the variables (easier) or expecting the student to identify the quantities of interest and choose variables to represent then (harder), (c) intending equations of the form "expression = number" (easier, as in the example given here) or "expression = expression" (harder), or (d) varying the difficulty of extracting the expressions involved in the equation from the context given.

Example Item 2A.3a

Primary Target 2A (Content Domain A-REI), Secondary Target 1J (CCSS A-REI.6), Tertiary Target 2D

A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special.

- On Thursday, the restaurant collected $467 selling
  21 vegetarian specials and 40 chicken specials.

- On Friday, the restaurant collected $484 selling
  28 vegetarian specials and 36 chicken specials.

Enter the cost, in dollars, of the **vegetarian** lunch special.

Rubric: (1 point) The student correctly determines the cost of the vegetarian special (7).

Response Type: Equation/Numeric

Commentary on Example Item 2A.3a: The task given here, while not naming the variables, provides a clear structure for setting up the two equations by bulleting out the two days corresponding to equations.

Example Item 2A.3b
Primary Target 2A (Content Domain A-REI), Secondary Target 1J (CCSS A-REI.C)
(Source: Adapted from Illustrative Mathematics 8.EE Quinoa Pasta 1)

A type of pasta is made of two ingredients, quinoa and corn. The pasta company is not disclosing the amount of each ingredient in the pasta, but we know that the quinoa in the pasta contains 16.2% protein, and the corn in the pasta contains 3.5% protein. Overall, each 57 gram serving of pasta contains a total of 4 grams of protein.

How many grams of quinoa and how many grams of corn is in one serving of the pasta?

Enter the number of grams of quinoa in the first response box, Enter the number of grams of corn in the second response box. Round each amount to the nearest gram.

Quinoa: [   ]

Corn: [   ]

Rubric: (2 points) The student enters the correct number of grams for quinoa and corn into the response boxes (16 and 41). (1 point) Either the student reverses the answers (confused between quinoa and corn) or only answers one correctly.

Response Type: Equation/Numeric (2 response boxes)

Commentary on Example Item 2A.3b: This task illustrates a more difficult variant of this task model. In this task, one of the equations is $q + c = 57$, where q is the number of grams of quinoa and c is the number of grams of corn. This equation might not be apparent to the student because the coefficients of the variables are not evident in the problem statement. In Example Item 2A.3a, by contrast, the numbers 21, 40, 28, and 36 were all evident.

Example Item 2A.3c

Primary Target 2A (Content Domain G-SRT), Secondary Target 1O (CCSS G-SRT.C)

Melissa drew a right triangle.

- The length of the hypotenuse is $\sqrt{130}$ units.
- The perimeter is $14 + \sqrt{130}$ units.

Find the other two side lengths of Melissa's triangle. Enter one side length into each response box.

Rubric: (1 point) The student enters the correct side lengths in the response boxes (3 and 11).

Response Type: Equation/Numeric (2 response boxes)

Task Model 2A.4

Expectations:
- The student creates an equation in two variables or builds a function to represent a relationship in a mathematical or real-world context and uses the equation or function to answer a question about the context, by evaluating the function at one or more inputs or recognizing and using some feature of the function or its graph (such as the fact that the values are always positive, or that the graph is linear with a negative slope).
- Some items in this task model should reward looking for and making use of structure. For example, the student should select the form of the expression that defines the function that is appropriate for the purpose of the task.
- Dimensions along which to vary the task include (a) varying the context, (b) naming the independent or dependent variables (easier) or expecting the student to identify the quantities of interest and choose variables to represent them (harder), (c) varying the type of function built (linear, quadratic, exponential, power, or a combination of these), (d) naming (easier) or not naming (harder) the type of function to be used, or giving an explicit form for it (easiest), or (e) varying the difficulty of extracting the function from the context given (quite difficult in this case).

Example Item 2A.4a

Primary Target 2A (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A)

(Source: Adapted from Illustrative Mathematics A-SSE Course of Antibiotics)

Susan has an ear infection. Her doctor prescribes an antibiotic. The doctor tells Susan to take a 250-milligram dose of the antibiotic every 12 hours for the next 10 days.

- Susan finds out that 4% of the antibiotic is still in her body after 12 hours.
- Assume that each dose is exactly 250 milligrams and that Susan takes one dose every 12 hours.

Part A

How much of the antibiotic, in milligrams, is in Susan's body immediately after taking the **2nd** dose? Enter your answer in the first response box.

Part B

How much of the antibiotic, in milligrams, is in Susan's body immediately after taking the **10th** dose? Enter your answer in the second response box.

Rubric: (2 points) The student enters the correct amount of antibiotic in Susa's body for Part *A* and *B* (e.g., 260, 260.4167). Note: An acceptable range for Part *B* is 260.4-260.422.

(1 point) The student enters the correct amount for Part *A* or Part *B*, but not both.

Response Type: Equation/Numeric (2 response boxes)

Example Item 2A.4b

Primary Target 2A (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A), Tertiary Target 2D

$P(x)$ represents the cost that an online bookstore charges for shipping items and packaging material that together weigh x pounds. The packaging material weighs 1 pound.

A competitor charges the same rate per pound but does not charge for the weight of the packaging material. However, the competitor does charge an additional $5 processing fee for each shipment.

Which expression represents the cost of shipping x pounds with the competitor?

   A. $P(x + 5) + 1$
   B. $P(x + 5) - 1$
   C. $P(x + 1) + 5$
   D. $P(x - 1) + 5$

Rubric: (1 point). The student selects the correct answer choice (D).

Response Type: Multiple choice, single correct response.

Commentary on Example Item 2A.4b: This task requires the student to interpret information given about a function in a context and express that information using function notation. Students must determine the relationship between the price to ship a certain weight for two different companies, and so must draw on the skill set identified in Target 2D.

Task Model 2A.5

Expectations:

- The student solves a well-posed problem involving geometric measurement.
- Dimensions along which to vary the task include (a) varying the measurement context (circles, squares, triangles, compound figures composed out of these; cubes, rectangular prisms, pyramids, cones, spheres, compound figures composed out of these), (b) naming the unknown quantities explicitly and/or labeling them in a diagram (easier) or expecting the student to identify the quantities of interest and choose variables to represent them (harder), (c) requiring students to draw auxiliary lines in the figure, or (d) requiring students to use congruence or similarity concepts in the course of solving the problem.
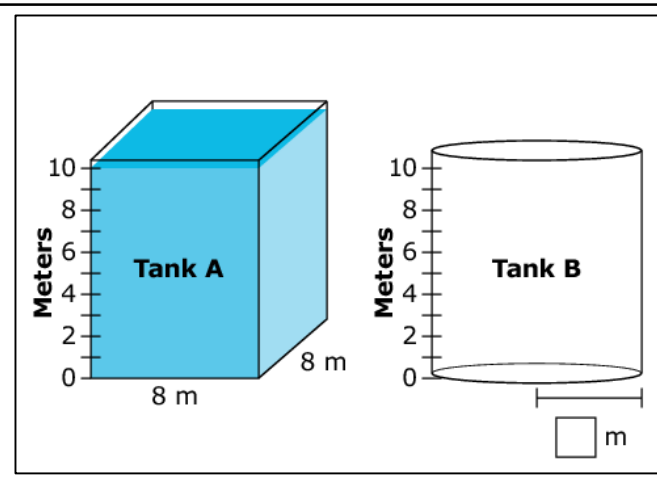
Example Item 2A.5a

Primary Target 2A (Content Domain G), Secondary Target 1I (CCSS 8.G.C), Tertiary Target 2D

Two water tanks are shown. Tank A is a rectangular prism and Tank B is a cylinder. The tanks are not drawn to scale.

Tank A is filled with water to the 10-meter mark.

Click Tank A to change the water level. The volume of water that leaves Tank A is transferred to Tank B, and then the height of the water in Tank B is shown.

Find the radius of Tank B, rounded to the nearest meter. Enter Your answer in the response box.

Rubric: (1 point). The student enters the correct radius in the response box (5).

Response Type: Equation/numeric with animation.

Task Model 2A.6

Expectations:
- The student solves a well-posed problem about geometric figures in the plane that require geometric reasoning; for example performing constructions or applying transformations.
- Dimensions along which to vary the task include (a) varying the types of geometric figure used (lines, circles, triangles, quadrilaterals, polygons), (b) varying the number of steps required to extract the desired feature, or (c) including a coordinate system or not.
- Note that tasks of this type can also be considered for Claim 3B, constructing autonomous chains of reasoning, depending on the balance between the problem solving aspect (making sense of the problem, for example) and the reasoning aspect (the depth and complexity of reasoning required to extract the desired feature).

Example Item 2A.6a

Primary Target 2A (Content Domain G), Secondary Target 1X (CCSS G-C.A)

A circle with center (6, 7) includes the point (1, 4). A second circle also include the point (1, 4), and contains the same area but has a different center.

Enter the ordered pair that corresponds to the center of the second circle.

([   ] , [   ])

Rubric: (1 point) The student correctly enters the ordered pair in the response box [(–4, 1)].

Response Type: Fill-in Table

Commentary on Item 2A.6a: A more difficult variant on this item is the following: A circle has its center at (6, 7) and goes through the point (1, 4). A second circle is tangent to the first circle at the point (1, 4) and has one-fourth the area. What are the coordinates for the center of the second circle? Show your work or explain how you found your answer.

Target 2B: Select and use appropriate tools strategically.

General Task Model Expectations for Target 2B

- The student uses a tool or makes a strategic selection of tools in the course of solving a problem.
- Mathematical information is presented in a table, graph, diagram, or equation or is extracted from a verbal description or pictorial representation of a context.
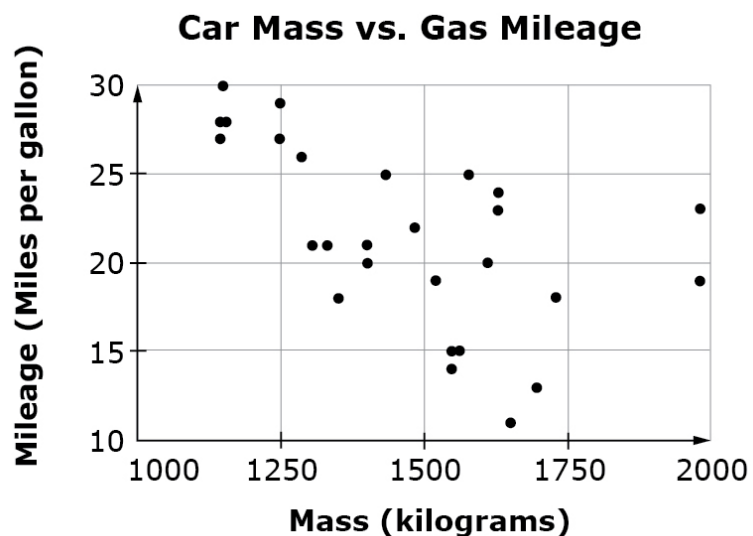- Tools include drawing tools, graphing tools, and geometric transformation tools.

Task Model 2B.1

Expectations:

- The student is asked to use a given tool to
  - produce a desired figure or graph given a mathematical or real-world context, or
  - produce a desired geometric transformation.
- Dimensions along which to vary tasks include (a) varying the tool used (b) varying the mathematical or real-world context and (c) varying the amount of supplemental reasoning and calculation necessary to achieve the task.
- Tasks have DOK Level 1 or 2.

Example Item 2B.1a

Primary Target 2B (Content Domain S-ID), Secondary Target 1P (CCSS 8.SP.A), Tertiary Target 2C

The scatterplot shows the weight and gas mileage for 31 cars.

Car Mass vs. Gas Mileage

Mileage (Miles per gallon) vs. *Mass (kilograms)*

Part A: Use the Add Arrow tool to create a line of best fit on the scatterplot.

Part B: What is the meaning of the slope of the line of best fit in terms of the situation?

    A. For every additional kilogram of mass, the gas mileage is predicted to increase 0.013 miles per gallon.
    B. For every additional kilogram of mass, the gas mileage is predicted to decrease 0.013 miles per gallon.
    C. For every additional kilogram of mass, the gas mileage is predicted to increase 3 miles per gallon.
    D. For every additional kilogram of mass, the gas mileage is predicted to decrease 3 miles per gallon.

Rubric: (1 points) The student adds a line of best fit and selects the correct interpretation of the slope (e.g., see graph on the following page; answer choice B).

Response Type: Graphing, Hot Spot

Car Mass vs. Gas Mileage

Example Item 2B.1b

Primary Target 2B (Content Domain F-IF), Secondary Target 1M (CCSS F-IF.C), Tertiary Target 2C

> The function f models the amount of a chemical that can be extracted from a mixture given the percent of ethanol used in the extraction process, x, for $0 \leq x \leq 100$. What value of x between 0 and 100 gives the maximum value for this function?
>
> $$f(x) = -0.002x^3 + 0.255x^2 - 4.5x + 165$$
>
> Enter your answer in the response box.

Interaction: The student uses the graphing calculator and estimates the value for x that produces the maximum value of the function.
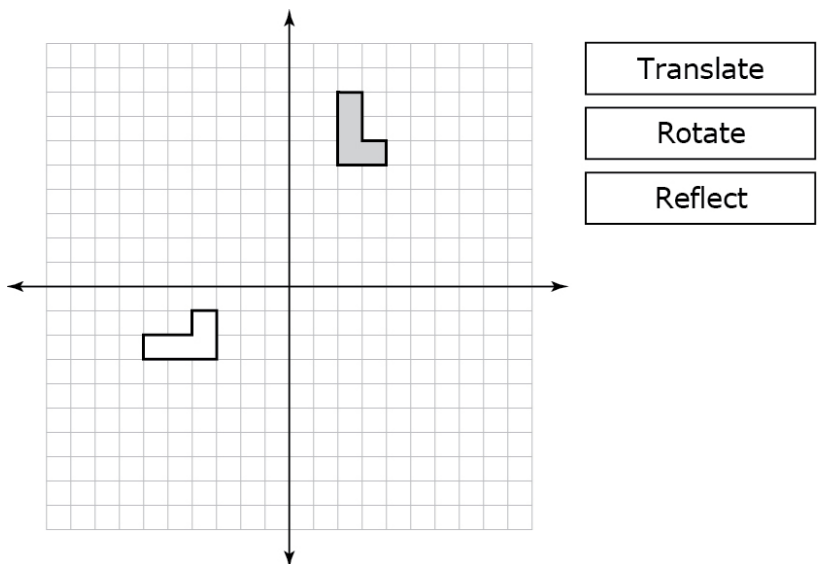
Rubric: (1 point) The student enters the correct value for x, with a tolerance of 5 percentage points (e.g., 75+/-5)

Response Type: Equation/Numeric

Commentary on Example Item 2B.1b: While a student in calculus could answer this question without technology, even then it would be computationally burdensome and prone to error. Using a graphing device to answer the question is the only way a student who does not know calculus can answer this question, although the concept of the maximum value of a function over a given domain is one that students learn about starting in grade 8.

Primary Target 2B (Content Domain G-CO), Secondary Target 1G (CCSS G-CO.B)

Use the tools to the right of the graph to choose a sequence of transformations that demonstrate that the shaded figure is congruent to the unshaded figure.

Interaction: The student uses the Translate, Rotate, and Reflect Tools to execute a series of translations, reflections, and rotations around the origin that would move the shaded figure onto the unshaded figure. An example of the interaction can found at this link: https://www.khanacademy.org/preview/content/items/x3a8afe95369d317e

Rubric: (1 point) The student selects a sequence of transformations where the shaded figure maps onto the unshaded figure (e.g., Step 1: Rotate 90 degrees counterclockwise around the origin. Step 2: Translate to the right two units. Step 3: Translate up one unit.)

Response Type: Graphing/Transformation (slated for field-testing in 2017 or later)

Commentary on Example Item 2B.1c: This task requires the student to use a transformation tool to produce a desired geometric figure from a given geometric figure. The required figure is not described explicitly, but must be extracted from a mathematical or real-world context. In this task, the student must understand the definition of congruence in terms of transformations; this item would not have been suitable for this model if it had told the student explicitly to move the shaded figure onto the hollow figure.
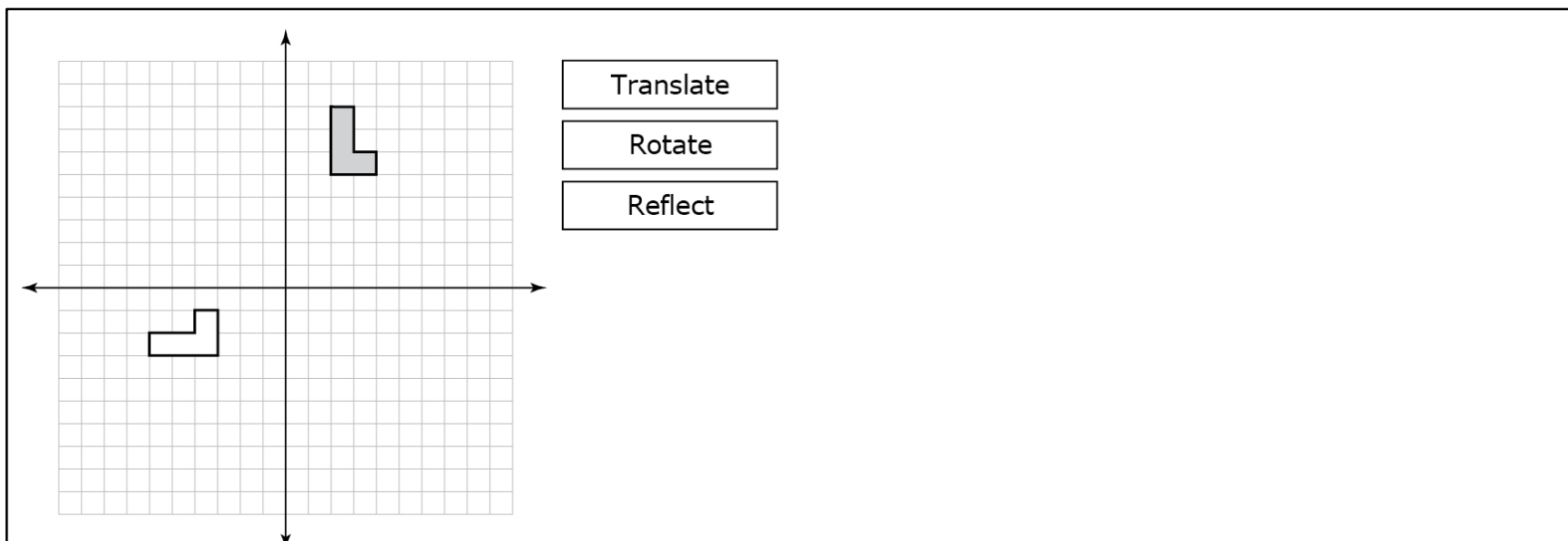
Expectations:

- Student is asked to decide whether or not to use a tool to produce a desired figure or graph, or to produce a desired geometric transformation. The task can be correctly performed with or without the available tools.
- Mathematical information is presented in a table, graph, diagram, or equation or is extracted from a verbal description or pictorial representation of a context.
- Tools include drawing tools, graphing tools, and geometric transformation tools.

Example Item 2B.2a

Primary Target 2B (Content Domain G-CO), Secondary Target 1G (CCSS G.CO.B)

Use the drop down-menus below to choose a sequence of transformations that would move the shaded figure onto the unshaded figure. If you wish, you can use the tools to the right of the graph to experiment before submitting your answer.

Interaction: The student chooses to use the Translate, Rotate, and Reflect Tools before entering the sequence of transformations.

Rubric: (1 point) The student selects a correct series of transformations from the dropdown menu (e.g., Step 1: Rotate 90 degrees counterclockwise around the origin. Step 2: Translate to the right two units. Step 3: Translate up one unit.)

Response Type: Drop-down menu.

Commentary on Example Item 2B.2a: This task requires the student to describe a transformation that produces a desired geometric figure from a given geometric figure. The required figure can be described explicitly, in contrast with Item 2B.1c. The emphasis in this task model is on providing the student with the option to choose the tool or not, so the complexity required in presenting the figure is correspondingly reduced. More complex variations may still require the desired figure to be extracted from a mathematical or real-world context.

Example Item 2B.2b

Primary Target 2B (Content Domain A-APR), Secondary Target 1F (CCSS A-APR.B)

Find all the zeros of the following polynomial function and enter them into the boxes.

$f(x) = x^5 - 37x^3 - 24x^2 + 180x$

Zeros:

Interaction: The student has access to a graphing calculator which can be used to identify the zeros of the polynomial.

Rubric: (1 point). Student enters the correct zeros in the response boxes (−6, −2, 0, 3, 5; the numbers may be in any order).

Response type: Fill-in Table

Variations on this item: This item requires students to give information about a function, expression, or equation that can be obtained either algebraically or graphically. Students can choose whether or not to use the graphing calculator to answer the question. Dimensions along which to vary the item include (a) varying the desirability of using the tool or not (for example, this item favors using the calculator because of the difficulty of factoring a degree 5 polynomial; another item might require the student to find the exact roots of $f(x) = x^3 - 3x$, which would favor factorization, since it would be difficult to recognize $\sqrt{3}$ exactly from the graph), (b) varying the type of object presented and the information required (e.g., factors of a polynomial, maximum of a quadratic function, solutions to an equation of the form $f(x) = g(x)$).

Target 2C: Interpret results in the context of a situation.

General Task Model Expectations for Target 2C
- Student is asked to solve a well-posed problem arising in a context from everyday life, society, or the workplace, and then to interpret the solution in terms of the context.
- Possible interpretations include: giving the units of an answer and explaining their meaning, interpreting parameters in a function or parts of an expression, interpreting the solution to an equation, interpreting a statement involving function notation. Problems involving interpreting data are more likely to fit Claim 4 than Claim 2C
- Because the focus is on interpreting the solution, items in this task model will generally have lower cognitive demand in the problem solving phase than items in Task Models 1 and 2.
- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.
- Solving the problem requires either using units, writing an expression in an equivalent form, setting up and solving an equation or system of equations, or interpreting and building functions, performing a geometric construction or transformation, or calculating geometric measures.
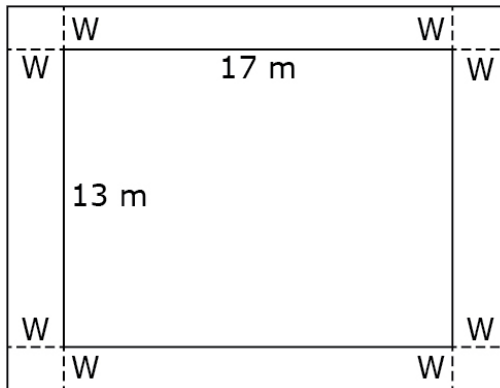- Tasks have DOK Level 2.

Task Model 2C.1

Expectations:
- The student sets up an equation arising from a thin or real-world context.
  - o If the equation is in one variable, the student solves the equation and interprets the solution in terms of the context.
  - o If the equation represents a function, the student interprets a parameter in the context.
- The wording of the problem should not reveal the answer to the interpretation step.
- Dimensions along which to vary the item include, (a) varying the context, (b) varying the type of equation to be solved (linear, quadratic, other), or (c) varying the complexity of the interpretation asked for.

Primary Target 2C (Content Domain A-CED), Secondary Target 1G (CCSS A-CED.A), Tertiary Target 2A

A rectangular garden, shown in the diagram, measures 13 meters by 17 meters. The garden has a cement walkway around its perimeter, as shown. The width, W, of the walkway remains constant on all four sides, as shown in the figure. The garden and walkway have a combined area of 396 square meters.

Part A
Enter an equation in the first response box that can be used to find the width, W.

Part B
Determine the width, W (in meters), of the walkway. Enter your answer in the second response box.

Rubric: (2 points) The student enters the correct equation and width in the response boxes [$(13+2W)(17+2W)=396$; 2.5].
(1 point) The student answers only one part correctly.

Response Type: Equation/Numeric (2 response boxes)

Commentary on Example Item 2C.1a: A variation of this item would ask for the dimensions of the garden rather than asking for the width).

Example Item 2C.1b
Primary Target 2C (Content Domain F-LE), Secondary Target 1K (CCSS F-LE.B), Tertiary Target 2A

For his science experiment, Julio placed a pan of water in his refrigerator to see how long it would take to evaporate. After 2 days, there were 280 ml of water in the pan, and after 4 days, there were 240 ml of water in the pan. In his report, he noted that the same amount of water evaporated each day.

How many milliliters of water were in the pan to start?
Enter your answer in the first response box.

At this rate, how many days until no water remains in the pan?
Enter your answer in the second response box.

Rubric: (1 point) The student correctly enters the initial amount of water and the time until the water is completely evaporated (320 and 16).

Response Type: Equation/Numeric (2 response boxes)

Commentary on Example Item 2C.1b: The amount of water in the pan can be modeled by linear relationship. The two questions correspond to interpreting the two intercepts.

Example Item 2C.1c
Primary Target 2C (Content Domain F-BF), Secondary Target 1N (CCSS F-IF.C), Tertiary Target 2A

Lisa plans to build a rectangular fenced garden. She has 500 feet of fencing to use for the project. She determines that if her garden has a width of w meters, then the area of her garden $A(w)$, in square meters, is given by the following function.

$A(w) = 250w - w^2$

Which of the following expressions is equivalent to $250w - w^2$?

  A. $-(w - 125)^2$
  B. $-(w - 250)^2$
  C. $-(w - 125)^2 + 15{,}625$
  D. $-(w - 250)^2 + 62{,}500$

What is the largest garden area, in square meters, that Lisa can enclose with 500 meters of fencing?
Enter your answer in the response box.

Area: ☐

Rubric: (2 points) The student selects the correct expression (C) and identifies the largest area possible (15625).
(1 point) The student is able to correctly identify the expression **or** identify the largest area.

Response Type: Multiple choice/single answer and Equation/numeric

Commentary on Example Item 2C.1c: Natural variations on this item include presenting some of the information about the function with a graph; for example, showing the parabola with the x-intercepts labeled.

Task Model 2C.2

Task Expectations:

- The student is presented with an expression, a solution to an equation, or a statement involving function notation, where the expression, equation or function represents a context from everyday life, society, or the workplace, and asked to interpret it in terms of the context.
- Possible interpretations include: giving the units of an answer and explaining their meaning; interpreting parts of an expression, interpreting the solution to an equation, interpreting a statement involving function notation. Problems involving interpreting data are more likely to fit into Claim 4B than Claim 2C.
- Mathematical information from the context is presented in a table, graph, or diagram, or is extracted from a verbal description or pictorial representation of the context.

Example Item 2C.2a
Primary Target 2C (Content Domain F-IF), Secondary Target 1K (CCSS F-IF.A), Tertiary Target 2D

Kiki starts her run at 5:00 p.m. Let $f(t)$ represent Kiki's speed, in miles per hour, t hours after she starts running for $0 \leq t \leq 4$. The following information holds for the function f:

- $f(1) = 5$
- $f(4) = 2$
- $f(2) = 8$
- $f(1) < f(3)$.

Kiki's fastest speed occurs at 7:00 p.m. Given this information, which of the following **must** be true? Select **all** that apply.

A. At 8:00 p.m., Kiki's speed was at least 5 miles per hour.
B. At 6:30 p.m., Kiki's speed was at most 8 miles per hour.
C. Kiki ran at most 32 miles.

Rubric: (1 point) The student selects all three options.

Response Type: Multiple-choice, multiple-select response

Example Item 2C.2b
Primary Target 2C (Content Domain F-IF), Secondary Target 1K (CCSS F-IF.B)

The height, in meters, of a golf ball t seconds after it was hit is given by the function $h(t) = -9.8(t - 8)^2 + 36$. The number 36 appears in the expression that defines the function. What does it tell you about the golf ball?

    A. The time it takes for the golf ball to hit the ground.
    B. The time it takes for the golf ball to reach it's greatest height.
    C. The greatest height the golf ball reaches.
    D. The speed at which the golf ball is traveling.

Rubric: The student selects the appropriate answer choice (C).

Response Types: Multiple Choice, single correct response

Example Item 2C.2c
Primary Target 2C (Content Domain F-IF), Secondary Target 1K (CCSS F-IF.B), Tertiary Target 2D

The relationship between the Fahrenheit (F) and Kelvin (K) scales for measuring temperatures can be represented by a linear function. A temperature of 68° F corresponds to 293.15° K, and a temperature of 185° F corresponds to 358.15° K.

Which statement is the best interpretation of the slope of the graph of this function?

A.   A temperature of 0° K corresponds to −459.67° F.

B.   A temperature of 0° F corresponds to 255.37° K.

C.   For each change of one degree in the Fahrenheit scale, there is a change of $\frac{5}{9}$ degree in the Kelvin scale.

D.   For each change of one degree in the Fahrenheit scale, there is a change of $\frac{9}{5}$ degree in the Kelvin scale.

Rubric: The student selects the appropriate answer choice (C).

Response Types: Multiple Choice, single correct response.

Target 2D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 2D identifies a key step in the modeling cycle, and is thus frequently present in problems with real-world contexts. Note that Target 2D is rarely the primary target for an item, but is frequently a Secondary or Tertiary Target for an item with primary alignment to 2A, 2B, or 2C; see example items for many of the task models in those Targets.