High School Mathematics Item Specification Claim 4
"Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making." (p.72, CCSSM)
Primary Claim 4: Modeling and Data Analysis   Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s): Items/tasks written primarily to assess Claim 4 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or 3 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.
Secondary Content Domain(s): While tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the N and S domains in the high school grades can be used to construct higher difficulty items for the adaptive pool. The integration of the A, F, and G domains with N or S allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.

DOK Levels	1, 2, 3, 4
Allowable Response Types	**Response Types:** Multiple Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA), Fill-in Table (TI) No more than six choices in MS and MA items. Short Text – Performance tasks and select CAT items **Scoring:** Scoring rules and answer choices will focus students' ability to use the appropriate reasoning. For some problems, multiple correct responses and/or strategies are possible. • MC and MS will be scored as correct/incorrect (1 point) • If MA items require two skills, they will be scored as:

	o All correct choices (2 points); at least ½ but less than all correct choices (1 point) o Justification[1] for more than 1 point **must be** clear in the scoring rules o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly • EQ, GI, and TI items will be scored as: o Single requirement items: will be scored as correct/incorrect (1 point) o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) o Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to students at the targeted grade level. Target-specific stimuli will be derived from the Claim 1 targets used in the problem situation. All real-world problem contexts will be relevant to the age of the students. Stimulus guidelines specific to task models are given below.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine Construct Relevant Vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice #5 and others can be found in the language of specific standards.
Target-Specific Attributes:	CAT Items should take from 3 to 8 minutes to solve; Claim 4 items that are part of a performance task may take 5 to 15 minutes.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references

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[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	CAT items/tasks generating evidence for Claim 4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up through that grade, though more complex problem-solving tasks may draw upon knowledge and skills from lower grade levels. Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 4. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 4. Distinguishing between Claim 4 and Claims 1 and 2: • In early grades when equations are still new to students, an important distinction between Claim 2 and Claim 4 is requiring a model that would lead to a problem's solution. • In Claim 2 problems are well posed, while in Claim 4 they may have extraneous or missing information. • In Claims 1 and 2, measurements of objects or figures can be accurately determined. In Claim 4, modeling is used to make approximations. • In Claim 1, data analysis is straightforward procedural. In Claim 4, the analysis should be tied to some useful purpose in the real world. At least 80% of the items written to Claim 4 should primarily assess the standards and clusters listed in the table that follows.

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[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

High School	
N-Q.A	S-ID.A
A-SSE.B	S-ID.B
A-CED.A	S-IC.A.1
A-REI.A	S-IC.B
A-REI.B	F-LE.A
A-REI.C	F-LE.B
F-IF.B	F-TF.B.5
F-IF.C	G-GMD.A.3
F-BF.A	G-MG.A

REMINDER: Claim 4 tasks may also ask students to apply content from prior grades in sophisticated applications.

Assessment Targets: Any given item/task should provide evidence for two or more Claim 4 assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Problems used to assess this target for Claim #4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)
 At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still give them time to think and explain. *For a minority of these tasks*, subtasks may be constructed to facilitate entry and assess students' progress toward expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Target C: State logical assumptions being used. (DOK 1, 2)
Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Interpret results in the context of a situation. (DOK 2, 3)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. (See Claim 2, Target C for further explication.)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data. Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future).
Longer constructed response items and extended performance tasks should be used to assess this target.

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
Unlike Claim 2 where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be embedded in a larger context for items/tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to

Claim 4 Targets A, B, E, and G.

Target G*: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)

Especially in extended performance tasks (those requiring 1-2 class periods to complete), students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed response items should incorporate "hyperlinked" information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

*Measured in Performance Tasks only

What sufficient evidence looks like for Claim 4 (Modeling and Data Analysis)[4]:

"A key feature of items and tasks in Claim 4 is that the student is confronted with a contextualized, or 'real world' situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, 'real world' situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical application context. In this way, items and tasks in Claim 4 differ from those in Claim 2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

"Items/tasks in Claim 4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim 4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim 1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower—normally met by content first taught in earlier grades, consistent with the emphases described under Claim 1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected-response and short constructed-response items will be appropriate to use.

"Modeling and data analysis in the Common Core State Standards trace a visible arc of growing prominence across the grades, showing low prominence in grades K-5, higher prominence in grades 6-8 (which is when the Statistics and Probability domain first appears), and highest prominence in High School (which is when Modeling appears as a content category with the full modeling cycle). Therefore to align to the Standards, Claim 4 will be more important on the assessment in high school, less important in grades 6-8, and the least important in grades 3-5. Again, to align to the Standards, Claim 4 tasks will be most sophisticated and complete in high school (cf. the modeling cycle in CCSSM pp. 72, 73), less sophisticated/more tied to specific content in middle school, and least sophisticated/most tied to specific content in grades 3-5."

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[4] Text excerpted from the Smarter Balanced Mathematics Content Specifications (p. 72-73).

High School Content Combinations:	The following standards can be effectively used in various combinations in High School Claim 4 items:
	Number and Quantities – Quantities (N-Q)
	N-Q.A: Reason quantitatively and use units to solve problems
	N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
	N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.
	N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	Algebra – Seeing Structure in Expressions (A-SSE)
	A-SSE.B: Write expressions in equivalent forms to solve problems
	A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
	a. Factor a quadratic expression to reveal the zeros of the function it defines.
	b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \square\square 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
	A-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★
	Algebra – Creating Equations (A-CED)
	A-CED.A: Create equations that describe numbers or relationships
	A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
	A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Algebra — Reasoning with Equations and Inequalities (A-REI)

A-REI.A: Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.B: Solve equations and inequalities in one variable

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A-REI.C: Solve systems of equations

A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = −3x and the circle x^2 + y^2 = 3.*

A-REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Functions – Interpreting Functions (F-IF)

F-IF.B: Interpret functions that arise in applications in terms of the context

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F-IF.C: Analyze functions using different representations

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*

F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions*). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Functions – Building Functions (F-BF)
F-BF.A: Build a function that models a relationship between two quantities

 F-BF.A.1 Write a function that describes a relationship between two quantities.★
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
  c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
 F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Functions – Trigonometric Functions (F-TF)
Model periodic phenomena with trigonometric functions

 F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Geometry
G-GMD.A: Explain volume formula and use them to solve problems
 G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

G-MG.A: Apply geometric concepts in modeling situations
 G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
 G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
 G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Statistics and Probability – Interpreting Categorical and Quantitative Data (S-ID)
S-ID.A: Summarize, represent, and interpret data on a single count or measurement variable

 S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
 S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median,

mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

S-ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables

S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Statistics and Probability – Making Inferences and Justifying Conclusions
S-IC.A.1: Understand and evaluate random processes underlying statistical experiments

S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC.B: Make inferences and justify conclusions from sample surveys, experiments, and observational studies

S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC.B.6 Evaluate reports based on data.

Range ALDs – Claim 4 High School	**Level 1** Students should be able to identify important quantities in the context of a familiar situation and translate words to equations or other mathematical formulation. When given the correct math tool(s), students should be able to apply the tool(s) to problems with a high degree of scaffolding.
	Level 2 Students should be able to identify important quantities in the context of an unfamiliar situation and to select tools to solve a familiar and moderately scaffolded problem or to solve a less familiar or a non-scaffolded problem with partial accuracy. Students should be able to provide solutions to familiar problems using an appropriate format (e.g., correct units, etc.). They should be able to interpret information and results in the context of a familiar situation.
	Level 3 Students should be able to map, display, and identify relationships, use appropriate tools strategically, and apply mathematics accurately in everyday life, society, and the workplace. They should be able to interpret information and results in the context of an unfamiliar situation.
	Level 4 Students should be able to analyze and interpret the context of an unfamiliar situation for problems of increasing complexity and solve problems with optimal solutions.

Target 4A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.

General Task Model Expectations for Target 4A
- The student is asked to solve a problem arising in everyday life, society, or the workplace using functions, geometric modeling, probability, or statistics.
- Information needed to solve the problem has a level of complexity that is not present in items within Claim 2 Target A. For example, the student may be required to
    - ignore extraneous information, and extract relevant information,
    - identify information that is not given in the problem and request or conduct research to find it,
    - make a reasoned estimate for one or more quantities and use that estimate to solve the problem, and/or
    - make strategic decisions about a solution that takes into account time, cost, or new information.
- The student must select a mathematical or statistical model themselves.
- Tasks in this model often have secondary alignments to other Claim 4 targets, in particular Target 4D, requiring the student to interpret results in the context of the problem, and Target 4F, requiring students to identify quantities and map relationships between them.
- Problems in this model may have more than one possible solution.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Students often have access to a calculator. Some items are best answered without a calculator, promoting strategic use of the calculator.
- Tasks have Depth of Knowledge (DOK) Level 1, 2 or 3.

Task Model 4A.1

Task Expectations
- The student solves a multi-step problem involving number and quantity, algebra, functions, or geometric modeling of real-world phenomena.
- The student identifies needed information and chooses which mathematical structure to employ. The student may
    - ignore irrelevant information,
    - request or conduct research to find missing information,
    - identify constraints that are not explicitly stated, and/or
    - make an estimate for one or more quantities and use that estimate to solve the problem.
- Example items from Task Model 4A.1 or 4A.2 for Grades 6-8 may be adapted to this task model by increasing the complexity of the algebraic structures or functions needed.
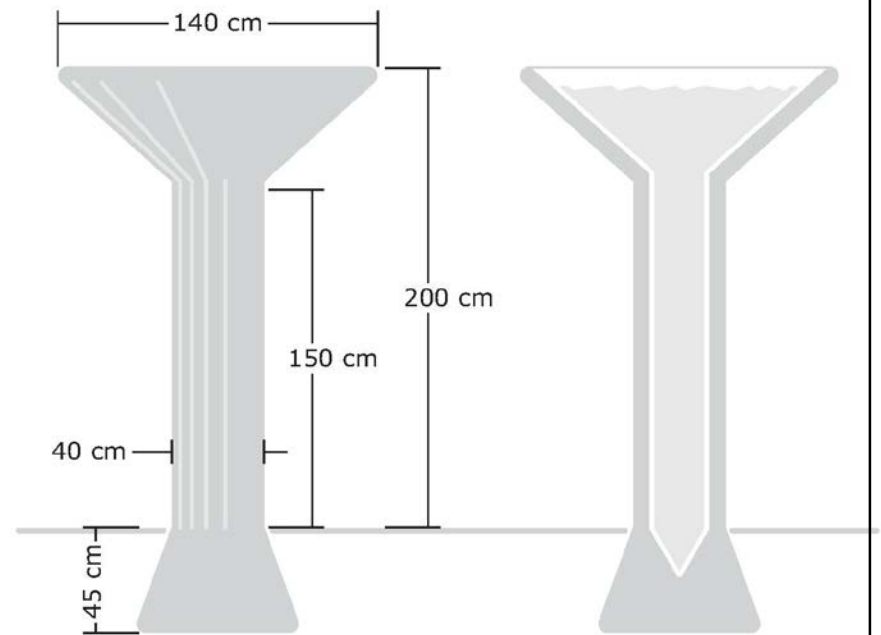
Example Item 4A.1a
Primary Target 4A (Content Domain G-MG), Secondary Target 1O, Tertiary Target 4B, Quaternary Target 4F

A rainwater cistern is shown in the figure.

Estimate the number of liters (*l*) of water the cistern can hold when full.

Enter your estimate in the response box.

[Click here for more information if you need it.[5]]

Interaction: The student can reveal the following list of conversions and formulas:

1 m = 100 cm	Volume of a sphere: $V = \frac{4}{3}\pi r^3$
1 cm = 10 mm	Volume of a cylinder: $V = \pi r^2 h$
1 cm^3 = 1 m*l*	Volume of a cone: $V = \frac{1}{3}\pi r^2 h$
1 *l* = 1000 m*l*	

Rubric: (1 point) The student enters a reasonable estimate for the amount of water in the cistern (between 340 and 700).

Response Type: Equation/Numeric

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[5] Pop-up windows with relevant information are not currently available as in interaction, but it is a planned enhancement for the item authoring tool in 2017.

Example Item 4A.1b
Primary Target 4A (Content Domain NQ), Secondary Target 1C (CCSS N-Q.A), Tertiary Target 4E, Quaternary Target 4D

Bear Lake straddles the border between Idaho and Utah. It is about 30 kilometers (km) long and about 11 km wide at the widest spot, as shown in the figure. The water is about 60 meters (m) deep at the deepest spot with an average depth of about 30 m. Use this information to estimate the number of liters (*l*) of water in Bear Lake.

[Click here for more information if you need it.]

Part A:
Enter your estimate in the response box using scientific notation.

Part B:
The surface area of Bear Lake is approximately 280 km². Use this information to improve your estimate of the number of liters contained in the lake if you can.

Enter your revised estimate in the response box using scientific notation.
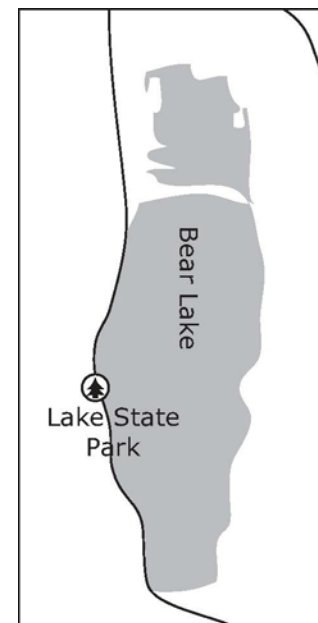
Interaction: The student can reveal the following list of conversions:
1 km = 1000 m   1 cm³ = 1 m*l*
1 m = 100 cm   1 *l* = 1000 m*l*
1 cm = 10 mm

The student must enter an answer in the first response box before seeing the second response box.

Rubric: (2 points) The student enters a reasonable estimate for Part A for the number of liters in the lake in the first response box (any number between 4.95×10^{12} and 9.9×10^{12}) and enters a more precise estimate for Part B that is closer to 8.4×10^{12}. (1 point) The student enters a reasonable estimate for Part A but does not enter an improved estimate in the second box or the student enters an unreasonable estimate in the first response box and 8.4×10^{12} in the second response box.

Response Type: Equation/Numeric (2 response boxes)

Note: Functionality for this item type does not currently exist. The stimulus can include the information that would be available upon request in the meantime.

Task Model 4A.2

Task Expectations
- The student solves a multi-step problem that requires applying statistics.
- The student identifies needed information and chooses which mathematical structure to employ. The student may
  - ignore irrelevant information,
  - request or conduct research to find missing information, and/or
  - identify constraints that are not explicitly stated.

Example Item 4A.2a

Primary Target 4A (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.B), Tertiary Target 4D, Quaternary Target 4E

This scatterplot shows the income per person (in U.S. dollars) versus the adult literacy rates for 30 countries throughout the world. The adult literacy rate is the percentage of people ages 15 and above who can, with understanding, read and write a short, simple statement about their everyday life.

Part A:

Use the slider to adjust the value for the base for the given exponential model so that it fits the data in the scatterplot.

$y = -100 \cdot b^x + 100$	✗
$b = 0.995$	✗
.995 •━━━━━━━━━━━━ .9999	

Part B:

Your model predicts the literacy rates in some countries better than others. Which country's literacy rate is least well predicted by your model?

Click on the point in the scatterplot to indicate your choice.

Comparison of Adult Literacy Rates to Income for 30 Countries

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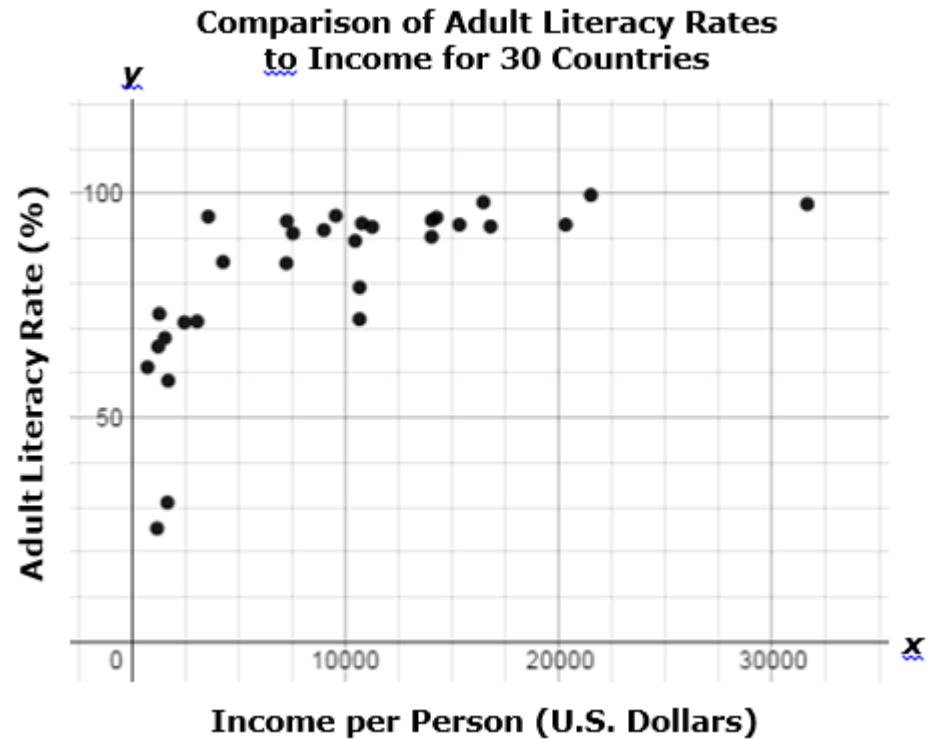
Interaction: The student moves the slider until the graph of the function fits the scatterplot reasonably well (see how this functionality might work here). When the student mouses over a point, the coordinates as well as the name of the country appear. When the student clicks on a point, the name of the country appears and remains on the graph.

Rubric: (1 point) the student adjusts the graph so it fits the scatterplot reasonably well (see exemplar below). At a minimum, at least five of the data points must be above the graph and at least five must be below the graph. The student then clicks on the point that has the greatest vertical distance from their graph.
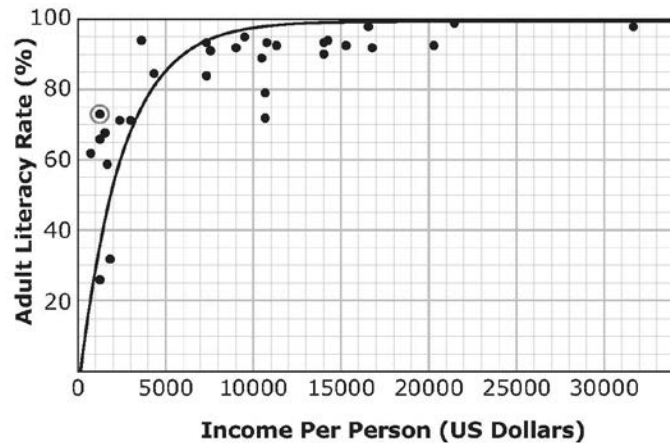
Response Type: Graphing

Note: The functionality for this item type does not currently exist, but it is planned for a future enhancement to the item-authoring tool.

Data for the scatterplot:

Country	Malawi	Guinea	Rwanda	Mali	Cameroon	Ghana	Honduras	El Salvador	Paraguay	Sri Lanka	Ecuador	China
Income Per Person	737	1,184	1,238	1,669	2,463	3,063	4,277	7,240	7,264	7,546	8,995	9,556
Adult Literacy Rate	61	25	66	31	71	71	85	84	94	91	92	95

Country	Dominican Rep.	Egypt	Colombia	Jordan	Brazil	Panama	Mexico	Malaysia	Russia	Spain	Bahrain	Oman	Singapore
Income Per Person	10,463	10,669	10,782	11,260	14,042	14,050	15,346	20,319	21,501	31,636	40,044	52,799	70,455
Adult Literacy Rate	90	72	93	93	90	94	93	93	100	98	95	87	96

Example Item 4A.2b

Primary Target 4A (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.A), Tertiary Target 4D, Quaternary Target 4B

Two airlines each made 30 flights. The dot plots shown compare how many minutes the actual arrival times were before or after the scheduled arrival times of these flights.
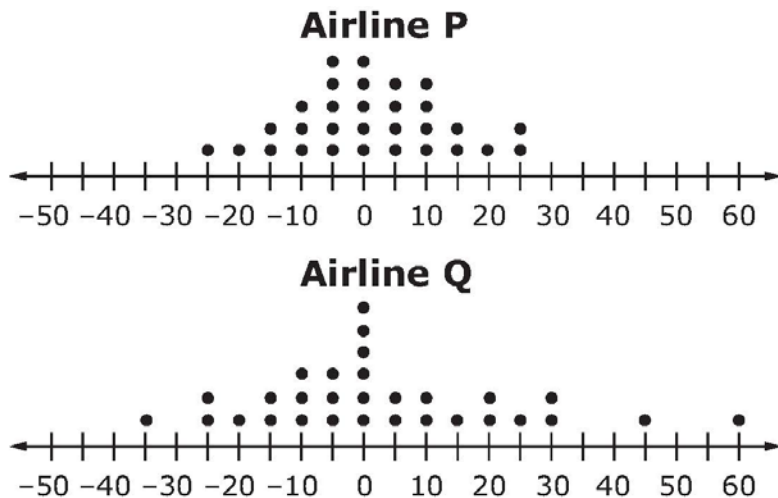
Airline P

Airline Q

Assuming you want to arrive as close to the scheduled time as possible, from which airline should you buy your ticket? Use all three drop-down menus to respond to this question.

I should choose [Airline P/Airline Q] because the distribution of flights for Airline P has a center that is [greater than/about the same as/less than] the center of the distribution of flights of for Airline Q, and the distribution of flights for Airline P has a spread that is [greater than/about the same as/less than] the spread of the distribution for Airline Q.

- Negative numbers represent the minutes the flight arrived before its scheduled time.
- Positive numbers represent the minutes the flight arrived after its scheduled time.
- Zero indicates the flight arrived at its scheduled time.

Interaction: The response would currently be in a GI box for the drop down functionality to work.

Rubric:
(2 points) The student chooses Airline P, about the same as, and less than.
(1 point) The student chooses Airline P and one of the correct choices for the other two menus.

Response Type: Drop-Down Menu (GI)

Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

General Task Model Expectations for Target 4B
- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

Target 4C: State logical assumptions being used.

General Task Model Expectations for Target 4C
- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - identifies information or assumptions needed to solve the problem,
  - researches additional information needed to solve the problem, or
  - provides a reasoned estimate of a quantity needed to solve the problem.

  It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model generally have either more information than is needed solve the problem (and students identify the relevant information) or not enough information (and students must provide a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks in this model sometimes ask students to choose one of two or more reasonable models, and then to draw the appropriate conclusion base on their choice without evaluating the appropriateness of that choice.
- Tasks for this target may also assess Target 4F.
- Tasks have DOK Level 1 or 2

Task Model 4C.1

Task Expectations:
- Students solve problems that involve using stated assumptions, definitions, and previously established results in developing their reasoning.

Example Item 4C.1a
Primary Target 4C (Content Domain S-IC), Secondary Target 1I (CCSS S-ID.A), Tertiary Target 4D
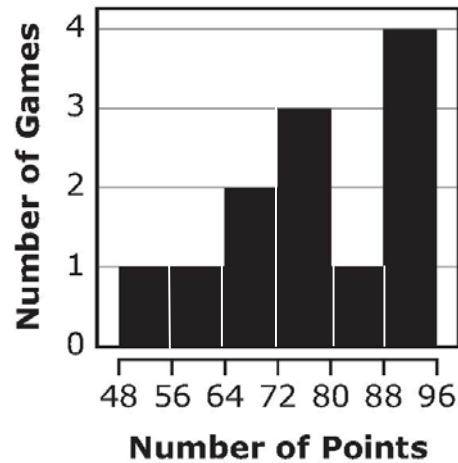

Dirk wants to compare the performance of the school basketball team last year with its performance this year. Decide whether he will use the mean or the median number of points earned in a basketball game to make the comparison. Either measure is acceptable, but you must choose one.

Dirk will use the [mean, median] number of points scored in a game.

Points Scored in Games Last Season
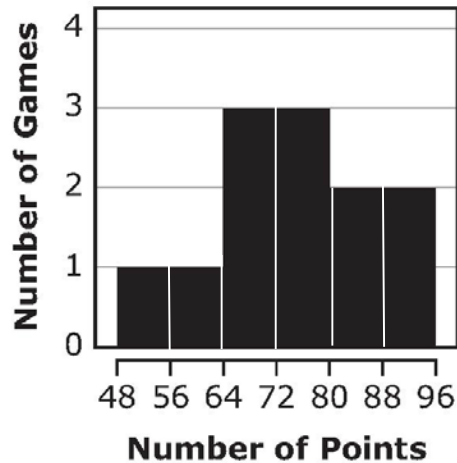
Points Scored in Games This Season

Based on the measure of center Dirk will use, which of the following is the best comparison between the performance of the school basketball team last season and this season? You can choose to view the data sets on which the histograms are based if you think it will help you choose.

[Click here to show the data set]

The basketball team performed [better, about the same, worse] this year than last year based on the fact that the [auto-populated with student's choice of *mean* or *median*] is [higher, about the same, lower] this year.

Interaction: The student selects mean or median, which then populates the answer choices. The student can choose to view the underlying dataset—this is not needed if the student chooses mean, but it is if he/she chooses median. The student can change the choice.

Rubric: (1 point) the student selects mean or median, and then selects the answer choice that is best given their selection. If the student selects **mean**, then the correct response choices are **better** and **higher**. If the student selects **median**, then **about the same** should be selected for both choices.

Response Type: Drop-down Menu

Note: Functionality for this item type does not currently exist. The stimulus can include the information that would be available upon request in the meantime. The item can be reconfigured into a hot-spot item by presenting the information on the left side and the choices to complete the statements on the right side, but it wouldn't be fully accessible.

Data Set that displays:

Game #	Scores Last Season	Scores This Season
1	75	73
2	68	71
3	89	94
4	61	67
5	74	72
6	83	85
7	92	94
8	74	62
9	55	54
10	66	75
11	85	90
12	71	90

Example Item 4C.1b
Primary Target 4C (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A), Tertiary Target 4D, Quaternary Target 4F


Between 1980 and 1995, there was a single surviving group of Florida panthers that ranged from 30 to 50 individuals in number. In 1995, two females from a closely related species were introduced into this population, and the number of Florida panthers increased to 87 by 2003.

Part A:
Would you model this data with a piece-wise linear function or an exponential function? Select one of these models. Either model is acceptable, but you must choose one. [piece-wise linear, exponential]

Part B:
You chose to model this data with a [auto-populates with student's choice]. This graph shows such a model.
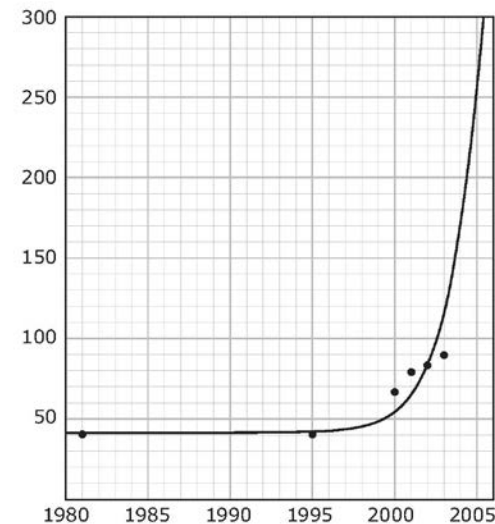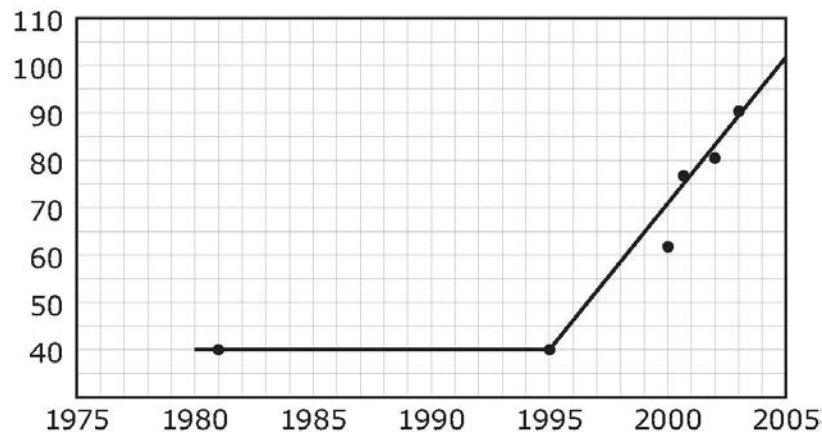
[If the student chose a piece-wise linear function, they see the first graph. If they chose an exponential function, they see second graph.]

What would the approximate population of Florida panthers have been in 2005 according to the model you chose?

Enter your answer in the response box.

Interaction: The student selects a model and then sees the corresponding graph. The student can change his/her choice.

Rubric: (1 point) Student selects piecewise linear or exponential, and then enters the appropriate estimate (between 98 and 100 if piecewise linear, and between 240 and 260 if exponential).

Response Type: Equation/Numeric

Note: Functionality for this item type does not currently exist. The stimulus can include both graphs in the meantime.

Example Item 4C.1c
Primary Target 4C (Content Domain N-NQ), Secondary Target 1C (CCSS NQ.A), Tertiary Target 4F

There is a traffic jam on a highway. A reporter is trying to estimate the number of vehicles involved in the traffic jam. Select all of the information that will help the reporter make a reasonable estimate of the number of vehicles in the traffic jam.

    A.   The cause of the traffic jam
    B.   The average length of a vehicle
    C.   The number of lanes on the highway
    D.   The average distance between vehicles
    E.   The average number of people in each vehicle
    F.   The distance from the beginning to the end of the traffic jam

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Rubric: (1 point) Student selects B, C, D, and F.

Response Type: Multiple Choice, multiple correct response

Task Model 4C.2

Task Expectations:
- Students solve problems that involve providing missing information by researching, and/or providing a reasoned estimate.

Example Item 4C.2
Primary Target 4C (Content Domain G-MG), Secondary Target 4F
[Adapted from Illustrative Mathematics 1794]

Eric is using a shovel to clear the snow from his driveway. He moves 8 shovelfuls of snow each minute.
After 60 minutes of hard work, Eric states, "I think I have shoveled more than a ton of snow."

Part A:
Estimate the weight of snow that Eric can move with each shovelful.

If you want to, you can use the table of weights of everyday objects below. A ton is 2000 pounds, and a pound is 16 ounces.

Object	Weight
Basketball	20 ounces
Apple	7 ounces
Bicycle	20 pounds
Car	1.5 tons
Pack of chewing gum	1 ounce

Eric can move [1 ounce/1 pound/10 pounds/0.25 tons] of snow with each shovelful.

Part B:
Use your estimate to decide if Eric's claim is correct or not. [Eric is correct/Eric is not correct]

Rubric: (1 point) Student selects 1 or 10 pounds, and then the corresponding answer to part B (Eric is not correct if one pound was selected, Eric is correct if 10 pounds is selected).

Response type: Drop-down Menu (This can alternately by a hot spot item; but it won't be fully accessible.)

Target 4D: Interpret results in the context of a situation.

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer as well as many problems where students construct an equation or a graph.

General Task Model Expectations for Target 4D
- The student is presented with a problem situation in everyday life, society, or the workplace or a mathematical model of such a situation. The student interprets the solution to the problem in terms of the context, in terms of the model, or compares the results of the model with the real-world data it represents.
    - Item types with a primary alignment to 4D focus on interpreting results in terms of the model or comparing the results of the model with the real-world data it represents.
    - It is not necessary for a student to generate a complete solution for problems with a primary alignment to this target.
- Tasks in Targets 4A, 4C, 4E, and 4F frequently have this target as a tertiary or quaternary alignment because students must interpret their results in terms of the context.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2 or 3.

Task Model 4D.1

- The student is presented with a mathematical model of real-world data.
- The student interprets the solution to the problem in terms of the model or compares the results of the model with the real-world data it represents.

Example Item 4D.1a
Primary Target 4D (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.2)

Lisa was throwing a dart at a target. She threw 50 times with her left hand and 50 times with her right hand. The histograms show the distance Lisa **missed** the target by each time.
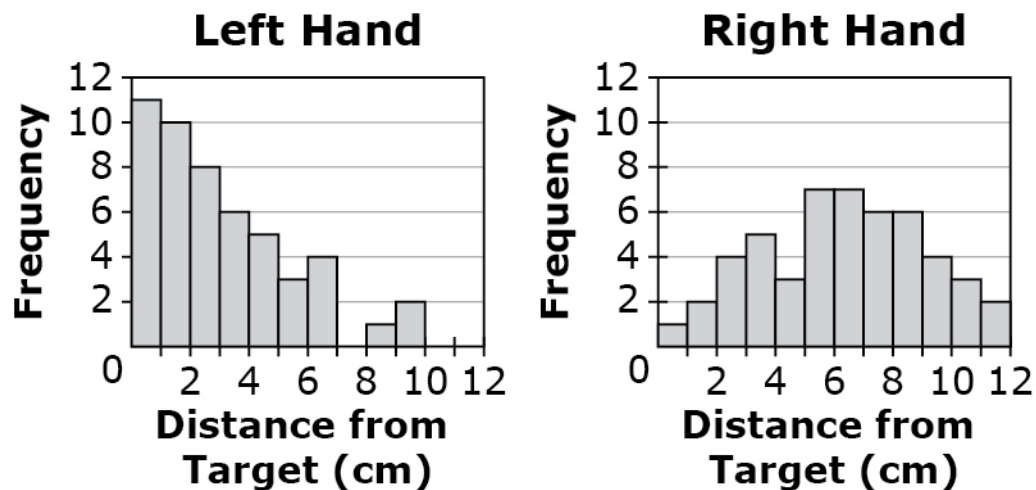
Which statement is an appropriate inference based on the median of each data set?

A.   Lisa has better aim with her left hand because the median for her left hand is greater than the median for her right hand.

B.   Lisa has better aim with her right hand because the median for her left hand is less than the median for her right hand.

C.   Lisa has better aim with her left hand because the median for her left hand is less than the median for her right hand.

D.   Lisa has better aim with her right hand because the median for her left hand is greater than the median for her right hand.

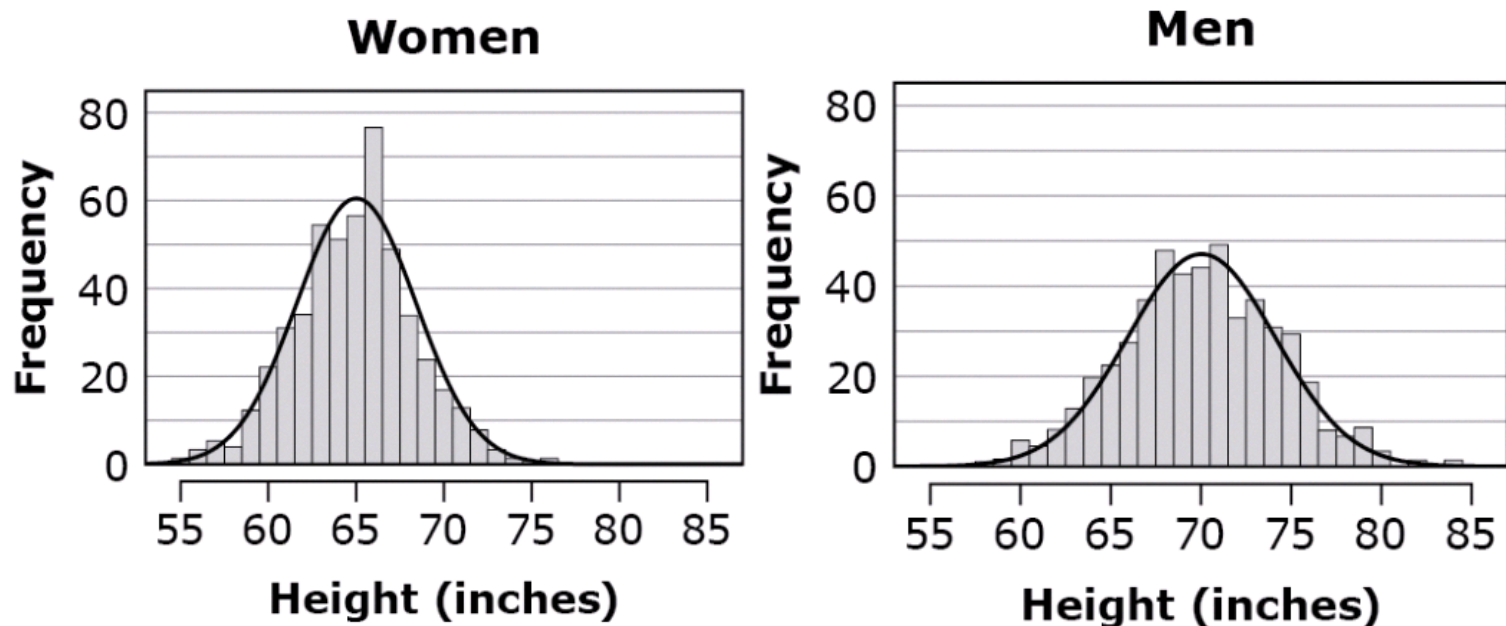**Rubric:** (1 point) The student selects the correct option (C).

Response Type: Multiple choice, single correct response

Example Item 4D.1b
Primary Target 4D (Content Domain S-ID), Secondary Target 1P (CCSS S-ID.A)

The distributions of heights of 1000 men and 1000 women selected at random from the population of a large metropolitan area are shown.

Which statement gives an accurate comparison of the heights of men and women shown?

A. The mean height for women is greater than for men and women's heights vary more than men's heights.

B. The mean height for women is greater than for men and women's heights vary less than men's heights.

C. The mean height for women is less than for men and women's heights vary more than men's heights.

D. The mean height for women is less than for men and women's heights vary less than men's heights.

Rubric: (1 point) The student selects the correct comparison statement (D).

Response Type: Multiple Choice, single correct response

Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

General Task Model Expectations for Target 4E

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem),
  - evaluates a partial or complete (possibly incorrect) solution to the problem, or
  - constructs a mathematical model to solve the problem.

  Note: It is not necessary that a student constructs a complete solution to the problem for this target.
- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task, not just a single step.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2, 3, or 4

Task Model 4E.1

Task Expectations:
- The student constructs a geometric, algebraic, or statistical model for a given context.
- Given data (table of values, scatterplot, etc.) the student will identify the type of function that might best model the situation.
- The student will assess the fit of a particular model being used, including models used in two and three-dimensional geometry.
- May use a simulation that mirrors the functioning of a formula-based online calculator.

Example Item 4E.1a

Primary Target 4E (Content Domain A-CED), Secondary Target 1G (CCSS A-CED.1), Tertiary Target 4B, Quaternary Target 4F

Maia deposited $5500 in a bank account. The money earns interest annually, and the interest is deposited back into her account.

Maia uses an online calculator to determine the amount of money she will have in the account at the end of each year. Follow these steps to use the calculator for each row in the table.

- Select number of years.
- Select "Find Amount."

The amount of money that Maia will have in her account at the end of the selected year, up to 12 years, will appear in the table.

You may use the calculator as many times as you need to help solve the problem.

Years (1 ▾)

[Find Amount]

Years	Money in Bank

Enter an equation that models the amount of money, *y*, Maia will have in the account at the end of *t* years.

Rubric: (1 point) The student is able to determine an equation to fit the situation [e.g., $y=5500(1.03)^t$].

**Response Type:** Equation/Numeric

Example Item 4E.1b:

Primary Target 4E (Content Domain G-MG), Secondary Target 1X (G-MG.1), Tertiary Target 4C

A researcher models the area of the surface of a pond using a rectangle, a semi-circle, and an isosceles triangle.

Drag each shape onto the scale diagram of the pond to show how the model fits.

Explain whether the researcher's model will estimate an area greater than, equal to, or less than the actual area of the pond's surface.

Use specific examples and mathematics to support your answer.

Pond

10 units   15 units   7 units

7 units
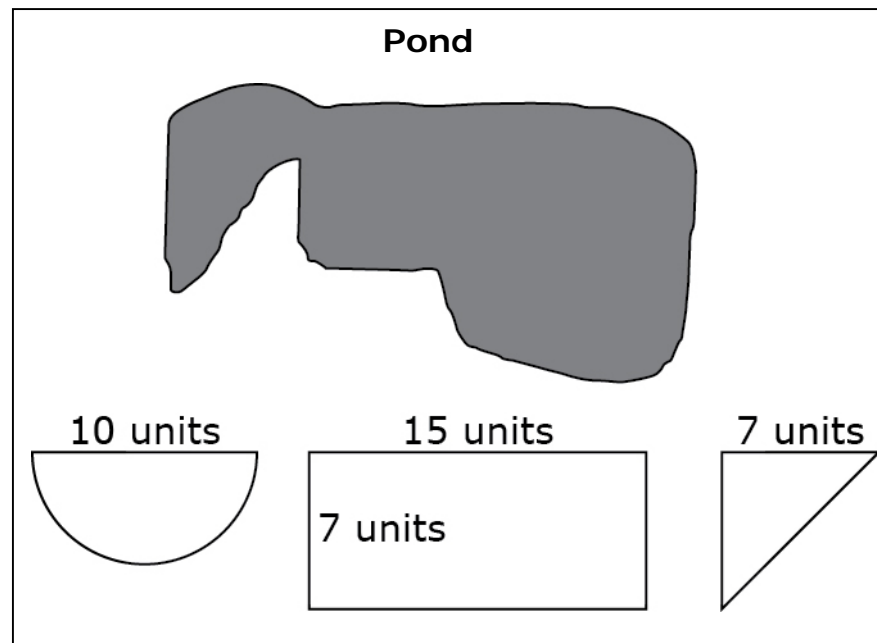
Interaction: The student drags the objects from the palette and places them on top of the pond to approximate the area.

Rubric: (2 points) The student is able to drag the shapes onto the pond in a way to model the best possible fit and make the determination that the pond is slightly larger than the combined areas of the three shapes. The student must supply an explanation that adjusts for the difference in size by either determining the areas of the shapes with specific values or stating how the pond is larger than the combined shapes by a certain portion of one of the shapes (e.g., the triangle needed to be about 2 units longer).

(1 point) The student is able to place the shapes onto the pond in a way to model the best possible fit, but is not able to draw a correct conclusion or support the conclusion.

Exemplar[6]: The area of the pond is slightly greater than the combined area of the three shapes. The semi-circle is the best fit, with a only small amount of the pond extending out the right side, but that is accounted for because of the gap between the pond and the semi-circle at the bottom left side. The rectangle is a good match to the main portion of the pond. However, the triangle is smaller than the remaining portion of the pond. Given the combined area of the three shapes is about 39.3 + 105 +24.5 or 168.8 sq units, I would estimate the pond to be about 175 sq units.
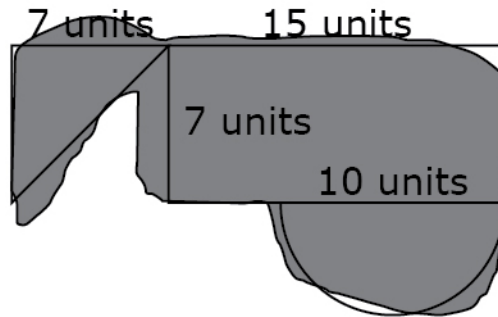
Response Type: Drag and Drop and Short Text[7] (hand scored)

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[6] An exemplar response represents only one possible solution. Typically, many other solutions/responses may receive full credit. The full range of acceptable responses is determined during rangefinding and/or scoring validation.

[7] The combination of item response types is currently not available, but it is a planned enhancement to the item authoring tool in 2017.

Task Model 4E.2

Task Expectations:
- Students choose between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.
- The student assesses the fit of a particular model being used.

Example Item 4E.2
Primary Target 4E (Content Domain S-IC), Secondary Target 1I (CCSS S-IC.B), Tertiary Target 4B, Quaternary Target 4C

Lana wrote an article for the school newspaper about the seniors at her school. There are over 1000 students in the senior class and over 4000 students at her school. Lana asked all 40 of the seniors on the swim team whether they intend to go to college, and 22 said yes. She reported in her article that 55% of this year's senior class intends to go to college.

How could Lana improve the accuracy of the information she reported in her article about the seniors at her school? Select **all** that apply.

A. She could report that this only applies to seniors who are swimmers.
B. She should check her arithmetic because she made a mistake computing the percentage.
C. She could ask the guidance counselors what percentage of the senior class went to college last year.
D. There is no need to improve the accuracy of the information she reported because she did everything correctly.
E. She could randomly select 40 students at the all-senior assembly and ask them whether they intend to go to college.

Rubric: (1 point) Student selects all correct answer choices (A and E).

Response Type: Multiple Choice, multiple select response

Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

Task Model 4F.1

Task Model Expectations
- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

Example Item 4F.1a
Primary Target 4F (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The relationship between Jack's distance from home and the time since he left home is linear, as shown in the table.

Time (hrs)	Distance (mi)
0	7.5
2	17.5
4	27.5

Based on the values in the table, determine whether each statement is true. Select True or False for each statement.

Statement	True	False
Jack's initial distance from home is 7.5 miles.		
Jack's distance increases by 5 miles every 1 hour.		
Jack's distance from home at 3 hours is 23.5 miles.		

**Rubric:** (1 point) Student determines each statement as being either true or false (TTF).

Example Item 4F.1b

Primary Target 4F (Content Domain F-BF), Secondary Target 1N (CCSS F-BF.A), Tertiary Target 4D

The scatterplot shown represents the winning times for the women's 100 meter freestyle race for the Olympic Games between 1912 and 2012.

Winning Times, Women's 100m Freestyle Race
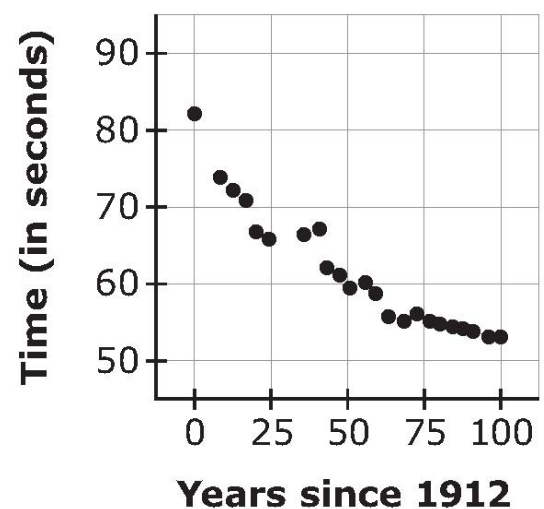
Part A:
Let x be the year since 1912 and $f(x)$ be the winning time in seconds. Enter either a linear or an exponential function that models the data in the response box.

Part B:
Greta Andersen from Denmark won the race in 1948 with a time of 66.3 seconds. What is the difference between your model's prediction and Greta's actual winning time? Enter your answer in the response box.

Part C:
What does your model predict the winning time will be in the 2016 Olympics? Enter your answer in the response box.

Interaction: The student has access to a Desmos-like graphing calculator with the data already pre-loaded, or some other appropriate graphing or statistical app.

Rubric: (2 points) The student enters an expression for $f(x)$ in the first response box such that $|f(0) - 82| \leq 10$ and $|f(100) - 53| \leq 10$. The student then enters $\pm(f(36)-66.3)$ within a reasonable tolerance in the second response box and enters $f(104)$ in the third response box.

(1 point) The student enters an expression for $f(x)$ in the first response box such that $|f(0) - 82| \leq 10$ and $|f(100) - 53| \leq 10$, or, the student enters an expression that does not meet these criteria but successfully enters either $\pm(f(36)-66.3)$ within a reasonable tolerance in the second response box or enters $f(104)$ in the third response box (or both).

Response Type: Equation/Numeric - label each response type as follows: Part A: $f(x) =$, Part B:, and Part C:.

Data for the scatterplot for example item **4A.1c**:

Year since 1912	0	8	12	16	20	24	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
Time	82.2	73.6	72.4	71	66.8	65.9	66.3	66.8	62	61.2	59.5	60	58.59	55.65	54.79	55.92	54.93	54.65	54.5	53.83	53.84	53.12	53